EL-MOASSER

By a group of supervisors

THE MAIN BOOK

3rd PREP.

FIRST TERM

Maths

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Contents

First Algebra and Statistics

UNIT

1 Relations and functions

Lessons of the unit :

1. Cartesian product.
2. Relation - Function (mapping).
3. The symbolic representation of the function - Polynomial functions.
4. The study of some polynomial functions.

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Unit Objectives :

By the end of this unit, student should be able to :

- recognize the concept of the Cartesian product of two finite sets.
- represent the Cartesian product of two finite sets by the arrow diagram and the graphical (Cartesian) diagram.
- recognize the concept of the Cartesian product of two infinite sets.
- find the Cartesian product of two intervals.
- recognize the concept of the relation from a set to another one.
- recognize whether the relation is a function or not.
- represent the function by the arrow diagram and the graphical (Cartesian) diagram.
- recognize the domain , the codomain and the range of the function.
- express the function symbolically.
- search the degree of the polynomial function.
- represent the linear function graphically.
- recognize the constant function and represent it graphically.
- represent graphically the quadratic function.
- find the vertex of the curve of the quadratic function.
- find the maximum or the minimum value of the quadratic function.
- find the equation of the axis of symmetry of the quadratic function.

LESSON 1

Cartesian product

In this lesson , we shall know the concept of the Cartesian product and how to find it and how to represent it graphically.

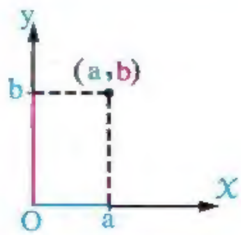
Before dealing with this subject , we shall remember together what we had studied about the ordered pair.

The ordered pair

(a , b) is called an ordered pair

- a is called the first projection
- b is called the second projection

and the ordered pair (a , b) could be represented by a point as shown in the opposite figure.

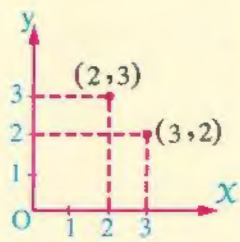


Remarks

- If $a \neq b$, then $(a , b) \neq (b , a)$

  For example: $(2 , 3) \neq (3 , 2)$

  and when representing them graphically as shown in the opposite figure , we find that they are represented by two different points.



- The ordered pair is not a set. **i.e.** $(a , b) \neq \{a , b\}$

- (a , a) is an ordered pair , while in the sets , we don't write $\{a , a\}$, but we write $\{a\}$ without repeating the element a
- There is an empty set of elements and denoted by the symbol $\varnothing$, but there is not an empty ordered pair.

The equality of two ordered pairs

If $(a , b) = (x , y)$, then $a = x$, $b = y$

For example:

- If $(a , b) = (3 , -4)$, then $a = 3$, $b = -4$
- If $(x , 2) = (-5 , y)$, then $x = -5$, $y = 2$

Example 1 Choose the correct answer from the given ones :

1 If $(3 , 8) = \left(3 , \sqrt{y}\right)$, then $\sqrt[3]{y} =$ ....

(a) -4 (b) 4 (c) 8 (d) 64

2 If $(32 , x + y) = (y^5 , 2)$, then $x =$ ....

(a) 0 (b) 2 (c) 4 (d) 5

3 If $(2^{x-1} , -3) = (1 , y)$, then $2x - y =$ ....

(a) -3 (b) -1 (c) 3 (d) 5

4 If $(x^2 - 1 , 4) = (48 , 2y)$, then $xy =$ ....

(a) -7 (b) 7 (c) 14 (d) ± 14

Solution

1 **(b) The reason :** $\because (3 , 8) = \left(3 , \sqrt{y}\right)$ $\therefore \sqrt{y} = 8$

$\therefore y = 8^2 = 64$ $\therefore \sqrt[3]{y} = \sqrt[3]{64} = 4$

2 **(a) The reason :** $\because (32 , x + y) = (y^5 , 2)$

$\therefore y^5 = 32$ $\therefore y = 2$ «because $2^5 = 32$»

, $x + y = 2$ substituting by $y = 2$ $\therefore x + 2 = 2$

$\therefore x = 0$

3 **(d) The reason :** $\because (2^{x-1} , -3) = (1 , y)$ $\therefore y = -3$

, $2^{x-1} = 1$, then $x - 1 = 0$ $\therefore x = 1$

$\therefore 2x - y = 2 \times 1 - (-3) = 2 + 3 = 5$

4 **(d) The reason :** $\because (x^2 - 1 , 4) = (48 , 2y)$ $\therefore x^2 - 1 = 48$

$\therefore x^2 = 49$

$\therefore x = \pm\sqrt{49} = \pm 7$, $2y = 4$ $\therefore y = \frac{4}{2} = 2$

$\therefore xy = \pm 7 \times 2 = \pm 14$

TRY *by yourself* **1** ***Find the values of x and y in each of the following :***

1. $(x+1, y^2) = (3, 9)$
2. $(x^3-5, 8) = (3, 3y-7)$
3. $(x^2-2, 2y) = \left(y, \sqrt[3]{64}\right)$

Final answers of try by yourself questions are at the end of each lesson to check your answer.

The Cartesian product of two finite sets

For any two finite and non empty sets X and Y , we get :

The Cartesian product of the set X by the set Y and it is denoted by $\boxed{X \times Y}$ is the set of all ordered pairs whose first projection of each of them belongs to X and the second projection of each of them belongs to Y

i.e. $X \times Y = \{(a, b) : a \in X, b \in Y\}$

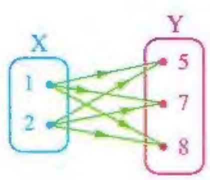
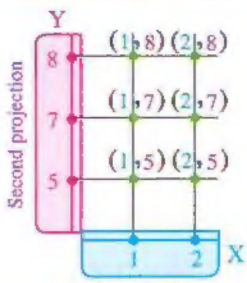
For example :

1 If $X = \{1, 2\}$, $Y = \{5, 7, 8\}$, then :

$$X \times Y = \{1, 2\} \times \{5, 7, 8\}$$

$$= \{(1, 5), (1, 7), (1, 8), (2, 5), (2, 7), (2, 8)\}$$

• We can represent $X \times Y$ by two ways as follows :

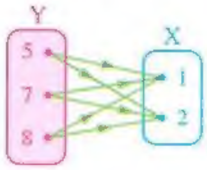
1^{st} way : The arrow diagram	2^{nd} way : The graphical (Cartesian) diagram
	
Where we draw an arrow going from each element representing the first projection (the elements of the set X) to each element representing the second projection (the elements of the set Y)	Where the elements of the set X are represented horizontally and the elements of the set Y are represented vertically and the points of intersection of the horizontal and vertical lines represent the Cartesian product of $X \times Y$

2 If $X = \{1, 2\}$, $Y = \{5, 7, 8\}$, then :

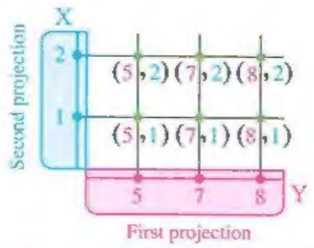
$$Y \times X = \{5, 7, 8\} \times \{1, 2\}$$

$$= \{(5, 1), (5, 2), (7, 1), (7, 2), (8, 1), (8, 2)\}$$

- Similarly , we can represent $Y \times X$ by two ways as follows :



The arrow diagram



The Cartesian diagram

The Cartesian product of a set by itself

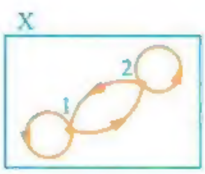
The Cartesian product of the set X by itself and we denote it by $\boxed{X \times X}$ or by $\boxed{X^2}$ (it is read X two) is the set of all ordered pairs whose first projections and second projections belong both to X

i.e. $X \times X = \{(a, b) : a \in X , b \in X\}$

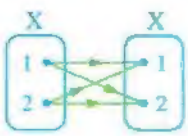
For example: If $X = \{1, 2\}$, then :

$$X \times X = \{1, 2\} \times \{1, 2\} = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$$

- We can represent $X \times X$ by two ways as follows :

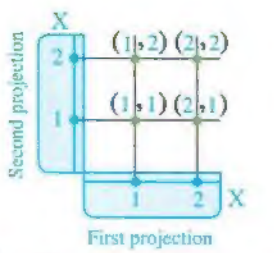


or



The arrow diagram

Notice that : The figure $\bigcirc_1$ is called a loop to show that the arrow goes from the point and returns to the same point.



The Cartesian diagram

Remarks

- For any two finite and non empty sets X and Y , then $X \times Y \neq Y \times X$ where $X \neq Y$
- For any set X , then $X \times \varnothing = \varnothing \times X = \varnothing$ where $\varnothing$ is the empty set.
- If $(a , b) \in X \times Y$, then $a \in X$, $b \in Y$

For example: If $(3 , 5) \in X \times Y$, then $3 \in X$, $5 \in Y$

Example 2

If $X = \{2 , 3 , 4\}$ and $Y = \{a , b\}$, **find each of :**

1 $X \times Y$ 2 $Y \times X$ 3 $X \times X$ 4 Y^2

Solution

1 $X \times Y = \{(2 , a) , (2 , b) , (3 , a) , (3 , b) , (4 , a) , (4 , b)\}$

2 $Y \times X = \{(a , 2) , (a , 3) , (a , 4) , (b , 2) , (b , 3) , (b , 4)\}$

3 $X \times X = \{(2 , 2) , (2 , 3) , (2 , 4) , (3 , 2) , (3 , 3) , (3 , 4), (4 , 2) , (4 , 3), (4 , 4)\}$

4 $Y^2 = \{(a , a) , (a , b) , (b , a) , (b , b)\}$

TRY by yourself 2

If $X = \{3 , 4 , 5\}$ and $Y = \{5 , 6\}$, find each of the following :

1 $Y \times X$ and represent it by an arrow diagram

2 X^2 and represent it by a Cartesian diagram

The number of the elements of the Cartesian product

If we denote the number of elements of the set X by n (X) and the number of elements of the set Y by n (Y) , then the number of elements of the Cartesian product $X \times Y$ is denoted by $n (X \times Y)$, and :

- $n (X \times Y) = n (Y \times X) = n (X) \times n (Y)$
- $n (X \times X) = n (X) \times n (X) = [n (X)]^2$
- $n (X \times \varnothing) = n (X) \times n (\varnothing)$
  $= 0$ [Because $n (\varnothing) = 0$]

Notice that :

If X , Y are two finite and non empty sets , $X \neq Y$, then $X \times Y \neq Y \times X$, but $n (X \times Y) = n (Y \times X)$

For example :

If $X = \{2 , -1 , 0\}$ and $Y = \{5 , -7\}$, then $n (X) = 3$, $n (Y) = 2$, then :

- $n (X \times Y) = 3 \times 2 = 6$
- $n (Y \times X) = 2 \times 3 = 6$
- $n (X^2) = 3^2 = 9$
- $n (Y^2) = 2^2 = 4$

Find the previous Cartesian products and verify the number of their elements.

Example 3 Choose the correct answer from the given ones :

1 If $X = \{0 , 2\}$, $n(Y) = 5$, then $n(X \times Y) = \dots\dots$

(a) 2 (b) 5 (c) 7 (d) 10

2 If $n(Y) = 4$, $n(X \times Y) = 8$, then $n(X) = \dots\dots$

(a) 2 (b) 4 (c) 8 (d) 32

3 If $n(X^2) = 9$, $n(Y^2) = 16$, then $n(Y \times X) = \dots\dots$

(a) 7 (b) 12 (c) 36 (d) 144

Solution

1 **(d) The reason :** $\because n(X) = 2$, $n(Y) = 5$

$\therefore n(X \times Y) = 2 \times 5 = 10$

2 **(a) The reason :** $n(X) = \frac{n(X \times Y)}{n(Y)} = \frac{8}{4} = 2$

3 **(b) The reason :** $\because n(X^2) = 9$ $\therefore n(X) = \sqrt{9} = 3$

$, \because n(Y^2) = 16$ $\therefore n(Y) = \sqrt{16} = 4$

$\therefore n(Y \times X) = 4 \times 3 = 12$

TRY by yourself 3

Choose the correct answer from the given ones :

1 If $n(X) = 3$, $n(X \times Y) = 12$, then $n(Y) = \dots\dots$

(a) 4 (b) 9 (c) 15 (d) 36

2 If $Y = \{-1 , 0 , 1\}$, $n(X \times Y) = 15$, then $n(Y^2) = \dots\dots$

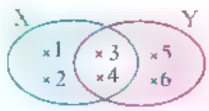
(a) 5 (b) 9 (c) 15 (d) 25

3 If $n(X^2) = 4$, $n(X \times Y) = 4$, then $n(Y^2) = \dots\dots$

(a) 1 (b) 2 (c) 4 (d) 16

Remember the operations on sets

If $X = \{1 , 2 , 3 , 4\}$, $Y = \{3 , 4 , 5 , 6\}$, then :

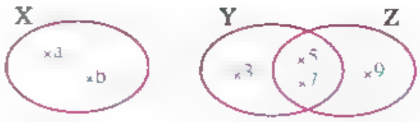


- $X \cap Y$ = the set of elements which are common in X and Y $= \{3 , 4\}$
- $X \cup Y$ = the set of all elements in X or Y without repeating $= \{1 , 2 , 3 , 4 , 5 , 6\}$
- $X - Y$ = the set of elements which are in X and not in Y $= \{1 , 2\}$
- $Y - X$ = the set of elements which are in Y and not in X $= \{5 , 6\}$

Example 4 If $X = \{a , b\}$, $Y = \{3 , 5 , 7\}$, $Z = \{5 , 7 , 9\}$

, **represent the sets X , Y and Z by Venn diagram , then find :**

1. $X \times (Y \cup Z)$, $(X \times Y) \cup (X \times Z)$
2. $X \times (Y \cap Z)$, $(X \times Y) \cap (X \times Z)$
3. $X \times (Z - Y)$, $(X \times Z) - (X \times Y)$



Solution

1 $\because Y \cup Z = \{3 , 5 , 7 , 9\}$

$\therefore X \times (Y \cup Z) = \{a , b\} \times \{3 , 5 , 7 , 9\}$

$= \{(a , 3) , (a , 5) , (a , 7) , (a , 9) , (b , 3) , (b , 5) , (b , 7) , (b , 9)\}$

$, X \times Y = \{a , b\} \times \{3 , 5 , 7\}$

$= \{(a , 3) , (a , 5) , (a , 7) , (b , 3) , (b , 5) , (b , 7)\}$ (1)

$, X \times Z = \{a , b\} \times \{5 , 7 , 9\}$

$= \{(a , 5) , (a , 7) , (a , 9) , (b , 5) , (b , 7) , (b , 9)\}$ (2)

From (1) and (2) :

$\therefore (X \times Y) \cup (X \times Z) =$

$\{(a , 3) , (a , 5) , (a , 7) , (a , 9) , (b , 3) , (b , 5) , (b , 7) , (b , 9)\}$

2 $\because Y \cap Z = \{5 , 7\}$

$\therefore X \times (Y \cap Z) = \{a , b\} \times \{5 , 7\}$

$= \{(a , 5) , (a , 7) , (b , 5) , (b , 7)\}$

From (1) and (2) :

$\therefore (X \times Y) \cap (X \times Z) = \{(a , 5) , (a , 7) , (b , 5) , (b , 7)\}$

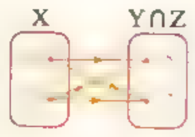
3 $\because Z - Y = \{9\}$

$\therefore X \times (Z - Y) = \{a , b\} \times \{9\} = \{(a , 9) , (b , 9)\}$

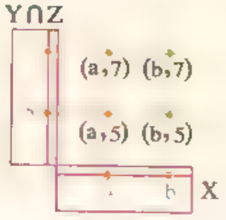
From (1) and (2) : $\therefore (X \times Z) - (X \times Y) = \{(a , 9) , (b , 9)\}$

! Remark

In the previous example , we can represent $X \times (Y \cap Z)$ by an arrow diagram and a Cartesian diagram as follows :



The arrow diagram



The Cartesian diagram

TRY by yourself 4

If $X = \{2 , 3\}$, $Y = \{1 , 3 , 5\}$, $Z = \{2\}$

, ***represent each of X , Y and Z by Venn diagram , then find :***

1 $Z \times (X \cap Y)$ 2 $(Z \times X) \cup (Z \times Y)$

The Cartesian product of two infinite sets

- We know that if X is a finite set (having n elements) , then the Cartesian product $X \times X$ is also a finite set (having n^2 elements).

  For example: If $n(X) = 3$, then $n(X \times X) = 9$

- But if X is an infinite set , then $X \times X$ is an infinite set also

  As examples for that :

  $\mathbb{N} \times \mathbb{N} = \{(x , y) : x \in \mathbb{N} , y \in \mathbb{N}\}$, $\mathbb{Z} \times \mathbb{Z} = \{(x , y) : x \in \mathbb{Z} , y \in \mathbb{Z}\}$,

  $\mathbb{Q} \times \mathbb{Q} = \{(x , y) : x \in \mathbb{Q} , y \in \mathbb{Q}\}$, $\mathbb{R} \times \mathbb{R} = \{(x , y) : x \in \mathbb{R} , y \in \mathbb{R}\}$

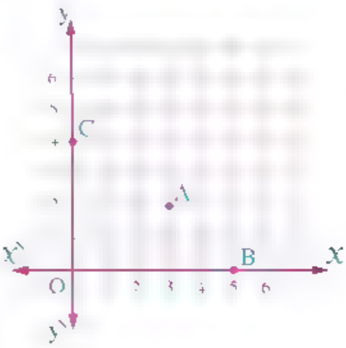
- We know that if X is a finite set , we represent the Cartesian product $X \times X$ graphically by a finite number of points.

- But if X is an infinite set , then the Cartesian product $X \times X$ is represented graphically by an infinite number of points.

The following is the graphical representation of each of : $\mathbb{N} \times \mathbb{N}$, $\mathbb{Z} \times \mathbb{Z}$, $\mathbb{R} \times \mathbb{R}$:

First Representing the Cartesian product $\mathbb{N} \times \mathbb{N}$ $(\mathbb{N}^2)$

- Represent the natural numbers on two perpendicular straight lines, one of them $\overleftrightarrow{XX'}$ is horizontal and the other $\overleftrightarrow{yy'}$ is vertical, where they intersect at the point which represents the number zero on each of them i.e. $O = (0 , 0)$



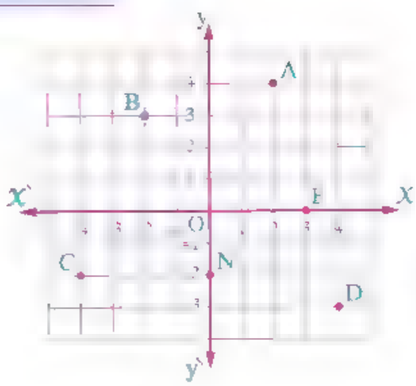
- The opposite figure shows a small part of the perpendicular graphical net of the Cartesian product $\mathbb{N} \times \mathbb{N}$ which consists of the vertical and the horizontal straight lines that pass through the points which represent the natural numbers on each of $\overleftrightarrow{XX'}$ and $\overleftrightarrow{yy'}$
- And each point of the points of this net represents an ordered pair of the Cartesian product $\mathbb{N} \times \mathbb{N}$

For example :

- The point A represents the ordered pair $(3 , 2)$
- The point B represents the ordered pair $(5 , 0)$
- The point C represents the ordered pair $(0 , 4)$
- The point O represents the ordered pair $(0 , 0)$

Second Representing the Cartesian product $\mathbb{Z} \times \mathbb{Z}$ $(\mathbb{Z}^2)$

- Represent the integers on each of $\overleftrightarrow{XX'}$ and $\overleftrightarrow{yy'}$ which are intersecting at $O (0 , 0)$



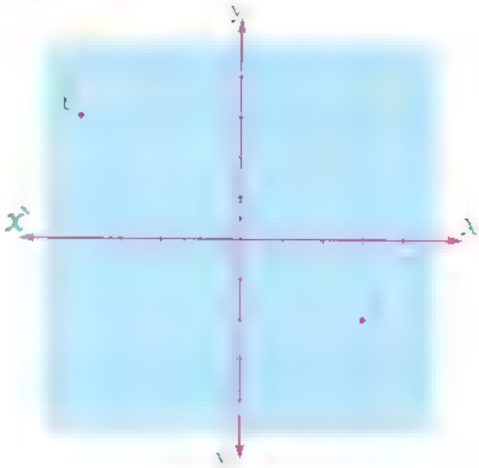
- The opposite figure shows a small part of the perpendicular graphical net of the Cartesian product $\mathbb{Z} \times \mathbb{Z}$
- And each point of its points represents an ordered pair of the Cartesian product $\mathbb{Z} \times \mathbb{Z}$

For example:

- The point A represents the ordered pair $(2 , 4)$
- The point B represents the ordered pair $(-2 , 3)$
- The point C represents the ordered pair $(-4 , -2)$
- The point D represents the ordered pair $(4 , -3)$
- The point E represents the ordered pair $(3 , 0)$
- The point N represents the ordered pair $(0 , -2)$

Third Representing the Cartesian product $\mathbb{R} \times \mathbb{R}$ $(\mathbb{R}^2)$

- The perpendicular graphical net of the Cartesian product $\mathbb{R} \times \mathbb{R}$ is an infinite extended surface from all sides and the opposite figure shows a small part of this region.
- Each point of this region represents an ordered pair of the Cartesian product $\mathbb{R} \times \mathbb{R}$

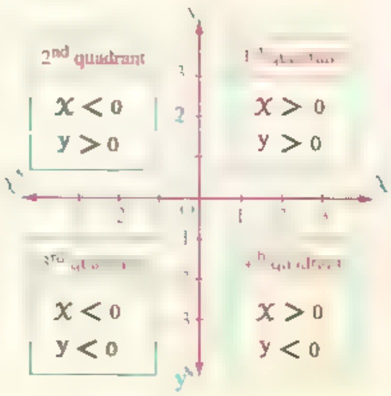


For example:

- The point A represents the ordered pair $(3 , -2)$
- The point B represents the ordered pair $(-4 , 3)$

Remarks

1. The horizontal straight line $\overleftrightarrow{XX'}$ is called X-axis or the horizontal axis and the vertical straight line $\overleftrightarrow{yy'}$ is called y-axis or the vertical axis.
2. The point of intersection of the two axes $\overleftrightarrow{XX'}$ and $\overleftrightarrow{yy'}$ is called the origin point.
3. If the point A represents the ordered pair (X , y) in the Cartesian product $\mathbb{R} \times \mathbb{R}$, then :
   - The first projection X is called the X-coordinate of the point A
   - The second projection y is called the y-coordinate of the point A
4. The two axes $\overleftrightarrow{XX'}$ and $\overleftrightarrow{yy'}$ divide the plane into four quadrants as shown in the opposite figure and we can determine the quadrant in which any point lies by knowing the signs of its two coordinates.



5. If the X-coordinate of the point $= 0$, then the point lies on y-axis.
6. If the y-coordinate of the point $= 0$, then the point lies on X-axis.

Example 5 Choose the correct answer from the given ones :

1 The point $(4 , -3)$ lies on the .... quadrant.

(a) first (b) second (c) third (d) fourth

2 Which of the following points lies on the third quadrant ?

(a) $(2 , 5)$ (b) $(2 , -5)$ (c) $(-2 , 5)$ (d) $(-2 , -5)$

3 If the point (a , 3 – a) lies on the X-axis , then a = ·········

(a) – 3 (b) 0 (c) 3 (d) 5

4 If b < 2 , then the point (b – 2 , 4) lies on the ·········· quadrant.

(a) first (b) second (c) third (d) fourth

5 If the point (X – 3 , 4 – X) where $X \in \mathbb{Z}$ lies on the fourth quadrant , then X = ···········

(a) 2 (b) 3 (c) 4 (d) 5

Solution

1 **(d) The reason :** Because the X-coordinate is positive and the y-coordinate is negative.

2 **(d) The reason :** Because the X-coordinate and the y-coordinate of all the points on the third quadrant are negative.

3 **(c) The reason :** $\because$ (a , 3 – a) $\in \overleftrightarrow{XX}$

$\therefore$ 3 – a = 0 $\therefore$ a = 3

4 **(b) The reason :** $\because$ b < 2

$\therefore$ The X-coordinate of the point (b – 2 , 4) is negative and its y-coordinate is positive.

$\therefore$ (b – 2 , 4) lies on the second quadrant.

5 **(d) The reason :** Because at X = 5 , then (X – 3 , 4 – X) = (2 , – 1)

i.e. The X-coordinate is positive and the y-coordinate is negative.

TRY by yourself 5

Choose the correct answer from the given ones :

1 The point (– 2 , – 7) lies on the ·········· quadrant.

(a) first (b) second (c) third (d) fourth

2 If the point (b – 5 , b) lies on the y-axis , then b = ···········

(a) – 5 (b) 0 (c) 1 (d) 5

3 If $\left(X - 2 , \sqrt{9}\right)$ = (– 3 , y) , then the point (y , X) lies on the ·········· quadrant.

(a) first (b) second (c) third (d) fourth

4 The point (X^2 , y^2) where $X \neq 0$, $y \neq 0$ lies on the ·········· quadrant.

(a) first (b) second (c) third (d) fourth

The Cartesian product of two intervals

We studied that the interval is a subset of the set of the real numbers (ℝ) and then the Cartesian product of two intervals is a subset of the Cartesian product ℝ × ℝ and we can explain that in the following example.

Example 6 If $X = [0 , 3]$, $Y = [1 , 3]$

, **represent graphically using the perpendicular graphical net of the Cartesian product ℝ × ℝ the region which represents each of :**

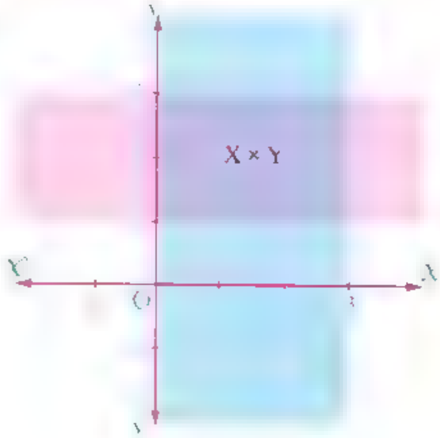
1 $X \times Y$ **2** $X \times X$ **3** $Y \times Y$

, then show , in each case , which of the following points belongs to the previous Cartesian products : $(2 , 2)$, $(1 , 0)$, $(0 , 3)$

Solution

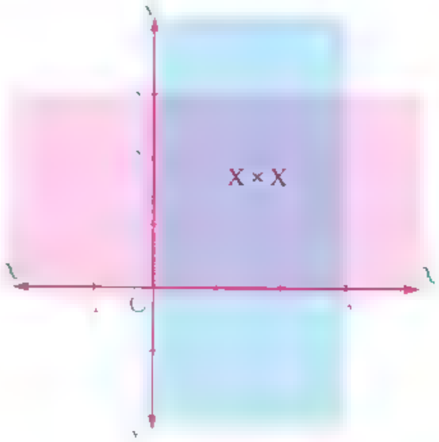
1 To represent X × Y graphically , do as follows :

- Represent the interval X on x-axis
- Represent the interval Y on y-axis
- The intersection region of the two colours represents $X \times Y$
- $(2 , 2) \in X \times Y$ because it belongs to the region which represents $X \times Y$
- $(1 , 0) \notin X \times Y$ because it lies outside the region which represents $X \times Y$
- $(0 , 3) \in X \times Y$



2 To represent X × X graphically , do as follows :

- Represent the interval X one time on x-axis and another time on y-axis.
- The intersection region of the two colours represents $X \times X$
- $(2 , 2) \in X \times X$, $(1 , 0) \in X \times X$ and $(0 , 3) \in X \times X$

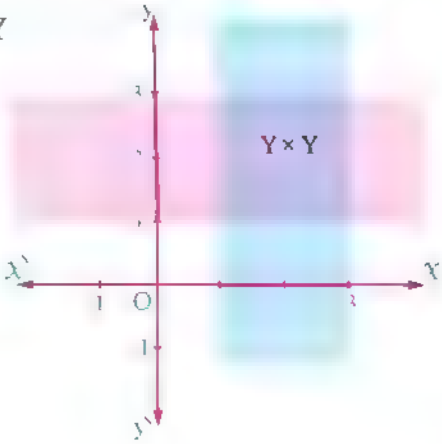


3 **Similarly , you can represent Y × Y as shown in the opposite figure :**

- $(2 , 2) \in Y \times Y$

  , $(1 , 0) \notin Y \times Y$

  and $(0 , 3) \notin Y \times Y$



TRY by yourself **6**

If $X = [-2 , 1]$, $Y = [0 , 2]$

, find the region which expresses each of the following using the perpendicular square net of the Cartesian product $\mathbb{R} \times \mathbb{R}$:

1. $X \times X$

2. $X \times Y$ and show which of the following points belongs to $X \times Y$

   $A(1 , 2)$, $B(0 , -2)$, $C(3 , -1)$, $D(-2 , -2)$

Answers *of try by yourself*

1 1 $x = 2$, $y = \pm 3$ 2 $x = 2$, $y = 5$ 3 $x = \pm 2$, $y = 2$

2 1 $\{(5 , 3) , (5 , 4) , (5 , 5) , (6 , 3) , (6 , 4) , (6 , 5)\}$, represent by yourself.

2 $\{(3 , 3) , (3 , 4) , (3 , 5) , (4 , 3) , (4 , 4) , (4 , 5) , (5 , 3) , (5 , 4) , (5 , 5)\}$, represent by yourself.

3 1 (a)   2 (d)   3 (c)

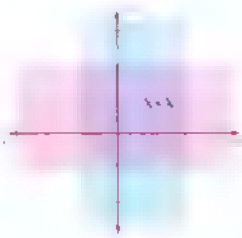
4 Represent by yourself.

1 $\{(2 , 3)\}$ 2 $\{(2 , 2) , (2 , 3) , (2 , 1) , (2 , 5)\}$

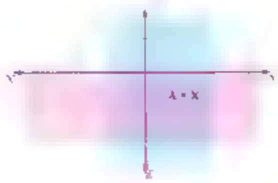
5 1 (c)   2 (d)

3 (d)   4 (a)

6 1



2



$A(1 , 2) \in X \times Y$

At the end of each lesson , you will find the final answers of try by yourself questions in the same form.

2 Relation - Function (mapping)

First The relation

The relation from set X to set Y is a connection that connects some or all the elements of set X with some or all the elements of set Y and it is denoted by "R"

- The relation R from X to Y is a set of ordered pairs whose first projection belongs to X and its second projection belongs to Y and the first projection is connected with the second projection by this relation.

  If $(a , b) \in R$ where $a \in X , b \in Y$

  So , we express this as "a R b"

- The relation R from set X to set Y is a subset of the Cartesian product $X \times Y$

  i.e. $R \subset X \times Y$

- The relation can be expressed by an arrow diagram or a Cartesian diagram (graphical).

Example 1 If $X = \{2, 5\}$, $Y = \{1, 4, 7\}$ and R is a relation from X to Y where "a R b" means "a < b" for every $a \in X$, $b \in Y$, state the relation R and represent it by an arrow diagram and by a Cartesian diagram.

Solution

$\because$ 2 is not less than 1 $\quad\therefore (2, 1) \notin R$

$\because 2 < 4 \quad\therefore (2, 4) \in R$

$\because 2 < 7 \quad\therefore (2, 7) \in R$

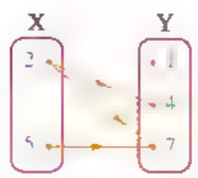
$\because$ 5 is not less than 1 $\quad\therefore (5, 1) \notin R$

$\because$ 5 is not less than 4 $\quad\therefore (5, 4) \notin R$

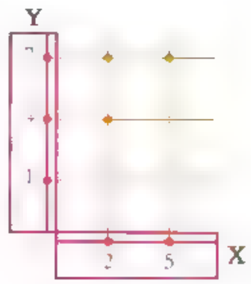
$\because 5 < 7 \quad\therefore (5, 7) \in R$

$\therefore$ The relation $R = \{(2, 4), (2, 7), (5, 7)\}$

The following figures represent the arrow diagram and the Cartesian diagram of this relation :



The arrow diagram



The Cartesian diagram

TRY by yourself 1

If $X = \{1, 2, 3\}$, $Y = \{3, 4, 5, 6\}$ and R is a relation from X to Y where "a R b" means "a + b = 6" for every $a \in X$ and $b \in Y$

, ***state the relation R and represent it by an arrow diagram.***

! Remark

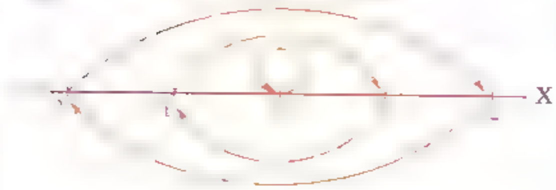
If R is a relation from X to X , then : R is a relation on X and the relation $R \subset X \times X$

Example 2 If $X = \{-2, -1, 0, 1, 2\}$ and R is a relation on X where "a R b" means "a is the additive inverse of the number b" for every $a \in X$ and $b \in X$, state R, then represent it by an arrow diagram and a Cartesian diagram.

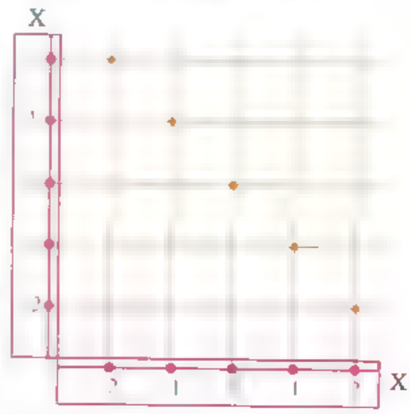
Solution $R = \{(-2, 2), (-1, 1), (0, 0), (1, -1), (2, -2)\}$

- **The arrow diagram :**

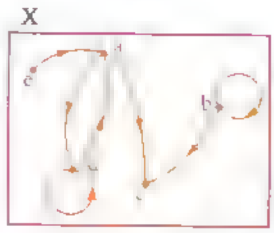
or



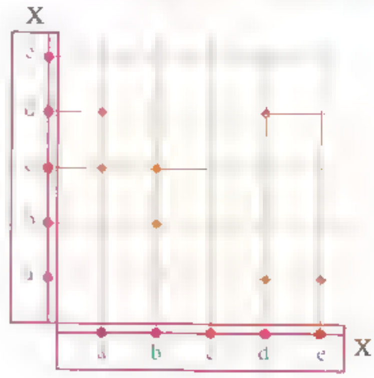
- **The Cartesian diagram :**



Example 3 If the opposite arrow diagram represents the relation R on X, state R, then represent it by a Cartesian diagram.



Solution $R = \{(a, c), (a, d), (b, b), (b, c), (d, d), (d, a), (e, a)\}$



TRY by yourself 2

If $X = \{1, 2, 4\}$ and R is a relation on X where "a R b" means "a is twice b" for every $a \in X$ and $b \in X$, ***state R and represent it by a Cartesian diagram.***

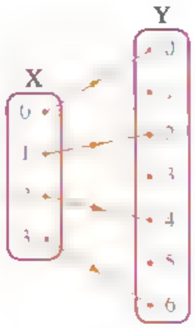
Second Function (Mapping)

Introductory example

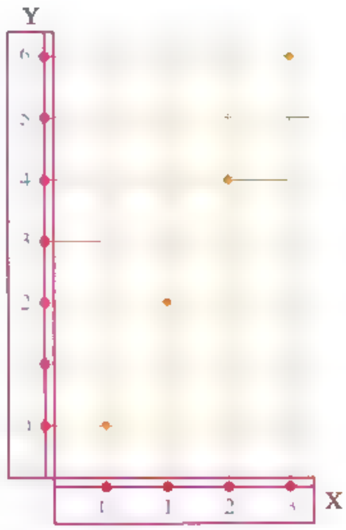
If $X = \{0, 1, 2, 3\}$, $Y = \{0, 1, 2, 3, 4, 5, 6\}$ and R is a relation from X to Y where "a R b" means "$a = \frac{1}{2}$ b " for each $a \in X$, $b \in Y$, write R and represent it by an arrow diagram and a Cartesian diagram.

Solution

$R = \{(0, 0), (1, 2), (2, 4), (3, 6)\}$



The arrow diagram

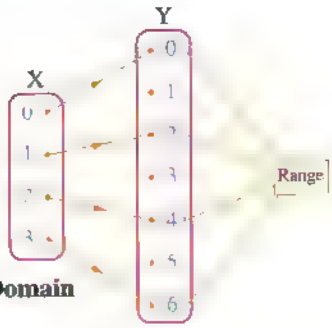


The Cartesian diagram

In the previous relation, we notice that :

Each element of the set X has been connected with **one and only one element** of the elements of the set Y

Such as , this relation is called **a function or (mapping).**



- The set $X = \{0, 1, 2, 3\}$ is called "the domain of the function".
- The set $Y = \{0, 1, 2, 3, 4, 5, 6\}$ is called "the codomain of the function".
- The set $\{0, 2, 4, 6\}$ is called "the range of the function" and it is a subset from the codomain of the function.

Generally

A relation from X to Y is said to be a function if one of the following cases is satisfied :

1 In the relation , **each element** of the set X appears **only once** as a first projection in one of the ordered pairs of **the relation**.

2 In the arrow diagram which represents the relation , **each element** of X has **one and only one arrow** going out of it to one element of Y

3 In the Cartesian diagram which represents the relation , **each vertical line** has **one and only one point** lying on it of the points which represent the relation.

Example 4

If $X = \{1 , 2 , 3 , 4\}$, $Y = \{1 , 3 , 5 , 7\}$

, show which of the following relations represents a function from X to Y and if it is a function , mention its range :

- $R_1 = \{(2 , 3) , (1 , 1) , (3 , 5) , (3 , 7) , (4 , 3)\}$
- $R_2 = \{(1 , 7) , (2 , 5) , (4 , 1)\}$
- $R_3 = \{(2 , 3) , (3 , 3) , (1 , 5) , (4 , 7)\}$

Solution

- R_1 is not a function because the element $3 \in X$ appears as a first projection twice in two ordered pairs of the relation (3 , 5) and (3 , 7)
- R_2 is not a function because the element $3 \in X$ does not appear as a first projection in any ordered pair of the relation.
- R_3 is a function because each element of X appeared only once as a first projection in an ordered pair of the relation , the range of R_3 is $\{3 , 5 , 7\}$

Example 5

If $X = \{3 , 5 , 7 , 9\}$

, show which of the following arrow diagrams represents a function on X (i.e. from X to X) and if it is a function , mention its range :

F_1

F_2

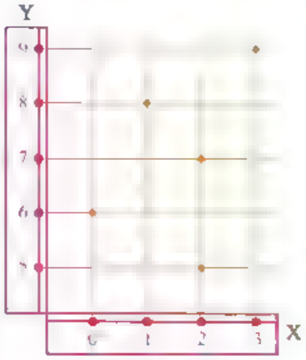
F_3

Solution

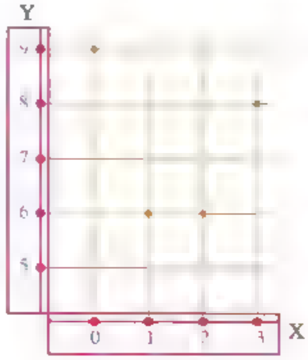
- F_1 is a function because each element of X has only one arrow going out of it to one element of X , the range of the function F_1 is $\{3, 7, 9\}$
- F_2 is not a function because for the element $5 \in X$ there are no arrows going out of it or because the element $3 \in X$ has two arrows going out of it.
- F_3 is not a function because the element $7 \in X$ has two arrows going out of it.

Example 6 If $X = \{0, 1, 2, 3\}$, $Y = \{5, 6, 7, 8, 9\}$

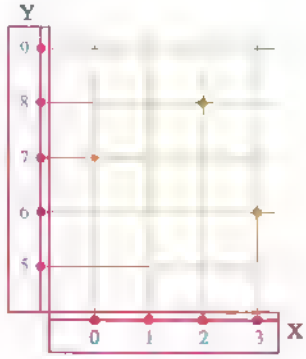
, show which of the following Cartesian diagrams represents a function from X to Y and if it is a function , mention its range :



R_1



R_2



R_3

Solution

- R_1 is not a function because there are two points lying on the vertical line which passes through the element $2 \in X$
- R_2 is a function because each vertical line has only one point lying on it , the range of the function R_2 is $\{6, 8, 9\}$
- R_3 is not a function because there is no point on the vertical line which passes through the element $1 \in X$

Example 7 If $X = \{0, 1, 2, 3\}$, $Y = \{2, 3, 4, 5, 6\}$ and R is a relation from X to Y where "a R b" means "$a + b = 5$" for each $a \in X$, $b \in Y$

, write the relation R and represent it by an arrow diagram.

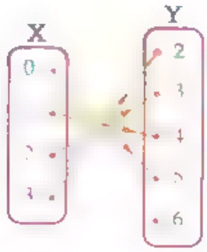
Mention giving reasons if R is a function from X to Y or not.

And if it is a function , find its range.

Solution

- $R = \{(0 , 5) , (1 , 4) , (2 , 3) , (3 , 2)\}$
- R represents a function from X to Y because each element of X is connected with only one element of Y

The range of the function $= \{5 , 4 , 3 , 2\}$

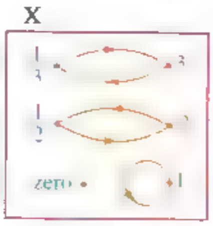


Example 8

If $X = \{3 , 2 , 1 , \text{zero} , \frac{1}{2} , \frac{1}{3}\}$ and R is a relation on X where "a R b" means "a is the multiplicative inverse of b" for each $a \in X , b \in X$, write R and represent it by an arrow diagram and mention giving reasons if R represents a function or not.

Solution

- $R = \{(3 , \frac{1}{3}) , (2 , \frac{1}{2}) , (1 , 1) , (\frac{1}{2} , 2) , (\frac{1}{3} , 3)\}$
- R does not represent a function because the element zero $\in X$ is not connected with any element in X (There is no arrow going out from zero in the arrow diagram which represents the relation)



TRY by yourself **3**

If $X = \{1 , 2 , 3\}$, $Y = \{1 , 4 , 6 , 9\}$ and R is a relation from X to Y where "a R b" means "$a = \sqrt{b}$" for each $a \in X , b \in Y$

, write the relation R and represent it by an arrow diagram. Mention giving reasons if R is a function from X to Y or not , and if it is a function , mention its range.

Answers of try by yourself

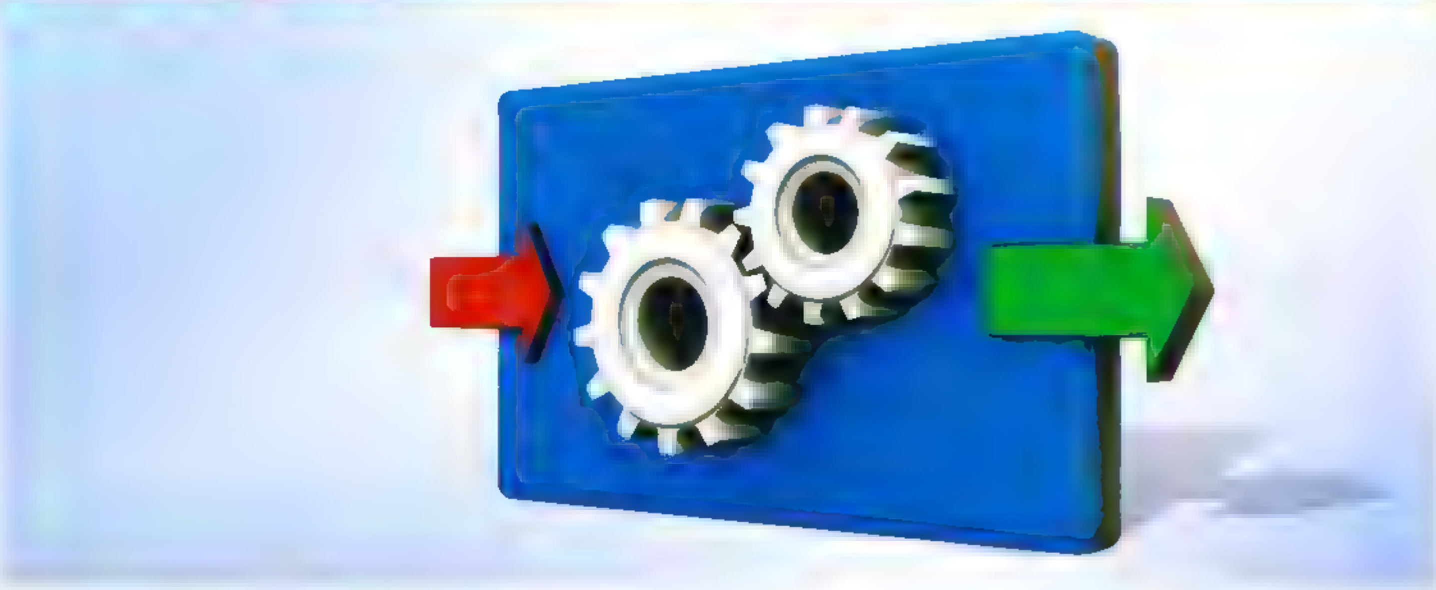
1 $R = \{(1 , 5) , (2 , 4) , (3 , 3)\}$, represent by yourself.

2 $R = \{(2 , 1) , (4 , 2)\}$, represent by yourself.

3 $R = \{(1 , 1) , (2 , 4) , (3 , 9)\}$, represent by yourself.

Yes , R is a function from X to Y because each element of X has one image in Y , its range $= \{1 , 4 , 9\}$

3 The symbolic representation of the function - Polynomial functions



The symbolic representation of the function

WATCH VIDEO

- The function is usually denoted by one of the letters f or g or k or ...

  and the function f from the set X to the set Y is written mathematically as :

  $f : X \longrightarrow Y$ and is read as f is a function from X to Y

  or $g : X \longrightarrow Y$ and is read as g is a function from X to Y and so on ...

- If the ordered pair (x , y) belongs to the function , then the element y is called the image of the element x by the function f and we express it by one of the following two forms :

  $f : x \longmapsto y$ and it is read as f maps x to y

  or $f : f(x) = y$ and it is read as f is a function where $f(x) = y$

For example:

If $f : X \longrightarrow Y$ where $f : x \longmapsto x^2$, then $f : 3 \longmapsto 9$

, also can be written in the form : $f(x) = x^2$, hence $f(3) = 9$

! Remark

The mathematical form $f(x) = x^2$ is called the rule of the function f, and it is used to find the image of any element of the domain by the function f

Remember that

If f is a function from the set X to the set Y i.e. $f : X \longrightarrow Y$, then :

1. X is called the **domain** of the function f
2. Y is called the **codomain** of the function f
3. The set of images of the elements of the set X by the function f is called the **range** of the function f which is a subset of the codomain Y

Example 1

If $X = \{-1 , 0 , 1\}$, $Y = \{0 , -1 , -2\}$ and the function $f : X \longrightarrow Y$ where $f(x) = x^2 - 1$, find the set of the function f and represent it by an arrow diagram , then write its range.

Solution

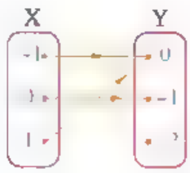
$\because f(x) = x^2 - 1$

$\therefore f(-1) = (-1)^2 - 1 = 0$ $\qquad \therefore (-1 , 0) \in$ the set of the function f

, $f(0) = (0)^2 - 1 = -1$ $\qquad \therefore (0 , -1) \in$ the set of the function f

, $f(1) = (1)^2 - 1 = 0$ $\qquad \therefore (1 , 0) \in$ the set of the function f

$\therefore$ The set of the function $f = \{(-1 , 0) , (0 , -1) , (1 , 0)\}$



The range of the function $f = \{0 , -1\}$

Remark

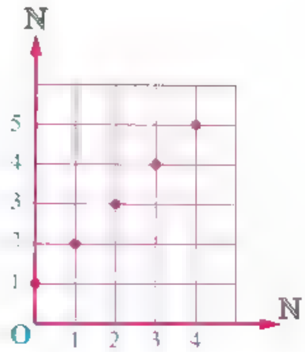
If f is a function from the set X to itself : i.e. $f : X \longrightarrow X$, then we say «f is a function on X»

Example 2 If $f : \mathbb{N} \longrightarrow \mathbb{N}$ where $\mathbb{N}$ is the set of natural numbers and $f(X) = X + 1$ find $f(0)$, $f(1)$, $f(2)$, $f(3)$ and $f(4)$, then graph a part of the square net of the Cartesian product $\mathbb{N} \times \mathbb{N}$ and represent on it five elements of this function. What is the range of the function f ?

Solution

$f(X) = X + 1$ for each $X \in \mathbb{N}$

means that the image of any natural number by the function f is "the number + 1"



$\therefore f(0) = 0 + 1 = 1$

, $f(1) = 2$

, $f(2) = 3$

, $f(3) = 4$

, $f(4) = 5$

$\therefore (0 , 1) , (1 , 2) , (2 , 3) , (3 , 4) , (4 , 5)$

are five elements of f

- The range of f is all the natural numbers except zero. (because there is no natural number added 1 gives zero)

i.e. The range of $f = \mathbb{N} - \{0\}$

TRY by yourself 1

If $X = \{2 , 4 , 6 , 8\}$

, $Y = \{1 , 2 , 3 , 4 , 5 , 6\}$

and the function $f : X \longrightarrow Y$ where $f(X) = \frac{1}{2} X$

, write the set of the function f and represent it by a Cartesian diagram , then find its range.

Polynomial functions

Definition

The function $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(X) = a_0 + a_1 X + a_2 X^2 + \cdots + a_n X^n$

where a_0 , a_1 , a_2 , $\cdots$, $a_n \in \mathbb{R}$, $n \in \mathbb{N}$ is called a polynomial function.

i.e. **The polynomial function is a function whose rule is a term or an algebraic expression in condition that the following should be identified :**

1. Each of the domain and the codomain of the function is the set of real numbers.
2. The power (the index) of the variable X in any of its terms is a natural number.

For example: The following functions are all polynomial functions :

- $f : f(X) = 2X + 5$
- $g : g(X) = X^2 - 2X + 1$
- $k : k(X) = 8$
- $n : n(X) = 1 + \sqrt{2}\,X - 9X^3$

! Remark

If the domain or the codomain of a function is not the set of real numbers , then that function is not a polynomial function.

For example :

- $f : f(X) = \sqrt{X}$ is not a polynomial function because $f(X)$ doesn't exist in $\mathbb{R}$ if X equals a negative number.

  For example : $f(-1) \notin \mathbb{R}$ because $\sqrt{-1} \notin \mathbb{R}$

  , so the domain of the function f is not the set of real numbers.

- $h : h(X) = \frac{1}{X}$ is not a polynomial function

  because $h(X)$ doesn't exist in $\mathbb{R}$ if X equals zero. i.e. $h(0) \notin \mathbb{R}$

  , so the domain of the function h is not the set of real numbers.

Remark

When we search if the function is a polynomial or not , we do not simplify its rule.

For example:

The function $f_1 : f_1(x) = x\left(x + \frac{1}{x}\right)$ doesn't represent a polynomial function

because $f_1(0) \notin \mathbb{R}$ while the function $f_2 : f_2(x) = x^2 + 1$ represents a polynomial function.

And notice that: $x\left(x + \frac{1}{x}\right) = x^2 + 1$ for all real numbers except 0

TRY by yourself 2 ***Which of the functions defined by the following rules represents a polynomial function :***

1. $f_1(x) = x(x^2 - 3)$
2. $f_2(x) = x\left(\frac{2}{x} + 5\right)$
3. $f_3(x) = x^2 - \sqrt{x} + 1$
4. $f_4(x) = x^2 - (x^2 - 4)$

The degree of the polynomial function

The degree of the polynomial function is the highest power of the variable in the function rule.

For example:

- The function $f_1 : f_1(x) = 3x - \frac{1}{2}$ is of the first degree (a linear function)
- The function $f_2 : f_2(x) = \sqrt{5}\,x^2 - 3x + 4$ is of the second degree (a quadratic function)
- The function $f_3 : f_3(x) = x^3 - 5x^2 + 4$ is of the third degree (a cubic function)

Remarks

- The function $f : f(x) = a$ where $a \in \mathbb{R} - \{0\}$

  is a polynomial function of zero degree (a constant function) as $f : f(x) = 3$

  In the case of $a = 0$ i.e. When $f(x) = 0$, then the function f has no degree.
- When you want to determine the degree of the function you should simplify its rule to the simplest form before telling its degree.

Example 3 **Choose the correct answer from the given ones :**

1 The function $f : f(x) = x^2 (2 + x)^2$ is a polynomial function of the .... degree.

(a) first (b) second (c) third (d) fourth

2 The function $f : f(x) = x^2 - (x - 5)^2$ is a polynomial function of the .... degree.

(a) zero (b) first (c) second (d) fourth

3 The function $f : f(x) = x^4 - (x^2 + 1)(x^2 - 1)$ is a polynomial function of the .... degree.

(a) zero (b) first (c) second (d) fourth

4 If $f(x) = x^2 - x - 2$, then $f(-3) =$ ....

(a) -3 (b) 4 (c) 10 (d) 14

5 If $f(x) = x^2 - 2x + 5$, then $f(0) =$ ....

(a) 2 (b) 4 (c) 5 (d) 7

6 If $f(x) = x^2 - \sqrt{3}\, x$, then $f\left(-\sqrt{3}\right) =$ ....

(a) 0 (b) 3 (c) 6 (d) $2\sqrt{3}$

7 If $f(x) = x^3$, then $f(3) + f(-3) =$ ....

(a) 54 (b) 27 (c) 6 (d) 0

8 If $f(x) = a\,x - 6$, $f(2) = 0$, then $a =$ ....

(a) -6 (b) -3 (c) 3 (d) 0

Solution

1 **(d) The reason :** $\because f(x) = x^2 (4 + 4x + x^2) = 4x^2 + 4x^3 + x^4$

$\therefore f$ is a function of the fourth degree.

2 **(b) The reason :** $\because f(x) = x^2 - (x^2 - 10x + 25) = x^2 - x^2 + 10x - 25$

$= 10x - 25$

$\therefore f$ is a function of the first degree.

3 **(a) The reason :** $\because f(x) = x^4 - (x^4 - 1) = x^4 - x^4 + 1 = 1$

$\therefore f$ is a function of the zero degree.

4 **(c) The reason :** Substituting by $x = -3$ at the function rule

$\therefore f(-3) = (-3)^2 - (-3) - 2 = 9 + 3 - 2 = 10$

5 (c) **The reason :** Substituting by $x=0$ at the function rule

$$\therefore f(0)=0^2-2(0)+5=0-0+5=5$$

6 (c) **The reason :** Substituting by $x=-\sqrt{3}$ at the function rule

$$\therefore f\left(-\sqrt{3}\right)=\left(-\sqrt{3}\right)^2-\left(\sqrt{3}\right)\left(-\sqrt{3}\right)=3+3=6$$

7 (d) **The reason :** $\because f(3)=3^3=27$, $f(-3)=(-3)^3=-27$

$$\therefore f(3)+f(-3)=27+(-27)=0$$

8 (c) **The reason :** $\because f(2)=0$ $\quad\therefore a\times 2-6=0$

$$\therefore 2a=6 \qquad \therefore a=3$$

TRY by yourself 3

Choose the correct answer from the given ones :

1 The function $f : f(x)=x(x^3-2)$ is a polynomial function of the .... degree.

(a) first (b) second (c) third (d) fourth

2 If $f(x)=3-5x$, then $f(-2)=$ ....

(a) 1 (b) 5 (c) 7 (d) 13

3 If $f(x)=x^2+x-1$, then $f(1)+f(-1)=$ ....

(a) -2 (b) 0 (c) 2 (d) 3

4 If $f(x)=4x+k$, $f(2)=15$, then $k=$ ....

(a) 2 (b) 4 (c) 7 (d) 15

Example 4 If $f(x)=x^2-2x+5$

, **prove that :** $f\left(2\sqrt{2}+1\right)=2f\left(1-\sqrt{2}\right)$

Solution $\because f\left(2\sqrt{2}+1\right)=\left(2\sqrt{2}+1\right)^2-2\left(2\sqrt{2}+1\right)+5$

$$=8+1+4\sqrt{2}-4\sqrt{2}-2+5=12 \qquad (1)$$

$$, f\left(1-\sqrt{2}\right)=\left(1-\sqrt{2}\right)^2-2\left(1-\sqrt{2}\right)+5$$

$$=1+2-2\sqrt{2}-2+2\sqrt{2}+5=6 \qquad (2)$$

From (1) and (2) : $\therefore f\left(2\sqrt{2}+1\right)=2f\left(1-\sqrt{2}\right)$

Example 5 If $f(X) = 2X + b$ and $g(X) = X^2 + b$ and if $f(2) + g(-4) = 30$,
then find : $f(-2) - g(2)$

Solution

$\because f(2) = 2 \times 2 + b = 4 + b$, $g(-4) = (-4)^2 + b = 16 + b$

, $\because f(2) + g(-4) = 30$ $\qquad \therefore 4 + b + 16 + b = 30$

$\therefore 20 + 2b = 30$ $\qquad \therefore 2b = 30 - 20 = 10$

$\therefore b = \frac{10}{2} = 5$

$\boxed{\therefore f(X) = 2X + 5}$, $\boxed{g(X) = X^2 + 5}$

$\therefore f(-2) = 2 \times (-2) + 5 = 1$, $g(2) = 2^2 + 5 = 9$

$\therefore f(-2) - g(2) = 1 - 9 = \;\; 8$

TRY by yourself 4 If $f(X) = 2X + 5$ and $g(X) = X - 6$, ***then prove that :*** $f(2) + 3g(3) = 0$

Answers of try by yourself

1 R = {(2 , 1) , (4 , 2) , (6 , 3) , (8 , 4)} , represent by yourself.
The range of the function f = {1 , 2 , 3 , 4}

2 The polynomial functions are : f_1 and f_4

3 1 (d) 2 (d) 3 (b) 4 (c)

4 Prove by yourself.

Unit 4

The study of some polynomial functions

First The linear function

Definition

The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = ax + b$, $a \in \mathbb{R} - \{0\}$, $b \in \mathbb{R}$ is called a linear function (it is a polynomial function of the first degree).

Examples of linear functions :

- $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(x) = x - 1$
- $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(x) = 2x + 1$
- $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(x) = 3x$

Notice that :
In each of the shown functions , the index of x is 1, therefore each of them is a function of the first degree.

The graphical representation of the linear function

- The linear function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = ax + b$, $a \in \mathbb{R} - \{0\}$, $b \in \mathbb{R}$ is represented graphically by a straight line intersecting :

  – The y-axis at the point (0 , b) – The x-axis at the point $\left(\frac{-b}{a}, 0\right)$

- To represent a linear function , it is enough to find two ordered pairs belonging to the function.
- You can find a third ordered pair to check that the three points are on the same straight line.

Example 1 **Graph each of the following linear functions :**

1 $f : f(x) = 2x - 3$ **2** $r : r(x) = -\frac{1}{2}x$

Solution 1 Determine three ordered pairs belonging to the function.

$\because f(x) = 2x - 3$

$\therefore f(-1) = 2(-1) - 3 = -5$ $\therefore (-1, -5) \in f$

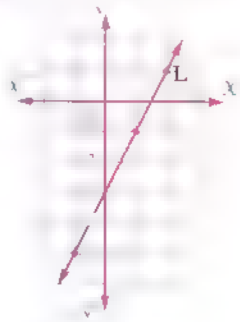
$, f(1) = 2 \times 1 - 3 = -1$ $\therefore (1, -1) \in f$

and $f(2) = 2 \times 2 - 3 = 1$ $\therefore (2, 1) \in f$

You can arrange these ordered pairs in the opposite table :

x	-1	1	2
$y = f(x)$	-5	-1	1

Locate these three points which represents the three ordered pairs in the Cartesian plane and draw the straight line L which passes through any two points of them.
Then check that the third point lies on the same straight line.
Then this straight line is the graphical representation of this function.



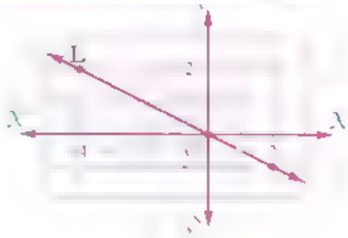
Notice that :

- The point of intersection with y-axis $= (0, b) = (0, -3)$
- The point of intersection with x-axis $= \left(-\frac{b}{a}, 0\right) = \left(\frac{3}{2}, 0\right)$

2 $\because r(x) = -\frac{1}{2}x$ $\therefore$

x	0	2	-4
$y = r(x)$	0	-1	2

From the opposite graph notice that , the straight line L passes through the origin point O (0 , 0)



Generally

The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = a x , a \in \mathbb{R}^*$
is represented graphically by a straight line passing through the origin point (0 , 0)

TRY 1

Represent graphically each of the following linear functions :

1 $f : f(x) = 3x - 3$

2 $f : f(x) = 2x$

Example 2

1. If the point $(a , -a)$ lies on the straight line representing the function $f : f(X) = X - 6$, find the value of a

2. If the straight line representing the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = aX + b$ intersects the y-axis at $(0 , 3)$ and $f(2) = 7$, find the value of each of a , b

Solution

1. $\because$ $(a , -a)$ lies on the straight line representing the function f

$\therefore$ $(a , -a)$ satisfies the function

$\therefore a - 6 = -a$ $\qquad$ $\therefore 2a = 6$ $\qquad$ $\therefore \boxed{a = 3}$

2. $\because$ The straight line intersects the y-axis at $(0 , 3)$

$\therefore$ $(0 , 3)$ satisfies the function $\qquad$ $\therefore 3 = a \times 0 + b$

$\therefore \boxed{b = 3}$ $\qquad$, $\because f(2) = 7$ $\qquad$ $\therefore 7 = 2a + 3$

$\therefore 2a = 4$ $\qquad$ $\therefore \boxed{a = 2}$

TRY by yourself 2

If the straight line representing the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = 4X - a$ intersects the X-axis at $(2 , b)$, find the value of each of a , b

Second The constant function

Definition

The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = b$, $b \in \mathbb{R}$ is called a constant function.

For example:

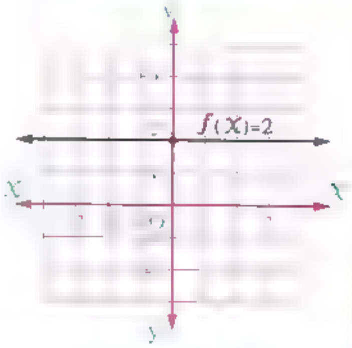
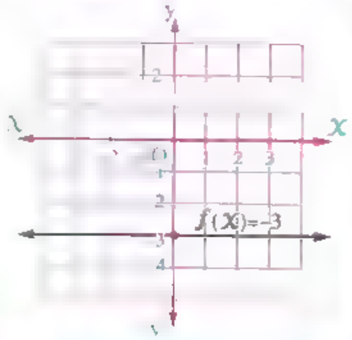
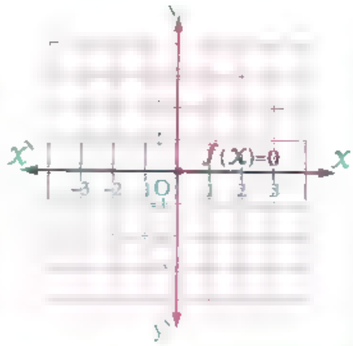
$f : f(X) = 5$ is a constant function where $f(1) = 5$, $f(0) = 5$, $f(-2) = 5$, ... and so on.

The graphical representation of the constant function

The constant function $f : f(X) = b$ (where $b \in \mathbb{R}$) is represented by a straight line parallel to X-axis and passing through the point $(0 , b)$ **and this line is :**

- **above** X-axis if $b > 0$
- **below** X-axis if $b < 0$
- **coincident** with X-axis if $b = 0$

The following examples illustrate that :

$f : f(X) = 2$	$f : f(X) = -3$	$f : f(X) = 0$
		
The straight line is above X-axis and passes through $(0 , 2)$	The straight line is below X-axis and passes through $(0 , -3)$	The straight line is coincident with X-axis and passes through $(0 , 0)$

Example 3 **Choose the correct answer from the given ones :**

1 The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = -3$ is represented by a straight line intersecting y-axis at the point ....

(a) $(-3 , 0)$ (b) $(0 , -3)$ (c) $(3 , 0)$ (d) $(0 , 3)$

2 If $f(X) = 4$, then $f(2)$ .... $f(3)$

(a) $<$ (b) $>$ (c) $=$ (d) $\neq$

3 If $f(X) = 5$, then $2 f(3) =$ ....

(a) 6 (b) $f(6)$ (c) 10 (d) $3 f(2)$

4 If $f(X) = 7$, then $f(7) + f(-7) =$ ....

(a) -14 (b) -7 (c) 7 (d) 14

5 If $f(X) = 2$, then $f(X - 2) =$ ....

(a) -2 (b) 0 (c) 2 (d) 4

Solution

1 (b)

2 (c) **The reason :** $\because$ f is a constant function $\therefore f(2) = f(3) = 4$

3 (c) **The reason :** $\because$ f is a constant function $\therefore 2\,f(3) = 2 \times 5 = 10$

4 (d) **The reason :** $\because$ f is a constant function

$\therefore f(7) + f(-7) = 7 + 7 = 14$

5 (c) **The reason :** $\because$ f is a constant function $\therefore f(X - 2) = f(X) = 2$

TRY by yourself 3

Represent graphically $f : f(X) = -1$, then find the following :

1 The degree of the function f

2 $f(5)$

3 $f(2) + f(-2)$

4 $f(-X)$

Third The quadratic function

Definition

The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = a\,X^2 + b\,X + c$

where a , b and c are real numbers , $a \neq 0$

is called a quadratic function (it is a polynomial function of the second degree).

Examples of quadratic functions :

- $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(X) = X^2$
- $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(X) = X^2 - 2$
- $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(X) = 3\,X^2 - 7\,X + 2$
- $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(X) = 6 - X^2 + X$

Notice that :

In each of the shown functions , the highest index of X is 2 , therefore each of them is a function of the 2^{nd} degree.

The graphical representation of the quadratic function

We know that the domain of the quadratic function is the set of real numbers $\mathbb{R}$ which is an infinite set. So , to represent this function graphically , we should represent it on a certain interval by determining some of ordered pairs which belong to the function. Then we draw the curve (paved curve) passing through the points which represent these ordered pairs.

The following examples illustrate that :

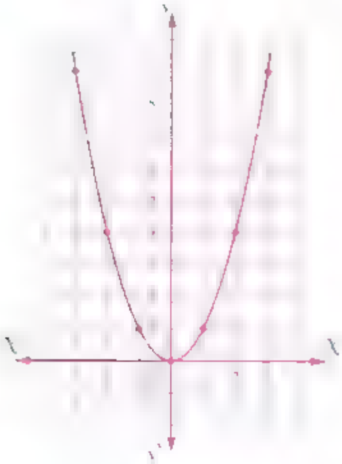
Example 4 **Graph each of the following quadratic functions :**

1. $f : f(x) = x^2$, taking $x \in [-3, 3]$
2. $f : f(x) = -x^2$, taking $x \in [-3, 3]$

Solution

1. $f(x) = x^2$

x	-3	-2	-1	0	1	2	3
$f(x)$	9	4	1	0	1	4	9



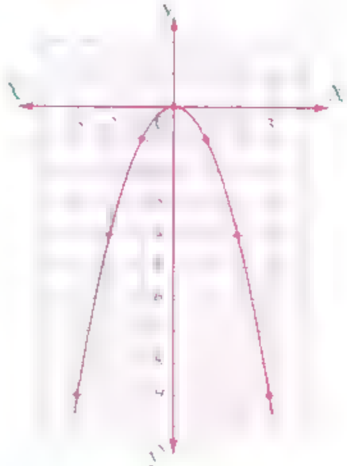
Notice that :

The coefficient of $x^2 > 0$

- The point (0 , 0) is the point of the vertex of the curve , it is considered as **a minimum value** point of the curve because the whole curve **lies up on it.**
- **The minimum** value of the function is zero which is the y-coordinate of the vertex of the curve.
- The curve is symmetric about y-axis
  i.e. the y-axis is the line of symmetry of the curve and its equation is $x = 0$

2. $f(x) = -x^2$

x	-3	-2	-1	0	1	2	3
$f(x)$	-9	-4	-1	0	-1	-4	-9



Notice that :

The coefficient of $x^2 < 0$

- The point (0 , 0) is the point of the vertex of the curve , it is considered as **a maximum value** point of the curve because the whole curve **lies below it.**
- **The maximum** value of the function is zero which is the y-coordinate of the vertex of the curve.
- The curve is symmetric about y-axis
  i.e. the y-axis is the line of symmetry of the curve and its equation is $x = 0$

Generally

The quadratic function $f : f(x) = a\,x^2 + b\,x + c$ where a , b and c are real numbers , $a \neq 0$ has the following properties :

1. The vertex of the curve $= \left(\frac{-b}{2a} , f\left(\frac{-b}{2a}\right)\right)$
2. If a (the coefficient of x^2) is positive , then the curve is open upwards and the function has a minimum value equals $f\left(\frac{-b}{2a}\right)$
3. If a (the coefficient of x^2) is negative , then the curve is open downwards and the function has a maximum value equals $f\left(\frac{-b}{2a}\right)$
4. The curve of the function is symmetric about the vertical line which passes through the vertex of the curve and the equation of that line is : $x = \frac{-b}{2a}$ and it is called the axis of symmetry of the curve.

Example 5

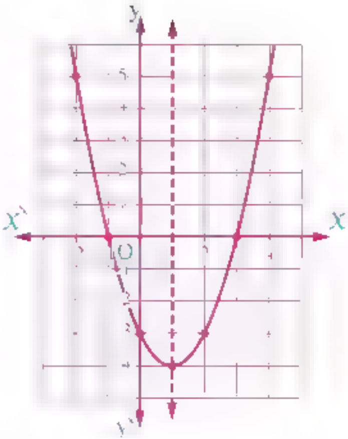
Graph the function $f : f(x) = x^2 - 2x - 3$, taking $x \in [-2 , 4]$, then from the graph , find :

1. The point of the vertex of the curve.
2. The equation of the line of symmetry.
3. The maximum or minimum value of the function.

Solution

$f(x) = x^2 - 2x - 3$

x	-2	-1	0	1	2	3	4
$f(x)$	5	0	-3	-4	-3	0	5



From the graph , we deduce that :

1. The vertex of the curve is $(1 , -4)$
2. The equation of the line of symmetry is $x = 1$, it is a straight line parallel to y-axis and passing through the vertex of the curve.
3. The minimum value of the function $= -4$

! Remark

We can form the table used in graphing the function $f : f(X) = X^2 - 2X - 3$ where $X \in [-2 , 4]$ by using the scientific calculator which supports (Table) as follows :

1. Turn the calculator on (Table) as follows : Press MODE SETUP , then choose TABLE
2. Input data : Write the rule of the previous function , press successively the following buttons : Start → ALPHA) X^2 − 2 ALPHA) − 3
3. Press the button = , then at the beginning of the interval START? write (−) 2 , then press =
4. At the end of the interval END? write the number 4 , then press =
5. To determine the length of the interval STEP? write 1 , then press =

The table is formed in the display , you can move by using button up or down.

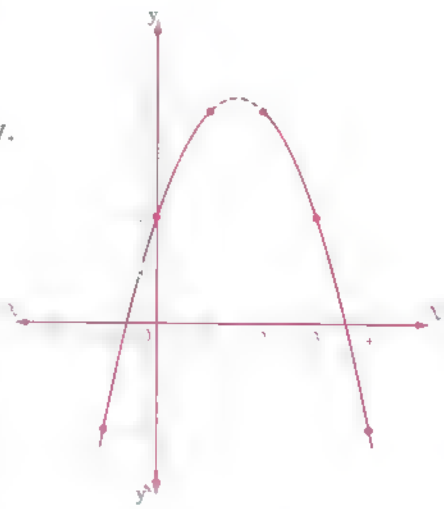
	X	F(X)
1	-2	5
2	-1	0
3	0	-3
4	1	-4
5	2	-3
6	3	0
7	4	5

• To exit the program , press successively the buttons : Start → MODE SETUP 1

Example 6

Graph the function $f : f(X) = -X^2 + 3X + 2$, taking $X \in [-1 , 4]$, then find :

1. The maximum value or minimum value of the function.
2. The equation of the line of symmetry.



Solution

X	−1	0	1	2	3	4
$f(X)$	−2	2	4	4	2	−2

When we represent these ordered pairs , we notice that the point of the vertex of the curve is not among these points which makes the drawing of the dotted part in the opposite figure is inaccurate , so the studying of the curve will be difficult , then we should find the vertex point of the curve algebraically as the following :

Finding the vertex point

At the point of the vertex of the curve of the quadratic function , it will be :

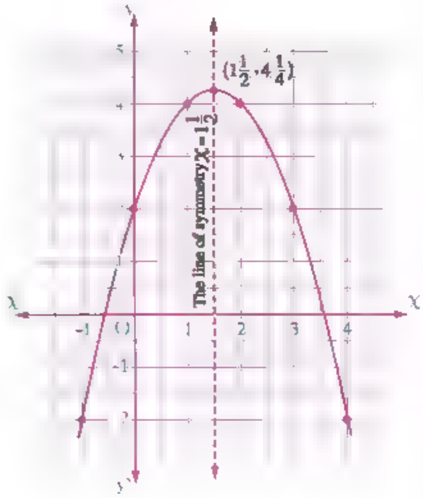
- The X-coordinate $= \frac{-b}{2a}$
- The y-coordinate $= f\left(\frac{-b}{2a}\right)$

where b is the coefficient of X , a is the coefficient of X^2

$\therefore$ X at the vertex of the curve $= \frac{-3}{2 \times -1} = \frac{-3}{-2} = 1\frac{1}{2}$

, $\because f\left(1\frac{1}{2}\right) = -\frac{9}{4} + \frac{9}{2} + 2 = 4\frac{1}{4}$

$\therefore$ The vertex of the curve is $\left(1\frac{1}{2}, 4\frac{1}{4}\right)$



From the vertex of the curve , we find that :

1. The maximum value $= 4\frac{1}{4}$
2. The equation of the line of symmetry is $X = 1\frac{1}{2}$

TRY by yourself 4

Graph the curve of the function f : f $(X) = X^2 + 2X - 3$ on the interval $[-4, 2]$

From the graph, find :

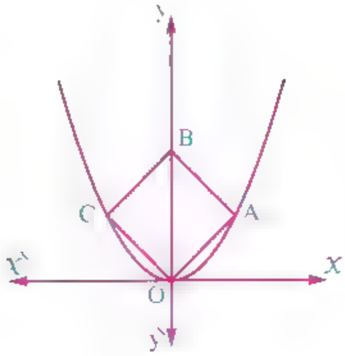
1. The maximum or minimum value of the function.
2. The equation of the line of symmetry.

Example 7

In the opposite figure :

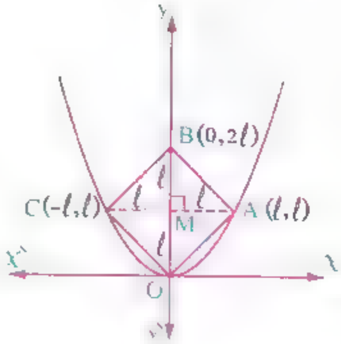
ABCO is a square and the curve represents the function $f : f(X) = X^2$

Find the coordinates of the points : A , B and C



Solution

Draw the square diagonal $\overline{AC}$ to intersect the another diagonal $\overline{BO}$ at the point M



$\because$ The two diagonals of the square are equal in length and bisect each other.

$\therefore$ MA = MB = MC = MO and let : MA = ℓ

$\therefore$ MA = MB = MC = MO = ℓ

$\therefore$ A (ℓ , ℓ) , C $(-\ell , \ell)$, B $(0 , 2\ell)$

$\because$ A $(\ell , \ell) \in$ the function $f : f(X) = X^2$

By substituting in the rule of the function

$\therefore \ell = \ell^2$ $\qquad \therefore \ell^2 - \ell = 0$ $\qquad \therefore \ell(\ell - 1) = 0$

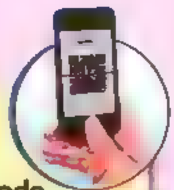
$\therefore \ell = 0$ (refused) $\qquad$ or $\ell - 1 = 0$ $\qquad \therefore \ell = 1$

$\therefore$ A (1 , 1) , B (0 , 2) and C (– 1 , 1)

UNIT 2
Ratio, proportion, direct variation and inverse variation

Lessons of the unit :

1. Ratio and proportion.
2. Follow properties of proportion.
3. Continued proportion.
4. Direct variation and inverse variation.



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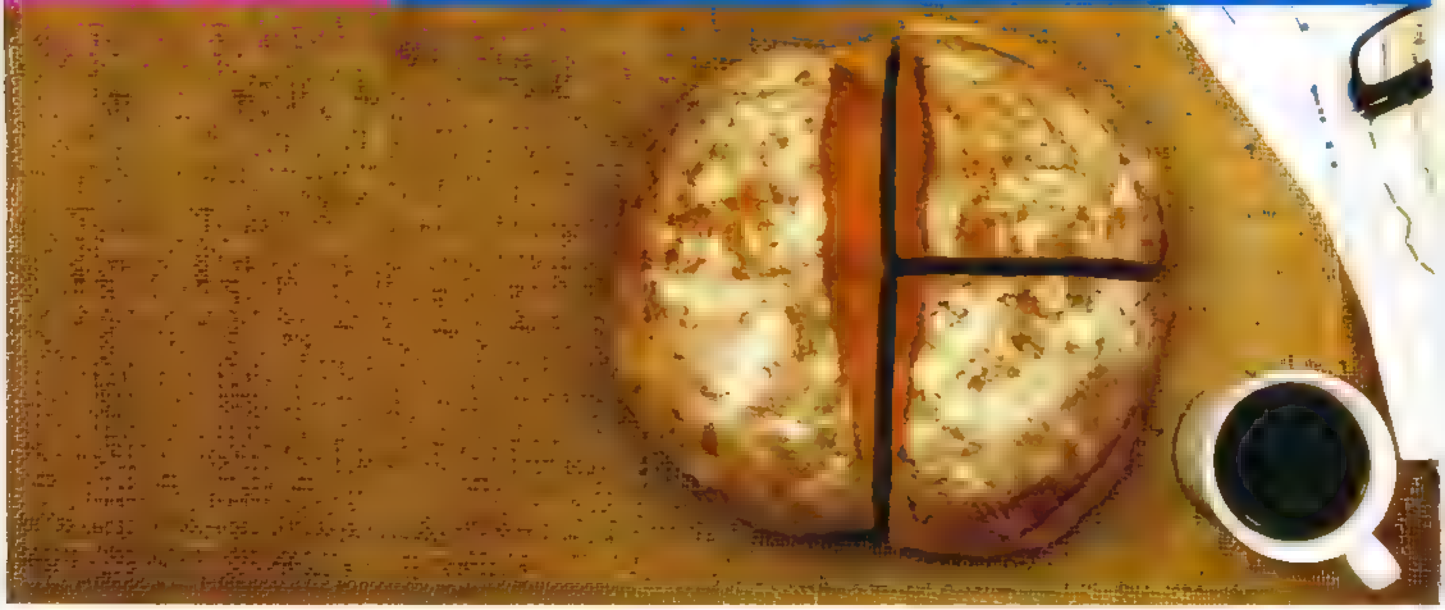
Unit Objectives :

By the end of this unit, student should be able to :

- recognize the concept of the ratio.
- recognize the properties of the ratio.
- recognize the concept of the proportion.
- recognize the properties of the proportion.
- recognize the concept of the continued proportion.
- use the properties of the ratio and the proportion for solving a lot of problems.
- recognize the concept of the direct variation.
- recognize the concept of the inverse variation.
- differentiate between the direct variation and the inverse variation.
- solve real life problems on the direct variation and the inverse variation.
- appreciate the role of mathematics in solving a lot of real life problems.

Unit 1

Ratio and proportion

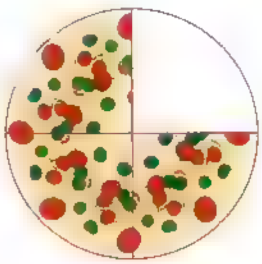


First Ratio

We have studied in the primary stage that the ratio is one of methods of comparison between two quantities.

For example:

If a pie is divided into four equal parts and Hany ate one part only of it , then :



- The ratio of what Hany ate to the whole pie is 1 : 4

  and it may written as $\frac{1}{4}$

- The ratio of what was left of the pie to the whole pie is 3 : 4

  and it may written as $\frac{3}{4}$

- The ratio of what Hany ate to which was left of the pie is 1 : 3

  and it may written as $\frac{1}{3}$

Generally

If a and b are two real numbers , then :

The ratio between a and b is written as a : b or $\frac{a}{b}$

and is read as a to b where :

a is called the antecedent of the ratio , b is called the consequent of the ratio , a and b are called together the two terms of the ratio.

Properties of the ratio

Property 1

The value of the ratio does not change if each of its terms is multiplied or divided by the same non-zero real number.

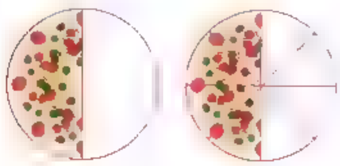
i.e.

$a : b = ak : bk \;, k \in \mathbb{R}^*$

For example:

$1 : 2 = 1 \times 4 : 2 \times 4$

i.e. $1 : 2 = 4 : 8$



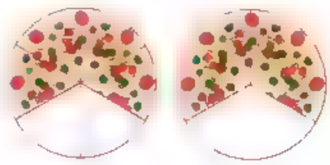
1 : 2 4 : 8

$a : b = \frac{a}{n} : \frac{b}{n} \;, n \in \mathbb{R}^*$

For example:

$4 : 6 = \frac{4}{2} : \frac{6}{2}$

i.e. $4 : 6 = 2 : 3$



4 : 6 2 : 3

Property 2

The value of the ratio ($\neq 1$) changes if we add or subtract (to or from) each of its two terms a non-zero real number.

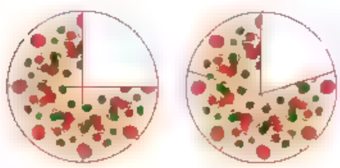
i.e.

$a : b \neq a + k : b + k \;, k \in \mathbb{R}^*$
where $a \neq b$

For example:

$3 : 4 \neq 3 + 1 : 4 + 1$

i.e. $3 : 4 \neq 4 : 5$



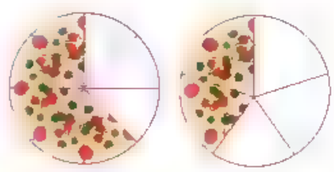
3 : 4 4 : 5

$a : b \neq a - k : b - k \;, k \in \mathbb{R}^*$
where $a \neq b$

For example:

$5 : 8 \neq 5 - 3 : 8 - 3$

i.e. $5 : 8 \neq 2 : 5$



5 : 8 2 : 5

Second Proportion

The opposite table shows two sets of numbers.

If we look at these sets , we can notice that :

$\frac{2}{8} = \frac{4}{16} = \frac{7}{28} = \frac{3}{12} = \frac{6}{24}$ each of them equals $\frac{1}{4}$

The set (A)	2	4	7	3	6
The set (B)	8	16	28	12	24

In this case , we say that the numbers of set (A) are proportional to the corresponding numbers in the set (B)

The previous form which expresses the equality of two ratios or more is called proportion.

Definition of proportion

It is the equality of two ratios or more.

i.e.

If $\frac{a}{b} = \frac{c}{d}$, then the quantities a , b , c and d are proportional.

And vice versa : If a , b , c and d are proportional , then : $\frac{a}{b} = \frac{c}{d}$

- $\boxed{a}$ is called the **first** proportional.
- $\boxed{b}$ is called the **second** proportional.
- $\boxed{c}$ is called the **third** proportional.
- $\boxed{d}$ is called the **fourth** proportional.

$\boxed{\text{a and d}}$ are called **extremes** and $\boxed{\text{b and c}}$ are called **means**.

For example: The numbers 1 , 4 , 7 and 28 are proportional numbers , because $\frac{1}{4} = \frac{7}{28}$

And : $\boxed{1}$ is the first proportional , $\boxed{4}$ is the second proportional , $\boxed{7}$ is the third proportional , $\boxed{28}$ is the fourth proportional , $\boxed{\text{1 and 28}}$ are the extremes of this proportion and $\boxed{\text{4 and 7}}$ are the means.

Properties of proportion

Property 1

If $\frac{a}{b} = \frac{c}{d}$, then : $a \times d = b \times c$ (the product of the extremes = the product of the means)

The reason : If we multiply each ratio by b d , we get : $\frac{a}{b} \times b\,d = \frac{c}{d} \times b\,d$

i.e. $a \times d = b \times c$

Example 1 Choose the correct answer from the given ones :

1. The third proportional for the quantities 2 , 4 and 20 is ....
   (a) 10 (b) 15 (c) 20 (d) 40
2. The fourth proportional for the numbers 4 , 12 and 16 is ....
   (a) 24 (b) ± 24 (c) 48 (d) ± 48
3. If 2 , x , 4 and 6 are proportional , then $x =$ ....
   (a) 1 (b) 3 (c) 5 (d) 8

Solution

1. (a) **The reason :** Let the third proportional be x

   $\therefore$ The quantities 2 , 4 , x and 20 are proportional

   $\therefore \frac{2}{4} = \frac{x}{20}$ $\therefore 2 \times 20 = 4 \times x$

   $\therefore 40 = 4x$ $\therefore x = 10$

2. (c) **The reason :** Let the fourth proportional be x

   $\therefore$ The numbers 4 , 12 , 16 and x are proportional

   $\therefore \frac{4}{12} = \frac{16}{x}$ $\therefore 4x = 12 \times 16$ $\therefore x = \frac{12 \times 16}{4} = 48$

3. (b) **The reason :** $\because$ 2 , x , 4 and 6 are proportional

   $\therefore \frac{2}{x} = \frac{4}{6}$ $\therefore 4x = 12$ $\therefore x = 3$

TRY by yourself 1 If the quantities x , 23 , 15 and 69 are proportional , ***find the value of :*** x

Example 2 Find the number that will be added to each of the numbers : 1 , 13 , 7 and 31 to get proportional numbers.

Solution Let the number be x $\therefore 1 + x , 13 + x , 7 + x , 31 + x$ are proportional.

$\therefore \frac{1 + x}{13 + x} = \frac{7 + x}{31 + x}$ $\therefore (x + 1)(x + 31) = (x + 7)(x + 13)$

$\therefore \cancel{x^2} + 32x + 31 = \cancel{x^2} + 20x + 91$ $\therefore 32x - 20x = 91 - 31$

$\therefore 12x = 60$ $\therefore x = 5$ $\therefore$ The required number $= 5$

Example 3 If $(2x + 5) : (3x - 3) = 5 : 4$, **find the value of :** x

Solution $\because \frac{2x + 5}{3x - 3} = \frac{5}{4}$ $\therefore 4(2x + 5) = 5(3x - 3)$

$\therefore 8x + 20 = 15x - 15$ $\therefore 20 + 15 = 15x - 8x$

$\therefore 35 = 7x$ $\therefore x = \frac{35}{7} = 5$

Example 4 Find the number that if we add to the two terms of the ratio 17 : 22 , the result will be 6 : 7

Solution Let the required number be x $\therefore \frac{17+x}{22+x}=\frac{6}{7}$

$\therefore 7(17+x)=6(22+x)$ $\therefore 119+7x=132+6x$

$\therefore 7x-6x=132-119$ $\therefore x$ (The required number) $=13$

TRY by yourself 2 Find the real number that if we subtract from both terms of the ratio $\frac{5}{6}$, it will become $\frac{3}{2}$

Property 2

If $a \times d = b \times c$, then $\frac{a}{b}=\frac{c}{d}$

The reason : If we divide each side by b d , we get : $\frac{a \times d}{b\,d}=\frac{b \times c}{b\,d}$ i.e. $\frac{a}{b}=\frac{c}{d}$

Also we can deduce that :

- If $a \times d = b \times c$, then $\frac{a}{c}=\frac{b}{d}$
- If $a \times d = b \times c$, then $\frac{b}{a}=\frac{d}{c}$
- If $a \times d = b \times c$, then $\frac{c}{a}=\frac{d}{b}$

Example 5 **In each of the following , find $\frac{x}{y}$ if :**

1. $12x=3y$
2. $4x-3y=0$

Solution

1. $\because 12x=3y$ $\therefore \frac{x}{y}=\frac{3}{12}=\frac{1}{4}$
2. $\because 4x-3y=0$ $\therefore 4x=3y$ $\therefore \frac{x}{y}=\frac{3}{4}$

Example 6 If $4x-3y : 2x+y=\frac{4}{7}$, find in the simplest form the ratio $x : y$

Solution

$\because \frac{4x-3y}{2x+y}=\frac{4}{7}$ $\therefore 7(4x-3y)=4(2x+y)$

$\therefore 28x-21y=8x+4y$ $\therefore 28x-8x=21y+4y$

$\therefore 20x=25y$ $\therefore \frac{x}{y}=\frac{25}{20}$ $\therefore \frac{x}{y}=\frac{5}{4}$

Example 7 If $2x^2 - 6y^2 = xy$, **find** : $x : y$

Solution $\because 2x^2 - 6y^2 = xy$ $\therefore 2x^2 - xy - 6y^2 = 0$

$\therefore (2x + 3y)(x - 2y) = 0$

, then $2x = -3y$ $\therefore 2x + 3y = 0$ $\therefore \boxed{\frac{x}{y} = -\frac{3}{2}}$

or $x - 2y = 0$, then $x = 2y$ $\therefore \boxed{\frac{x}{y} = \frac{2}{1}}$

i.e. $\frac{x}{y} = -\frac{3}{2}$ or $\frac{x}{y} = \frac{2}{1}$

TRY 3

1. If $2a - 5b = 0$, ***find*** : $\frac{a}{b}$
2. If $\frac{x + 2y}{4x - 3y} = \frac{7}{6}$, ***then prove that*** : $\frac{x}{y} = \frac{3}{2}$
3. If $4a^2 - 9b^2 = 0$, ***find*** : $a : b$

Property 3

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$

i.e. $\frac{\text{The antecedent of the first ratio}}{\text{The antecedent of the second ratio}} = \frac{\text{The consequent of the first ratio}}{\text{The consequent of the second ratio}}$

The reason : If we multiply each ratio by $\frac{b}{c}$, we get : $\frac{a}{\cancel{b}} \times \frac{\cancel{b}}{c} = \frac{\cancel{c}}{d} \times \frac{b}{\cancel{c}}$

i.e. $\frac{a}{c} = \frac{b}{d}$

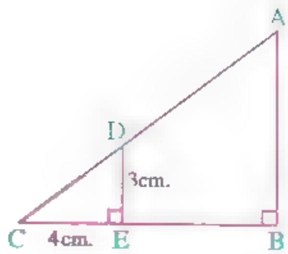
For example: If $\frac{a}{4} = \frac{b}{3}$, then $\frac{a}{b} = \frac{4}{3}$ and $\frac{b}{a} = \frac{3}{4}$

Example 8 **In the opposite figure :**

ABC is a right-angled triangle at B in which :

$D \in \overline{AC}$, $E \in \overline{BC}$ where $\overline{DE} \perp \overline{BC}$

, DE = 3 cm. and EC = 4 cm. **Find** AB : BC



Solution In $\Delta\Delta$ ABC , DEC : m ($\angle$ B) = m ($\angle$ DEC) = 90° , $\angle$ C is a common angle

$\therefore$ m ($\angle$ A) = m ($\angle$ EDC)

$\therefore \Delta ABC \sim \Delta DEC$, then we deduce that : $\frac{AB}{DE} = \frac{BC}{EC}$

$\therefore \frac{AB}{3} = \frac{BC}{4}$ $\therefore \frac{AB}{BC} = \frac{3}{4}$ (The req.)

Property 4

If $\frac{a}{b} = \frac{c}{d}$, then a = cm and b = dm (where m is a constant ≠ 0)

For example: If $\frac{a}{b} = \frac{3}{4}$, then : a = 3 m , b = 4 m (where m is a constant ≠ 0)

Example 9

If a : b = 3 : 5 , find the ratio 20 a – 7 b : 15 a + b

Solution

$\because \frac{a}{b} = \frac{3}{5}$ $\quad \therefore a = 3m \quad , \quad b = 5m$ (where m ≠ 0)

Substituting by a and b in terms of m :

$$\therefore \frac{20a - 7b}{15a + b} = \frac{60m - 35m}{45m + 5m} = \frac{25m}{50m} = \frac{1}{2}$$

Another solution :

By dividing the terms of the ratio $\frac{20a - 7b}{15a + b}$ by b

, then substituting by the value $\frac{a}{b} = \frac{3}{5}$

$$\therefore \frac{20a - 7b}{15a + b} = \frac{20\left(\frac{a}{b}\right) - 7}{15\left(\frac{a}{b}\right) + 1} = \frac{20 \times \frac{3}{5} - 7}{15 \times \frac{3}{5} + 1} = \frac{12 - 7}{9 + 1} = \frac{5}{10} = \frac{1}{2}$$

Example 10

If $\frac{a}{b} = \frac{2}{3}$ and $\frac{x}{y} = \frac{3}{5}$, **prove that :**

$(7ax + 4by)$, $(11ay + bx)$, 12 and 14 are proportional quantities.

Solution

$\because \frac{a}{b} = \frac{2}{3}$ $\quad \therefore a = 2m \ , \ b = 3m$ (where m ≠ 0)

$\because \frac{x}{y} = \frac{3}{5}$ $\quad \therefore x = 3k \ , \ y = 5k$ (where k ≠ 0)

[**Notice that :** We used two different constants m and k]

Substituting by a , b , x and y

$$\therefore \frac{7ax + 4by}{11ay + bx} = \frac{7 \times 2m \times 3k + 4 \times 3m \times 5k}{11 \times 2m \times 5k + 3m \times 3k}$$

$$= \frac{42mk + 60mk}{110mk + 9mk} = \frac{102mk}{119mk} = \frac{6}{7}$$

$\because \frac{12}{14} = \frac{6}{7}$

$\therefore (7ax + 4by)$, $(11ay + bx)$, 12 and 14 are proportional quantities.

TRY by yourself 4

If $\frac{x}{y} = \frac{2}{5}$, ***prove that*** : $(2x + y)$, $(x + 2y)$, 12 and 16 are proportional quantities.

Example 11 The ratio between two real numbers is 4 : 7
If we subtract 16 from each of them , then the ratio between the two obtained numbers is 2 : 5 Find the two numbers.

Solution Let the two numbers be a and b

$\therefore \frac{a}{b} = \frac{4}{7}$ $\therefore a = 4m$, $b = 7m$ (where $m \neq 0$)

$\therefore \frac{4m - 16}{7m - 16} = \frac{2}{5}$ $\therefore 14m - 32 = 20m - 80$

$\therefore 80 - 32 = 20m - 14m$ $\therefore 48 = 6m$ $\therefore m = \frac{48}{6} = 8$

$\therefore a = 4 \times 8 = 32$, $b = 7 \times 8 = 56$ i.e. The two numbers are 32 and 56

TRY by yourself 5

The ratio between two integers is 2 : 5 If 2 is subtracted from the first integer and 1 is added to the second , then the ratio becomes 1 : 4
Find the two integers.

Answers of try by yourself

1. 5
2. 8
3. (1) $\frac{5}{2}$ (2) Prove by yourself. (3) $\pm \frac{3}{2}$
4. Prove by yourself.
5. 6 , 15

LESSON 2

Follow properties of proportion

In this lesson , we will study the property (5) from properties of proportion , before studying this property , we will study an important remark in proportion to help us solving problems.

! Important remark

* If a , b , c and d are proportional quantities and we assume that : $\frac{a}{b} = \frac{c}{d} = m$, then

$a = bm$, $c = dm$

For example:

If $\frac{a}{b} = \frac{c}{d} = \frac{3}{4}$, then $a = \frac{3}{4}\,b$, $c = \frac{3}{4}\,d$

* ***Generally***

If a , b , c , d , e , f , ... are proportional quantities and we assume that :

$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \cdots = m$, then $a = bm$, $c = dm$, $e = fm$, ...

Example 1 If a , b , c and d are proportional quantities , **prove that :**

1. $\frac{2a + 3c}{7a - 5c} = \frac{2b + 3d}{7b - 5d}$

2. $\frac{a + c}{b + d} = \frac{a^2 + c^2}{ab + cd}$

Solution

1. Let $\frac{a}{b} = \frac{c}{d} = m$ $\therefore a = bm$, $c = dm$

$$\text{L.H.S.} = \frac{2\,bm + 3\,dm}{7\,bm - 5\,dm} = \frac{m\,(2\,b + 3\,d)}{m\,(7\,b - 5\,d)} = \frac{2\,b + 3\,d}{7\,b - 5\,d} = \text{R.H.S.}$$

2 Let $\frac{a}{b} = \frac{c}{d} = m$ $\therefore$ $a = bm$, $c = dm$

$\because \frac{a+c}{b+d} = \frac{bm+dm}{b+d} = \frac{m(b+d)}{(b+d)} = m$ (1)

, $\because \frac{a^2+c^2}{ab+cd} = \frac{(bm)^2+(dm)^2}{bm \times b + dm \times d} = \frac{b^2 m^2 + d^2 m^2}{b^2 m + d^2 m} = \frac{m^2(b^2+d^2)}{m(b^2+d^2)} = m$ (2)

From (1) and (2) we deduce that : $\frac{a+c}{b+d} = \frac{a^2+c^2}{ab+cd}$

Example 2 If a , b , c , d , e and f are positive proportional quantities ,

prove that : $\sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}} = \frac{a}{b}$

Solution Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = m$ $\therefore$ a $= bm$, c $= dm$, e $= fm$

$\therefore \sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}} = \sqrt{\frac{(bm)^2+(dm)^2+(fm)^2}{b^2+d^2+f^2}} = \sqrt{\frac{b^2 m^2 + d^2 m^2 + f^2 m^2}{b^2+d^2+f^2}}$

$= \sqrt{\frac{m^2(b^2+d^2+f^2)}{(b^2+d^2+f^2)}} = \sqrt{m^2} = m$

, $\because \frac{a}{b} = m$ $\therefore \sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}} = \frac{a}{b}$

TRY by yourself 1

If $\frac{a}{b} = \frac{c}{d}$, ***prove that :*** $\frac{5a-2c}{5b-2d} = \frac{4a+3c}{4b+3d}$

Property 5

We know that : $\frac{9}{15} = \frac{6}{10} = \frac{3}{5}$

- If we add the antecedents and consequents of the 1st and the 2nd ratios, we get the ratio $\frac{9+6}{15+10} = \frac{15}{25} = \frac{3}{5}$ which is one of the given ratios.
- Also if we add the antecedents and consequents of the 2nd and the 3rd ratios, we get the ratio $\frac{6+3}{10+5} = \frac{9}{15}$ = one of the given ratios.
- If we add the antecedents and consequents of the three given ratios, we get the ratio $\frac{9+6+3}{15+10+5} = \frac{18}{30} = \frac{3}{5}$ = one of the given ratios.

- Since the ratio does not change if we multiply its two terms by a non-zero real number , then if we multiply the two terms of the first ratio by any number as 2 and multiply the two terms of the second ratio by any other number as (– 4) , then the previous proportion stays true.

i.e. $\frac{18}{30} = \frac{-24}{-40} = \frac{3}{5}$

- If we add the antecedents and consequents of the first and the second ratios , we get the ratio $\frac{18-24}{30-40} = \frac{-6}{-10} = \frac{3}{5}$ = one of the given ratios.

- If we add the antecedents and consequents of the three ratios, we get the ratio $\frac{18-24+3}{30-40+5} = \frac{-3}{-5} = \frac{3}{5}$ = one of the given ratios.

From the previous points , we can say that :
If we have some equal ratios , then we can obtain many other ratios , each of them equals any of the initial ratios. This will happen by adding the antecedents and consequents of all the ratios or some of them directly or after multiplying the two terms of each ratio by a non-zero real number.

i.e.

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \cdots$ and m_1 , m_2 , m_3 , ⋯ are non-zero real numbers

, then $\frac{m_1 a + m_2 c + m_3 e + \cdots}{m_1 b + m_2 d + m_3 f + \cdots}$ = one of the given ratios.

Remark : The first problem in example (1) can be solved by using the previous property as follows :

$\because$ a , b , c and d are proportional quantities.

$\therefore \frac{a}{b} = \frac{c}{d}$ multiplying the two terms of the 1^{st} ratio by 2 and the 2^{nd} by 3

Then the sum of antecedents : The sum of consequents = one of the given ratios.

$\therefore \frac{2a+3c}{2b+3d}$ = one of the given ratios. (1)

Multiplying the two terms of the 1^{st} ratio by 7 and the 2^{nd} by (– 5) then

the sum of antecedents : the sum of consequents = one of the given ratio

$\therefore \frac{7a-5c}{7b-5d}$ = one of the given ratios. (2)

From (1) and (2) : $\therefore \frac{2a+3c}{2b+3d} = \frac{7a-5c}{7b-5d}$ $\qquad \therefore \frac{2a+3c}{7a \quad 5c} = \frac{2b+3d}{7b-5d}$

Example 3 If $\frac{a}{4} = \frac{b}{5} = \frac{c}{3}$,

find : $\frac{a-b+c}{a+b-c}$

Solution Multiplying the two terms of the 2nd ratio by (– 1) , then add the antecedents and the consequents of the three ratios :

$\therefore \frac{a-b+c}{4-5+3} = \frac{a-b+c}{2}$ = one of the given ratios. (1)

Multiplying the two terms of the 3rd ratio by (– 1) , then add the antecedents and the consequents of the three ratios :

$\therefore \frac{a+b-c}{4+5-3} = \frac{a+b-c}{6}$ = one of the given ratios. (2)

From (1) and (2) : $\therefore \frac{a-b+c}{2} = \frac{a+b-c}{6}$

$\therefore \frac{a-b+c}{a+b-c} = \frac{2}{6} = \frac{1}{3}$

Example 4 If $\frac{a+b}{11} = \frac{b+c}{9} = \frac{c+a}{4}$,

prove that : $\frac{a+b+c}{5a+4b+3c} = \frac{6}{25}$

Solution Adding the antecedents and consequents of the three ratios.

$\therefore \frac{\text{The sum of antecedents}}{\text{The sum of consequents}}$ = one of the given ratios.

$\therefore \frac{2a+2b+2c}{24}$ = one of the given ratios.

$\therefore \frac{a+b+c}{12}$ = one of the given ratios. (1)

Multiplying the two terms of the 1st ratio by 3 and the 3rd by 2 and adding the antecedents and consequents of the three ratios

$\therefore \frac{\text{The sum of antecedents}}{\text{The sum of consequents}}$ = one of the given ratios.

$\therefore \frac{3a+3b+b+c+2c+2a}{33+9+8}$ = one of the given ratios.

$\therefore \frac{5a+4b+3c}{50}$ = one of the given ratios. (2)

From (1) and (2) :

$\therefore \frac{a+b+c}{12} = \frac{5a+4b+3c}{50}$ $\therefore \frac{a+b+c}{5a+4b+3c} = \frac{12}{50} = \frac{6}{25}$

Example 5 If $\frac{a+4b}{x+2y}=\frac{4b+7c}{2y+5z}=\frac{7c+a}{5z+x}$,

prove that : $\frac{a}{2b}=\frac{x}{y}$

Solution Multiplying the two terms of the 2^{nd} ratio by (- 1) , then add the antecedents and the consequents of the three ratios :

$\therefore \frac{a+4b-4b-7c+7c+a}{x+2y-2y-5z+5z+x}=\frac{2a}{2x}=\frac{a}{x}$ = one of the given ratios. (1)

Multiplying the two terms of the 3^{rd} ratio by (– 1) , then add the antecedents and the consequents of the three ratios :

$\therefore \frac{a+4b+4b+7c-7c-a}{x+2y+2y+5z-5z-x}=\frac{8b}{4y}=\frac{2b}{y}$ = one of the given ratios. (2)

From (1) and (2) : $\therefore \frac{a}{x}=\frac{2b}{y}$ $\therefore \frac{a}{2b}=\frac{x}{y}$

TRY by yourself 2

If $\frac{x}{a-2b}=\frac{y}{b-2c}=\frac{z}{c-2a}$,

prove that : $\frac{x+2y-z}{3a-5c}=\frac{y+2z}{b-4a}$

Answers *of try by yourself*

1 Prove by yourself [Hint : a = bm , c = dm]

2 Prove by yourself [Hint : * Multiplying the terms of the 2^{nd} ratio by 2 , the 3^{rd} ratio by (– 1) and adding the antecedents and consequents of the three ratios

* Hence multiplying the terms of the 3^{rd} ratio by 2 and adding the antecedents and consequents of the 2^{nd} and the 3^{rd} ratios.]

LESSON **3**

Continued proportion

Definition

The quantities a , b and c are said to be in continued proportion if $\frac{a}{b} = \frac{b}{c}$ or $b^2 = a\,c$

WATCH VIDEO

In this proportion , a is called the **first proportional** , c is called the **third proportional** and b is called the **middle proportional (proportional mean)**.

For example:

The numbers 4 , 6 and 9 form a continued proportion because : $\frac{4}{6} = \frac{6}{9}$ or because : $(6)^2 = 4 \times 9$

where 6 is the middle proportional , 4 is the first proportional and 9 is the third proportional.

Notice that :

1 If a , b and c are in continued proportion , then : $b^2 = a\,c$ i.e. $b = \pm\sqrt{ac}$

and the two quantities a and c should be either both positive or both negative.

2 For any two positive numbers or any two negative numbers x and y , there are two middle proportional ($\sqrt{x\,y}$ and $-\sqrt{x\,y}$)

Example 1

Choose the correct answer from the given ones :

1 The middle proportional between 5 and 20 is ....

(a) – 10 (b) 10 (c) ± 10 (d) 100

2 The middle proportional between 3 and $\frac{1}{3}$ is ....

(a) ± 1 (b) 9 (c) $\frac{1}{9}$ (d) ± 9

3 The middle proportional between $3\,x^3$ and $27\,x$ is ....

(a) $9\,x^2$ (b) $\pm 9\,x^2$ (c) $9\,x^4$ (d) $\pm 9\,x^4$

4 The first proportional of 12 and 18 is ....

(a) 8 (b) ± 8 (c) 12 (d) 27

5 The third proportional of – 6 and 12 is ....

(a) – 24 (b) 6 (c) 18 (d) 72

Solution

1 **(c) The reason :** The middle proportional $= \pm\sqrt{5 \times 20} = \pm\sqrt{100} = \pm 10$

2 **(a) The reason :** The middle proportional $= \pm\sqrt{3 \times \frac{1}{3}} = \pm\sqrt{1} = \pm 1$

3 **(b) The reason :** The middle proportional $= \pm\sqrt{3x^3 \times 27x} = \pm\sqrt{81x^4}$

$= \pm 9x^2$

4 **(a) The reason :** Let the first proportional be a

$\therefore \frac{a}{12} = \frac{12}{18}$ $\therefore a = \frac{12 \times 12}{18} = 8$

5 **(a) The reason :** Let the third proportional be c

$\therefore \frac{-6}{12} = \frac{12}{c}$ $\therefore c = \frac{12 \times 12}{-6} = -24$

TRY by yourself 1

1 *Find the middle proportional between 32 and 18*

2 *Find the first proportional of 8 and 16*

! Remark

If a , b and c are in continued proportion and we assume that : $\frac{a}{b} = \frac{b}{c} = m$

, then $\frac{b}{c} = m$ $\therefore b = cm$ (1)

, $\because \frac{a}{b} = m$ $\therefore a = bm$

Substituting for b from (1) : $\therefore a = (cm)\, m$ $\therefore a = cm^2$

i.e.

If $\frac{a}{b} = \frac{b}{c} = m$, then $\begin{cases} b = cm \\ a = cm^2 \end{cases}$

Example 2

If a , b and c are in continued proportion ,

prove that : $\frac{4a^2 - 3b^2}{4b^2 - 3c^2} = \frac{a}{c}$

Solution Let $\frac{a}{b} = \frac{b}{c} = m$ $\therefore b = cm$, $a = cm^2$

$\therefore \frac{4\,a^2 - 3\,b^2}{4\,b^2 - 3\,c^2} = \frac{4\,(cm^2)^2 - 3\,(cm)^2}{4\,(cm)^2 - 3\,c^2} = \frac{4\,c^2\,m^4 - 3\,c^2\,m^2}{4\,c^2\,m^2 - 3\,c^2} = \frac{c^2\,m^2\,(4\,m^2 - 3)}{c^2\,(4\,m^2 - 3)} = m^2$ (1)

$, \because \frac{a}{c} = \frac{cm^2}{c} = m^2$ (2)

From (1) and (2) , we deduce that : $\frac{4\,a^2 - 3\,b^2}{4\,b^2 - 3\,c^2} = \frac{a}{c}$

Another solution :

$\because \frac{a}{b} = \frac{b}{c}$ $\therefore b^2 = ac$

$\therefore$ L.H.S. $= \frac{4\,a^2 - 3\,ac}{4\,ac - 3\,c^2} = \frac{a\,(4\,a - 3\,c)}{c\,(4\,a - 3\,c)} = \frac{a}{c} =$ R.H.S.

Example 3 If b is the middle proportional between a and c , **prove that :**

1. $\frac{a - b}{a} = \frac{a - c}{a + b}$
2. $ab - c^2 = (b - c)\,(a + b + c)$

Solution $\because$ b is the middle proportional between a and c

$\therefore$ a , b and c are in continued proportion

Let $\frac{a}{b} = \frac{b}{c} = m$

$\therefore b = cm$, $a = cm^2$

1 $\because \frac{a - b}{a} = \frac{cm^2 - cm}{cm^2} = \frac{cm\,(m - 1)}{cm^2} = \frac{m - 1}{m}$ (1)

$, \frac{a - c}{a + b} = \frac{cm^2 - c}{cm^2 + cm} = \frac{c\,(m^2 - 1)}{cm\,(m + 1)} = \frac{c\,(m - 1)\,(m + 1)}{cm\,(m + 1)} = \frac{m - 1}{m}$ (2)

From (1) and (2) , we deduce that :

$\frac{a - b}{a} = \frac{a - c}{a + b}$

2 $\because ab - c^2 = cm^2 \times cm - c^2 = c^2\,m^3 - c^2 = c^2\,(m^3 - 1)$ (1)

$, (b - c)\,(a + b + c) = (cm - c)\,(cm^2 + cm + c)$

$= c\,(m - 1) \times c\,(m^2 + m + 1)$

$= c^2\,(m - 1)\,(m^2 + m + 1) = c^2\,(m^3 - 1)$ (2)

From (1) and (2) , we deduce that : $ab - c^2 = (b - c)\,(a + b + c)$

Try 2 If a , b and c are in continued proportion , ***prove that :*** $\frac{3\,c^2 - 4\,b^2}{3\,b^2 - 4\,a^2} = \frac{c^2}{b^2}$

Generalizing the definition of the continued proportion

The quantities a , b , c , d , .. are in continued proportion if : $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \cdots$

For example:

The numbers 16 , 24 , 36 and 54 are in continued proportion

because : $\frac{16}{24} = \frac{24}{36} = \frac{36}{54}$ $\left(\text{each ratio} = \frac{2}{3}\right)$

Remark

If a , b , c and d are in continued proportion and we assume that : $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$, then :

$\frac{c}{d} = m$ $\therefore c = dm$ (1)

, $\frac{b}{c} = m$ $\therefore b = cm$

Substituting for c from (1) : $\therefore b = (dm)\, m$ $\therefore b = dm^2$ (2)

, $\frac{a}{b} = m$ $\therefore a = bm$

Substituting for b from (2) : $\therefore a = (dm^2)\, m$ $\therefore a = dm^3$

i.e.

If $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$, then $\boxed{c = dm}$, $\boxed{b = dm^2}$ and $\boxed{a = dm^3}$

Example 4

If a , b , c and d are in continued proportion

, **prove that :** $\frac{a+d}{b-c+d} = \frac{a-c}{b-c}$

Solution

Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$ $\therefore c = dm$, $b = dm^2$, $a = dm^3$

$$\therefore \frac{a+d}{b-c+d} = \frac{dm^3+d}{dm^2-dm+d} = \frac{d\,(m^3+1)}{d\,(m^2-m+1)}$$

$$= \frac{(m+1)\,(m^2-m+1)}{m^2-m+1} = m+1 \quad (1)$$

$$, \frac{a-c}{b-c} = \frac{dm^3-dm}{dm^2-dm} = \frac{dm\,(m^2-1)}{dm\,(m-1)} = \frac{(m-1)\,(m+1)}{(m-1)} = m+1 \quad (2)$$

From (1) and (2) , we deduce that : $\frac{a+d}{b-c+d} = \frac{a-c}{b-c}$

TRY by yourself 3

If a , b , c and d are in continued proportion , ***prove that :*** $\frac{a+2b}{b+2c} = \frac{c+a}{d+b}$

Example 5 If the quantities a , 2 b , 3 c and 4 d are in continued proportion , **prove that :** (2 b – 3 c) is the middle proportional between (a – 2 b) and (3 c – 4 d)

Solution

Let $\frac{a}{2b} = \frac{2b}{3c} = \frac{3c}{4d} = m$ $\therefore$ $3c = 4\,dm$, $2b = 4\,dm^2$, $a = 4\,dm^3$

Proving that : (2b – 3c) is the middle proportional between (a – 2b) and (3c – 4d)

means proving that : $(2b - 3c)^2 = (a - 2b)(3c - 4d)$

$\because (2b - 3c)^2 = (4\,dm^2 - 4\,dm)^2$

$= \left(4\,dm\,(m - 1)\right)^2 = 16\,d^2\,m^2\,(m - 1)^2$ (1)

, $(a - 2b)(3c - 4d) = (4\,dm^3 - 4\,dm^2)(4\,dm - 4d)$

$= 4\,dm^2\,(m - 1) \times 4d\,(m - 1) = 16\,d^2m^2\,(m - 1)^2$ (2)

From (1) and (2) , we deduce that : $(2b - 3c)^2 = (a - 2b)(3c - 4d)$

$\therefore$ (2b – 3c) is the middle proportional between (a – 2b) and (3c – 4d)

Another solution :

$\because$ a , 2b , 3c and 4d are in continued proportion.

$\therefore \frac{a}{2b} = \frac{2b}{3c} = \frac{3c}{4d}$

Subtracting the terms of the 2nd ratio from the terms of the 1st ratio

$\therefore \frac{a - 2b}{2b - 3c}$ = one of the given ratios. (1)

Subtracting the terms of the 3rd ratio from the terms of the 2nd ratio

$\therefore \frac{2b - 3c}{3c - 4d}$ = one of the given ratios. (2)

From (1) and (2) , we deduce that : $\frac{a - 2b}{2b - 3c} = \frac{2b - 3c}{3c - 4d}$

$\therefore$ (2b – 3c) is the middle proportional between (a – 2b) and (3c – 4d)

Answers Try by yourself

1. [1] ± 24 [2] 4
2. Prove by yourself [Hint : $a = cm^2$, $b = cm$]
3. Prove by yourself [Hint : $a = dm^3$, $b = dm^2$, $c = dm$]

Unit 4 Direct variation and inverse variation

First The direct variation

Definition

It is said that y varies directly as x and it is written $y \propto x$ if $\boxed{y = m\,x}$

$\boxed{\text{i.e. } \frac{y}{x} = m \text{ , where m is a constant} \neq 0}$

, the relation : $y = m\,x$ is represented graphically by a straight line passing through the origin point $(0 , 0)$

For example:

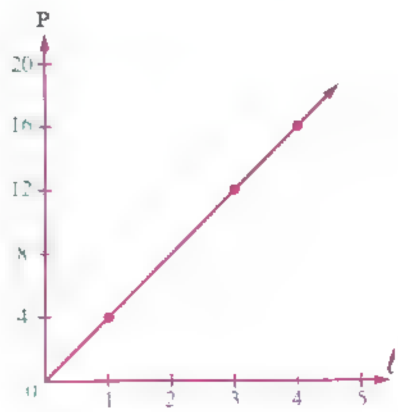
The perimeter of the square (P) is varying directly with its side length (ℓ) and it is written as $P \propto \ell$

Because : $P = 4\,\ell$ or $\frac{P}{\ell} = 4$

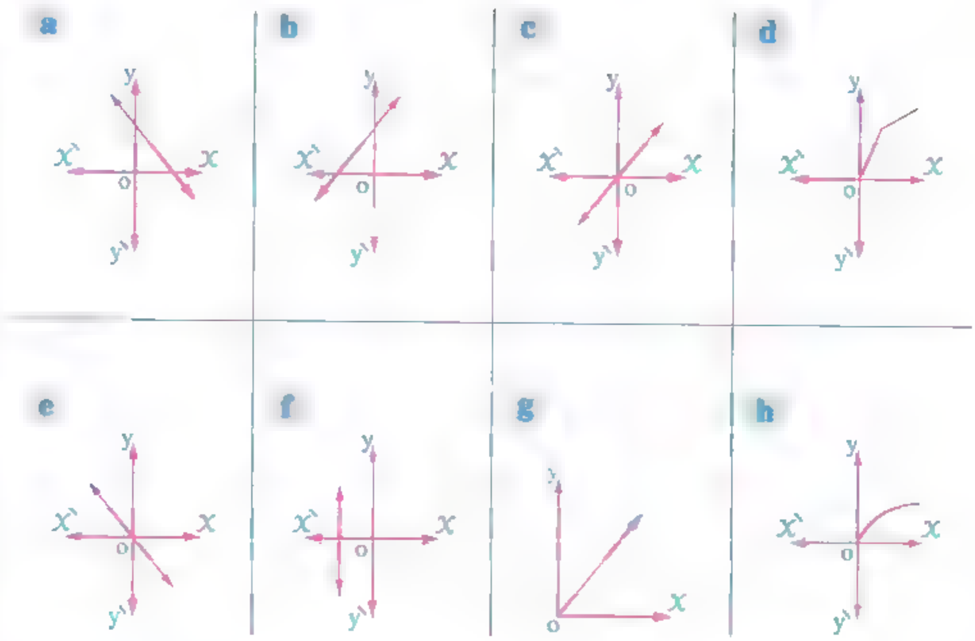
and the following table shows some values of ℓ and the values of P corresponding to them.

Side length (ℓ)	1	3	4
The perimeter (P)	4	12	16

and the opposite figure represents graphically the relation between P and ℓ



Example 1 **Show which of the following graphs represents a direct variation between X and y :**



Solution The graphs which represent a direct variation between X and y are :

c , e and g because in each of them , the straight line passes through the origin point.

Example 2 If $a^2 + 4b^2 = 4ab$, **prove that :** $a \propto b$

Solution To prove that $a \propto b$ we prove that $a = m\,b$ where m is a constant $\neq 0$

$\because a^2 + 4b^2 = 4ab$ $\quad\therefore a^2 - 4ab + 4b^2 = 0$

$\therefore (a - 2b)^2 = 0$ $\quad\therefore a - 2b = 0$

$\therefore \boxed{a = 2b}$ $\quad\therefore a \propto b$

TRY 1 If $\frac{3X - 5y}{3X - 9y} = \frac{1}{2}$ for every values of $X \in \mathbb{R}_+$, $y \in \mathbb{R}_+$, ***prove that :*** $X \propto y$

Property

If $y \propto X$, the variable X took the two values X_1 and X_2 and y took the two values y_1 and y_2 respectively , then : $\boxed{\frac{y_1}{y_2} = \frac{X_1}{X_2}}$

The reason : $\because y \propto x$ then $y = m\,x$ where m is a constant $\neq 0$

at $x = x_1$, $y = y_1$ then $y_1 = m\,x_1$ (1)

, at $x = x_2$, $y = y_2$ then $y_2 = m\,x_2$ (2)

Dividing (1) by (2) : $\therefore \frac{y_1}{y_2} = \frac{m\,x_1}{m\,x_2}$ $\therefore \frac{y_1}{y_2} = \frac{x_1}{x_2}$

Example 3

If $y \propto x$ and $y = 20$ when $x = 7$

, then find the value of y when $x = 14$

WATCH VIDEO

Solution

$\because y \propto x$ $\therefore \frac{y_1}{y_2} = \frac{x_1}{x_2}$

where $y_1 = 20$, $x_1 = 7$, $y_2 = ?$, $x_2 = 14$

$\therefore \frac{20}{y_2} = \frac{7}{14}$ $\therefore y_2 = \frac{20 \times 14}{7} = 40$

Another solution :

$\because y \propto x$ $\therefore y = m\,x$ (m is a constant $\neq 0$)

$\because y = 20$ as $x = 7$ $\therefore 20 = m \times 7$

$\therefore m = \frac{20}{7}$ $\therefore y = \frac{20}{7}\,x$

, when $x = 14$ $\therefore y = \frac{20}{7} \times 14$ $\therefore y = 40$

Example 4

If x and y are two variables where y varies directly as the multiplicative inverse of $\frac{1}{x^3}$, $y = 18$ when $x = 2$

, find the relation between x and y , then find the values of y when $x \in \{0 , 1 , 4\}$

Solution

$\because$ y $\propto$ the multiplicative inverse of $\frac{1}{x^3}$

$\therefore y \propto x^3$ $\therefore y = m\,x^3$ where m is a constant $\neq 0$

$\because y = 18$ as $x = 2$ $\therefore 18 = m \times (2)^3$ $\therefore m = \frac{18}{8} = \frac{9}{4}$

$\therefore y = \frac{9}{4}\,x^3$ This is the relation between x and y

as $x = 0$ $\therefore y = \frac{9}{4} \times 0 = 0$

as $x = 1$ $\therefore y = \frac{9}{4} \times 1 = \frac{9}{4} = 2\frac{1}{4}$

as $x = 4$ $\therefore y = \frac{9}{4} \times 64 = 144$

Example 5 If (V) denotes the volume of a right circular cone, its height is constant and if (V) varies directly as the square of radius length of the base of the cone (r) and the volume of the cone was 477 cm^3, when the radius length of its base = 15 cm.

Find the volume of the cone when the base radius length = 10 cm.

Solution $\because V \propto r^2$ $\therefore \frac{V_1}{V_2} = \frac{r_1^{\,2}}{r_2^{\,2}}$ $\therefore \frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^2$

where $V_1 = 477$ cm^3, $r_1 = 15$ cm., $V_2 = ?$, $r_2 = 10$ cm.

$\therefore \frac{477}{V_2} = \left(\frac{15}{10}\right)^2 = \frac{9}{4}$ $\therefore V_2 = \frac{477 \times 4}{9} = 212$ cm^3

TRY 2 If $x \propto y$ and $y = 2$ when $x = 40$, find the value of x when $y = 3$

Second The inverse variation

WATCH VIDEO

Definition

It is said that y varies inversely as x and it is written $y \propto \frac{1}{x}$ if $y = \frac{m}{x}$

i.e. $xy = m$, where m is a constant $\neq 0$

For example:

The uniform velocity (v) varies inversely as time (t) when the covered distance (d) is constant

Because : $v = \frac{d}{t}$ or $vt = d$

, in this case we say that the velocity varies directly as the multiplicative inverse of time and it is written as : $v \propto \frac{1}{t}$

Example 6 If $a^2 b^4 - 10\, ab^2 = -25$, **prove that :** a varies inversely as b^2

Solution To prove that a varies inversely as b^2 we prove that : $ab^2 = m$ where $m \neq 0$

$\because a^2 b^4 - 10\, ab^2 = -25$ $\therefore a^2 b^4 - 10\, ab^2 + 25 = 0$

$\therefore (ab^2 - 5)^2 = 0$ $\therefore ab^2 - 5 = 0$

$\therefore ab^2 = 5$ $\therefore$ a varies inversely as b^2

TRY 3 If $a^2 b^2 + 49 = 14\,ab$, ***prove that*** : $a \propto \frac{1}{b}$

Property

If $y \propto \frac{1}{x}$, the variable x took the two values x_1 and x_2 and as a result for that y took the two values y_1 and y_2 respectively, then : $\boxed{\frac{y_1}{y_2} = \frac{x_2}{x_1}}$

The reason : $\because y \propto \frac{1}{x}$, then $y = \frac{m}{x}$ where m is a constant $\neq 0$

at $x = x_1$, $y = y_1$, then $y_1 = \frac{m}{x_1}$ (1)

, at $x = x_2$, $y = y_2$, then $y_2 = \frac{m}{x_2}$ (2)

Dividing (1) by (2) :

$\therefore \frac{y_1}{y_2} = \frac{m}{x_1} \div \frac{m}{x_2} = \frac{\cancel{m}}{x_1} \times \frac{x_2}{\cancel{m}} = \frac{x_2}{x_1}$

Example 7 If the length of a rectangle (ℓ) varies inversely as its width (w), when the area is constant and $\ell = 12$ cm. as w = 8 cm., **find :** ℓ when w = 3 cm.

Solution

$\because \ell \propto \frac{1}{w}$

$\therefore \frac{\ell_1}{\ell_2} = \frac{w_2}{w_1}$, where $\ell_1 = 12$ cm., $w_1 = 8$ cm., $\ell_2 = ?$, $w_2 = 3$ cm.

$\therefore \frac{12}{\ell_2} = \frac{3}{8}$ $\qquad \therefore \ell_2 = \frac{8 \times 12}{3} = 32$ cm.

Another solution :

$\because \ell \propto \frac{1}{w}$

$\therefore \ell w = m$, where m is a constant $\neq 0$

$\because \ell = 12$ cm. as w = 8 cm. $\qquad \therefore m = 12 \times 8 = 96$ $\qquad \therefore \ell w = 96$

When w = 3 cm. $\qquad \therefore 3\ell = 96$ $\qquad \therefore \ell = \frac{96}{3} = 32$ cm.

Example 8 If y varies inversely as X and $y = 6$ as $X = 2.5$, find the relation between X and y , then find the value of y if $X = 5$

Solution

$\because y \propto \frac{1}{X}$ $\quad\therefore X y = m$, where m is a constant $\neq 0$

$\because y = 6$ as $X = 2.5$ $\quad\therefore m = 6 \times 2.5 = 15$

$\therefore$ The relation between X and y is $\boxed{X y = 15}$

, at $X = 5$ $\quad\therefore 5 y = 15$ $\quad\therefore y = \frac{15}{5} = 3$

Example 9 If $y = 1 + b$ where b varies inversely as X^2 and $y = 17$ as $X = \frac{1}{2}$, find the relation between X and y , then find the value of y when $X = 2$

Solution

$\because b \propto \frac{1}{X^2}$ $\quad\therefore b = \frac{m}{X^2}$, where m is a constant $\neq 0$ $\quad\therefore y = 1 + \frac{m}{X^2}$

, $\because y = 17$ as $X = \frac{1}{2}$ $\quad\therefore 17 = 1 + \frac{m}{\left(\frac{1}{2}\right)^2}$ $\quad\therefore 17 = 1 + \frac{m}{\frac{1}{4}}$

Subtracting 1 from both sides : $\therefore 16 = \frac{m}{\frac{1}{4}}$

$\therefore m = 16 \times \frac{1}{4} = 4$ $\quad\therefore \boxed{y = 1 + \frac{4}{X^2}}$

at $X = 2$: $\therefore y = 1 + \frac{4}{2^2} = 1 + \frac{4}{4} = 2$

TRY 4 If y varies inversely as X and $y = 2$ as $X = 6$, calculate the value of y as $X = 1$

UNIT

3 Statistics

Lessons of the unit :

1. Collecting data.
2. Dispersion.

Unit Objectives :

By the end of this unit, student should be able to :

- recognize the different resources of collecting data.
- recognize the methods of collecting data , and the advantages and the disadvantages of each method.
- recognize the concept of the sample.
- recognize the methods of selection of samples.
- recognize the types of the samples.
- choose the best method to select a sample for studying a certain phenomenon.
- use the calculator and the computer for generating random numbers used in the samples.
- recognize the dispersion measurements.
- recognize the advantages and the disadvantages of the range as one of the dispersion measurements.
- calculate the range of a set of individuals.
- calculate the standard deviation of a set of individuals.
- calculate the standard deviation of a simple frequency distribution.
- calculate the standard deviation of a frequency distribution of sets.
- use the calculator to calculate the standard deviation.

1 Collecting data

- The statistical investigator collects , classifies , represents and analyses data in purpose of deducing some results on which he depends in making the suitable decisions.
- The more data is accurate , the more the decisions will be true and reliable.
- Collecting data in such scientific methods will lead to get accurate outcomes when doing operations of statistical inference and proper decision making.
- Collecting statistical data demands knowing the resources of collecting it and determining the methods of collecting it.

Resources of collecting data is classified into

1 Primary resources (field resources) :

These are the resources from which we get data directly.

2 Secondary resources (historical resources) :

These are the resources from which we get data that previously collected and registered by some authorities , formal organisations or persons.

There are some examples for each resource with representing the advantages and the disadvantages of each one :

	1 Primary resources	2 Secondary resources
Examples :	• Personal interview. • Questionnaires (survey). • Observing and measuring.	• Central agency for public mobilization and statistics. • Mass-media and internet. • Documents of data of employees in a company.
Advantages :	Accuracy.	Saves time , effort and money.
Disadvantages :	It needs more time, effort and money besides it requires more investigators in large societies.	It is less accurate.

Methods of collecting data

- The method of collecting data depends on the aim of collecting these data and it also depends on the size of the statistical society under study.
- The statistical society is defined as all individuals which have general common characters.

For example:

- The workers in a factory represent a statistical society , whose individual is the worker.
- The pupils of a school represent a statistical society , whose individual is the pupil.

We will show two methods of collecting data :

1 Method of mass population :

It is based on collecting the data related to the phenomenon under study from all individuals of the statistical society.

2 Method of samples :

It is based on collecting data related to the phenomenon under study from a representative sample of the society , and applying the research on it , then generalizing the results on the whole society.

There are some examples for each method with representing the advantages and the disadvantages of each one :

	1 Method of mass population	**2 Method of samples**
Examples :	• Elections. • Census. • Setting up a data base of all employees in an organization.	• A sample of a patient's blood to make some clinical check up. • A sample of some products of a factory to find out if it matches the standard specifications.
Advantages :	• Accuracy. • Inclusiveness. • Neutrality. • Representing all the society individuals.	• Saving time , effort and money. • It is the only method for collecting data about large unlimited societies such as the search on contents of the desert sand. • It is the only method for collecting data about some limited societies in which mass population method leads to a great loss in it such as checking a sample of a patient's blood because of checking the whole blood of the patient leads to death.
Disadvantages :	• Sometimes it needs long time , great effort and a great cost.	• The results sometimes are not accurate specially if the sample doesn't represent the statistical society authentically , in this case the sample is called a biased sample.

In the following , we will explain the concept of the sample and its types and how we select it :

The concept of the sample

It is a small part from a large society that looks like the society and represents it well.

How can we select the sample ?

The types of samples according to the method of selecting it

- **The biased selection**
  not a random sample or deliberate sample
- **The randomly selection**
  random sample
  - **Simple** random sample (in case of the homogeneous societies)
  - **Layer** random sample (in case of the heterogeneous societies)

At the following , we explain each type in details :

First The biased selection (samples are not randomly selected)

- It means that we select the sample in a way to satisfy the objectives of the research. This is called **the deliberate sample**.

For example:

If we want to know how the students understood a lesson in algebra , we must analyze the outcomes of the test by considering the outcomes of a group of students studying the same topic without the other students , this is not a random selection.

- The biased selection is not representing the statistical society.

Second Random selection (random samples)

It means to select a sample such that every member of the population has an equal chance of having selected.

The following are the most important types of the random samples which are :

1 Simple random sample. 2 Layer random sample.

1 Simple random sample

- It is used for the homogeneous societies which are not naturally divided into groups or classes.
- It is selected by two ways according to the number of individuals of statistical society as the following.

A The first method : If the size of the society is small :

- **This method will be carried out as follows :**

1 Each individual of the society takes a number , this number is written on a card such that all cards are identical.
i.e. There is no difference in colour or size.

2 Each card is folded well such that the number does not appear , then they are put in a box and mixed well.

3 We select the sample by drawing one card from the box blindly , then we turned well the cards and select the next card , and so on till we reach the required number of the sample.

This method is suitable if , for example , we select a sample of 10 workers from a factory that has 50 workers.

B The second method : If the size of the society is large :

In this method , every individual of the society has a number , then we select the sample using the property of the random number in the scientific calculator as in the opposite picture.

- We press the following keys respectively from the left :

ON → SHIFT → Ran# (.) → =

then a decimal will appear on the display in the field from 0.000 to 0.999

- If we get a 1-decimal digit , add two zeroes to make it a part of 1000

For example: $(0.2 \to 0.200)$

- If we get a 2-decimal digit , add one zero to make it a part of 1000

For example: (0.64 → 0.640) and so on.

- Take the number neglecting the decimal point , then the individual who has this number is selected as a member of the sample , then repeat pressing on = to get more numbers.
- We will ignore the numbers which are greater than the number of society under study.
- And we ignore the repeated numbers which we selected before.
- The percentage 10% of the number of the society is suitable for holding the survey.

This method is more suitable for selecting a sample of 25 students from a school that has 900 students.

2 Layer random sample

- It is used in the statistical societies which are heterogeneous or made up of qualitative sets that are different in characteristics.
- In this case , we cannot select the sample by the simple random sample method because the sample will not represent the society well because it will not represent all the classes of the society.

Therefore we have to follow the following steps :

1. We divide the society into homogeneous sets according to the characteristics forming it , each set is called **a layer**.
2. We find the number of individuals of each layer , then we find its ratio referring to the total number of the society.
3. To form a sample , we select from each layer a certain number of individuals such that the ratio that represents each layer in the sample is the same ratio of the layer in the whole society , and this by using the following law :

The number of individuals of the layer in the sample

$$= \frac{\text{the total number of individuals in the layer}}{\text{the total number of individuals in the society}} \times \text{the number of individuals of the sample}$$

«approximated the result to the nearest unit»

For example:

When we want to study the educational level of the students of a school of 500 students (boys and girls) and if the ratio between the number of boys to the number of girls is 1 : 4 and we want to select a sample formed from 50 students , we should select 10 students from boys and 40 students from girls , for the sample representing all the society well.

Example 1 A factory has 300 workers. The people in charge of the monthly magazine of this factory want to develop this magazine by doing a survey of a sample representing 10% of the total number of the workers in this factory. Show how the selection of this sample can be carried out using the calculator.

Solution The number of workers in the factory = 300 workers.

$\therefore$ The number of the random sample $= \frac{10}{100} \times 300 = 30$ workers.

Then we want to select 30 workers to hold this survey.

The selection operation can be carried out as follows :

1. Each worker in the factory is given a number from 1 to 300
2. Use the calculator to select 30 numbers randomly , such that these numbers are included between 0 and 301 and the number that is above 300 should be ignored.

For example:

By pressing the keys ON → SHIFT → Ran# → = successively from left to right.

- If we get the decimal 0.049 , then the number of the selected person is 49
- If we get the decimal 0.132 , then the number of the selected person is 132
- If we get the decimal 0.12 , then the number of the selected person is 120
- If we get the decimal 0.453 , it must be ignored because 453 is above 300 and so on till we get 30 numbers.
- Assuming that the calculator gave us the shown numbers in the opposite table , then the workers who carry these numbers are the selected sample to carry out this survey.

49	132	120	141	249	272
254	256	4	213	74	198
131	2	156	47	172	13
8	3	85	82	9	38
41	14	34	279	118	103

Example 2 A factory produced 200 TV sets from the type A , 300 TV sets from the type B and 500 TV sets from the type C , if we want to select a layer sample formed from 50 TV sets such that it represents all the types to examine them.

Calculate the number of TV sets which should be selected from each kind.

Solution

- The total number of TV sets = 200 + 300 + 500 = 1000 TV sets.
- The number of TV sets of the type A in the sample = $\frac{200}{1000} \times 50 = 10$ TV sets.
- The number of TV sets of the type B in the sample = $\frac{300}{1000} \times 50 = 15$ TV sets.
- The number of TV sets of the type C in the sample = $\frac{500}{1000} \times 50 = 25$ TV sets.

TRY by yourself

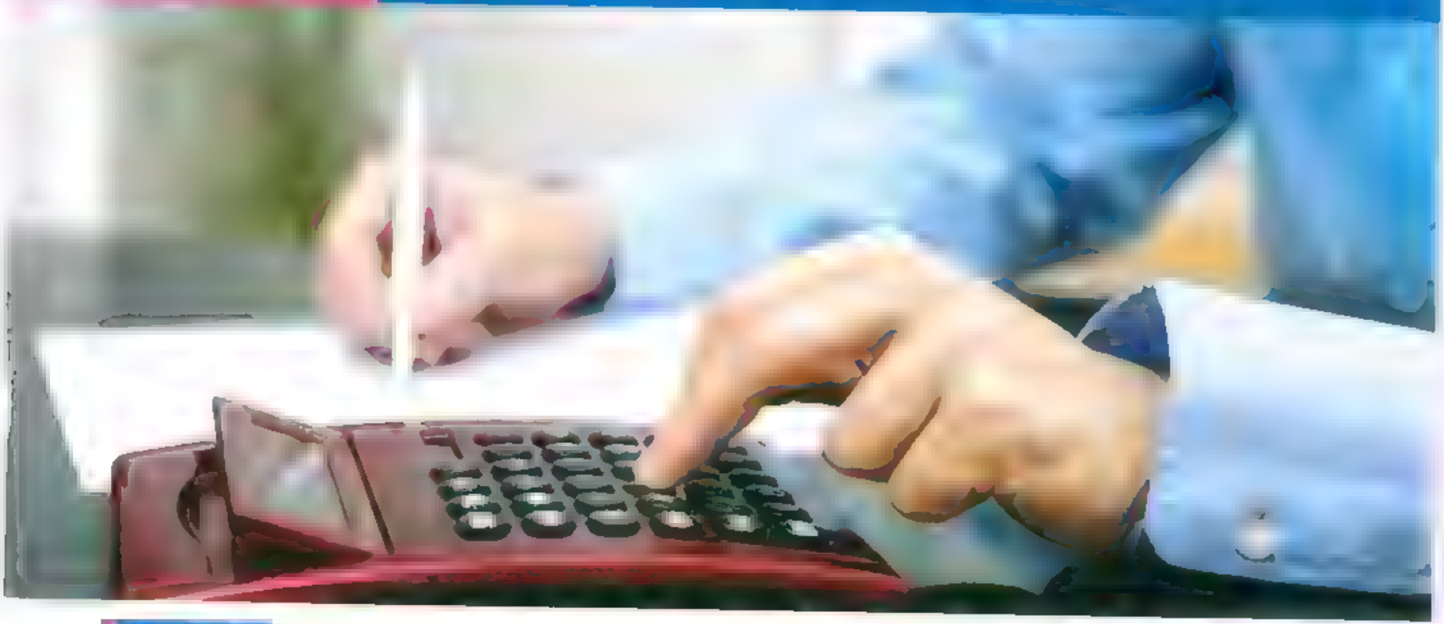
A school has 300 male students and 500 female students wanted to do a survey on a sample of 24 male and female students representing each layer according to its size. Calculate the number of students of each layer in the sample.

Answers of try by yourself

The number of male students in the sample is 9

The number of female students in the sample is 15

2 Dispersion



Prelude

- You studied before some of statistical measures which were known as "**measures of central tendency**" as the mean , the median and the mode.
- And we know that each of them describe the frequency distributions and the statistical data by identifying one numerical value , where the left values centralize about it.
- But in some cases the measures of central tendency are not enough to describe clearly the data.

To explain that , let's study the following case :

Two sets of 5 students each , an exam of maximum mark 50 marks is given for each sets , the marks of the students were as follows :

The set A : 29 , 26 , 35 , 35 , 35

The set B : 8 , 35 , 49 , 35 , 33

At calculating the mean , the median and the mode of the marks of the students in each set alone , we find the shown results in the following table :

	mean	median	mode
Set A	32	35	35
Set B	32	35	35

Remember that

- The mean $= \dfrac{\text{the sum of values}}{\text{the number of this values}}$
- The median of a set of values is the value which lies at the middle of the set of values after ordering them.
- The mode of a set of values is the most common value in the set.

- In the previous case , the two sets are different , and in spite of that , we found that they have the same mean , median and mode , which don't mean that these sets are necessarily homogeneous.
- Therefore , the measures of central tendency only are unable to describe all the characteristics a set of frequency distributions and statistical data.

  So we need besides the measures of central tendency that depends on determining one value that the other data centralize around it, another kind of measures which depends on determining a degree of convergence or divergence of data.

For example:

In the previous example , the marks of the set A are convergent because their values are included between 26 and 35 marks while the marks of the set B are divergent because their values are included between 8 and 49 marks.

i.e. The marks of the set B are more divergent than the marks of the set A

- These new measures are called the measures of dispersion. We will study each of the range and the standard deviation.

Dispersion of a set of values

It means the divergence or the differences among its values.

- The dispersion is small if the difference among the values is little while the dispersion is great if the difference among the values is great , the dispersion is zero if all the values are equal.

i.e. The dispersion of a set of values is a measure of the degree to which these values spread out and that expresses how much the sets are homogeneous.

Dispersion measurements

1 The range (the simplest measure of dispersion) :

It is the difference between the greatest value and the smallest value in the set.

The range = the greatest value – the smallest value

For example:

- If the values of set A are 60 , 58 , 62 , 61 and 59

  $\therefore$ The range = 62 – 58 = 4
- If the values of set B are 72 , 78 , 46 , 65 and 39

  $\therefore$ The range = 78 – 39 = 39

So the set B is more divergent than the set A

The advantages of range :

- It is an easy and simple method that gives a quick idea about the divergence or convergence of the values.
- It is considered as the simplest and the easiest method to measure dispersion.

The disadvantages of range :

- It does not reflect the influence of all values because its measure depends on the greatest and smallest values only , therefore it does not give a full idea of the dispersion of the set of values.
- It is influenced greatly by the outlier.

For example:

- The range of the set of values : 21 , 22 , 61 , 24 and 26 is (61 – 21 = 40)
- While if we ignore the value 61 from the set , then the range becomes (26 – 21 = 5)

i.e. The range equals $\frac{1}{8}$ the previous range , therefore the range is an approximated measure and we cannot depend on it.

2 Standard deviation :

It is the most important , common and accurate measure of dispersion. We can calculate it by calculating the positive square root of the average of squares of deviations of the values from their mean. It is denoted by σ and it is read as (sigma).

First Calculating the standard deviation of a set of values:

The standard deviation $\sigma = \sqrt{\dfrac{\Sigma (x - \overline{x})^2}{n}}$

Where :
x denotes a value of the values ,
$\overline{x}$ denotes the mean of the values and it is read as x bar ,
n denotes the number of values ,
Σ denotes the summation operation.

Example 1 Calculate the standard deviation of the values : 8 , 9 , 7 , 6 and 5

Solution

1 We find the mean of the values $(\overline{x}) = \dfrac{\Sigma x}{n} = \dfrac{8+9+7+6+5}{5} = 7$

2 We form the opposite table :

x	$x - \overline{x}$	$(x - \overline{x})^2$
8	8 – 7 = 1	1
9	9 – 7 = 2	4
7	7 – 7 = 0	0
6	6 – 7 = – 1	1
5	5 – 7 = – 2	4
Total		**10**

3 We calculate the standard deviation as follows :

The standard deviation $(\sigma) = \sqrt{\dfrac{\Sigma (x - \overline{x})^2}{n}} = \sqrt{\dfrac{10}{5}} = \sqrt{2} \simeq 1.41$

TRY by yourself 1

If 25 , 24 , 25 , 30 , 28 and 30 represent the marks of one of the pupils in examination of algebra in different months , ***find :***

1 The mean.
2 The standard deviation.

Second : Calculating the standard deviation of a frequency distribution :

For any frequency distribution : The standard deviation $\sigma = \sqrt{\dfrac{\Sigma (X - \overline{X})^2 k}{\Sigma k}}$

Where :

X represents the value or the centre of the set ,

k represents the frequence of the value or the set ,

Σ k is the sum of frequences and $\overline{X}$ (the mean) $= \dfrac{\Sigma (X \times k)}{\Sigma k}$

A : Calculating the standard deviation of a simple frequency distribution :

Example 2 **The following table shows the distribution of ages of 20 persons in years :**

The age	15	20	22	23	25	30	Total
Number of persons	2	3	5	5	1	4	**20**

Find the standard deviation of the ages.

Solution

1 We find the mean of the ages ($\overline{X}$) by using the following table :

The age (X)	Number of persons (k)	$X \times$ k
15	2	30
20	3	60
22	5	110
23	5	115
25	1	25
30	4	120
Total	**20**	**460**

The mean $(\overline{X}) = \dfrac{\Sigma (X \times k)}{\Sigma k} = \dfrac{460}{20} = 23$ years.

2 We form the following table :

x	k	$x - \overline{x}$	$(x - \overline{x})^2$	$(x - \overline{x})^2 \times k$
15	2	$15 - 23 = -8$	64	128
20	3	$20 - 23 = -3$	9	27
22	5	$22 - 23 = -1$	1	5
23	5	$23 - 23 = 0$	0	0
25	1	$25 - 23 = 2$	4	4
30	4	$30 - 23 = 7$	49	196
Total	**20**			**360**

3 We calculate the standard deviation as follows :

Standard deviation $(\sigma) = \sqrt{\dfrac{\sum (x - \overline{x})^2 \times k}{\sum k}} = \sqrt{\dfrac{360}{20}} = \sqrt{18} \simeq 4.24$ years.

TRY by yourself 2

The following frequency distribution shows the number of days of absentees in a class :

Number of absence days	0	1	2	3	4	Total
Number of pupils	5	7	7	5	6	**30**

Calculate the mean and the standard deviation for the number of days of absence.

B Calculating the standard deviation of a frequency distribution of sets :

Example 3 **The following is the frequency distribution of weekly incentives of 100 workers in a factory :**

Incentives in pounds	35 –	45 –	55 –	65 –	75 –	85 –
Number of workers	10	14	20	28	20	8

Find the standard deviation of this distribution.

Solution 1 We find the mean $(\overline{X})$ by using the following table :

Remember that

The centre of the set = $\frac{\text{lower limit + upper limit}}{2}$

Sets	Centres of sets (X)	Frequence (k)	$X \times k$
35 –	40	10	400
45 –	50	14	700
55 –	60	20	1200
65 –	70	28	1960
75 –	80	20	1600
85 –	90	8	720
Total		**100**	**6580**

$\therefore$ The mean $(\overline{X}) = \frac{\Sigma (X \times k)}{\Sigma k} = \frac{6580}{100} = 65.8$ pounds.

2 We form the following table :

X	k	$X - \overline{X}$	$(X - \overline{X})^2$	$(X - \overline{X})^2 \times k$
40	10	$40 - 65.8 = -25.8$	665.64	6656.4
50	14	$50 - 65.8 = -15.8$	249.64	3494.96
60	20	$60 - 65.8 = -5.8$	33.64	672.8
70	28	$70 - 65.8 = 4.2$	17.64	493.92
80	20	$80 - 65.8 = 14.2$	201.64	4032.8
90	8	$90 - 65.8 = 24.2$	585.64	4685.12
Total	**100**			**20036**

3 We calculate the standard deviation as follows :

Standard deviation $(\sigma) = \sqrt{\frac{\Sigma (X - \overline{X})^2 \times k}{\Sigma k}} = \sqrt{\frac{20036}{100}} \simeq 14.15$ pounds.

UNIT 3

! Remarks

- The standard deviation is influenced by all values not by the two terminal values only (the smallest and the greatest value) as the range , therefore it represents the dispersion better than the range.
- The standard deviation has the same measuring units of the original data.
- The values which are more homogeneous have less dispersion and their standard deviation is small.
- If the standard deviation equals zero that means the all values are equal , it is the perfect homogeneous case (the vanished dispersion)

TRY by yourself 3

For the following frequency distribution , calculate :

[1] The mean. [2] The standard deviation.

Sets	1 –	3 –	5 –	7 –	9 – 11
Frequency	7	3	5	3	2

Answers of try by yourself

1 [1] 27 marks [2] 2.45 marks (approximately)

2 The mean ($\overline{X}$) = 2 days

The standard deviation (σ) = 1.37 days

3 [1] 5 [2] 2.72 (approximately)

Using the calculator to calculate the standard deviation :

- We can use the calculator CASIO ($f\,X - 82$ ES , $f\,X - 85$ ES , $f\,X - 500$ ES , $f\,X - 95$ ES Plus , $f\,X - 991$ ES Plus) to calculate the standard deviation.
- The following steps show how to solve the previous example (example 3) using the calculator :
- We will use the calculator ($f\,X - 95$ ES Plus)

Step (1)

before inserting the data of the previous example, we should set the calculator system by pressing the following keys from left :

Then the screen will appear as in the opposite figure.

Step (2)

- we insert the values (X) in the case of simple frequency distribution or the centres of sets (X) in the case of frequency distribution of sets in the first column (X)
- With respect to the previous example :

We insert the centres of sets :

40 , 50 , 60 , 70 , 80 and 90 by pressing the following keys from left as follows :

Then the screen will appear as in the opposite figure.

Step (3)

Use the key to move to the second column (FREQ), then insert frequencies

10 , 14 , 20 , 28 , 20 and 8 by pressing the following keys from left as follows :

1 0 = 1 4 = 2 0 = 2 8 = 2 0 = 8 = AC

Thus we insert the data of the previous example on the calculator.

Step (4)

For finding the value of the standard deviation , we press the following keys from left :

SHIFT 1 4 (VAR) 3 (σx) =

Then the screen will appear as in the opposite figure.

$\therefore$ Standard deviation $\sigma \simeq 14.15$

Second

Trigonometry and Geometry

Pre-requirements

In this part , we present same important topics you have to study and remember before studying trigonometry and geometry this year. These topics are :

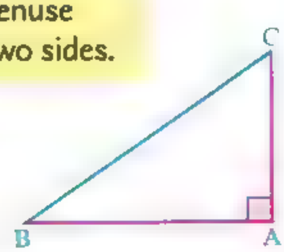
- Pythagoras' theorem.
- Converse of Pythagoras' theorem.
- Projections.
- Euclidean theorem.
- Classifying triangles according to their angles

Pythagoras' theorem

The sum of areas of the squares on the sides of the right angle of a right-angled triangle is the same as the area of the square on the hypotenuse.

We can also state this theorem as follows :

In a right-angled triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.



i.e. If ABC is a right-angled triangle at A , then :

$(BC)^2 = (AB)^2 + (AC)^2$

- **From the previous relation , we can deduce the following two relations :**

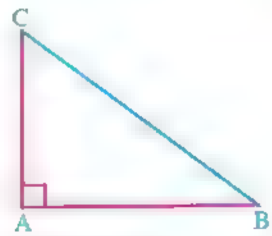
$(AB)^2 = (BC)^2 - (AC)^2$, $(AC)^2 = (BC)^2 - (AB)^2$

Converse of Pythagoras' theorem

In a triangle , if the sum of the areas of the two squares on two sides is equal to the area of the square on the third side , then the angle opposite to this side is a right angle.

i.e. If ABC is a triangle in which $(AB)^2 + (AC)^2 = (BC)^2$

, then $m(\angle A) = 90°$



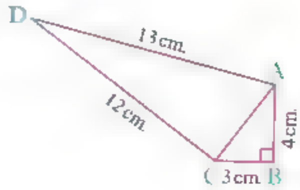
We can also state this theorem as follows :

In a triangle , if the square of the length of a side is equal to the sum of the squares of the lengths of the other two sides , then the angle opposite to this side is a right angle.

Corollary

In Δ ABC , if $\overline{BC}$ is the longest side and $(BC)^2 \neq (AB)^2 + (AC)^2$, then m ($\angle$ A) $\neq 90°$ and the triangle is not right-angled.

Example **In the opposite figure :**



ABCD is a quadrilateral in which :

m ($\angle$ B) = 90° , AB = 4 cm. , BC = 3 cm.

, CD = 12 cm. and DA = 13 cm.

Prove that : m ($\angle$ ACD) = 90°

Solution $\because$ ABC is a triangle in which : m ($\angle$ B) = 90°

$\therefore (AC)^2 = (AB)^2 + (BC)^2$ (Pythagoras' theorem)

$\therefore (AC)^2 = 16 + 9 = 25$ cm² $\therefore$ AC = 5 cm.

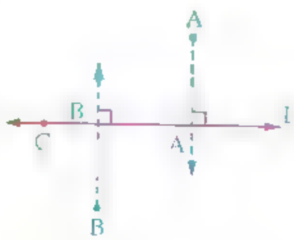
In Δ ACD : $\because (AD)^2 = (13)^2 = 169$ cm² , $(CD)^2 = (12)^2 = 144$ cm² , $(AC)^2 = (5)^2 = 25$ cm² $\therefore (AD)^2 = (AC)^2 + (CD)^2$

$\therefore$ m ($\angle$ ACD) = 90° (Converse of Pythagoras' theorem) (Q.E.D.)

Projections

1 The projection of a point on a straight line

In the opposite figure :



- The point $\grave{A}$ is the position of the perpendicular segment drawn from A to the straight line L and it is called

  the projection of the point A on the straight line L

- Also the point $\grave{B}$ is the position of the perpendicular segment drawn from B to the straight line L and it is called the projection of the point B on the straight line L

Special Case :

If the point C $\in$ L , then its projection on the straight line L is the same point C

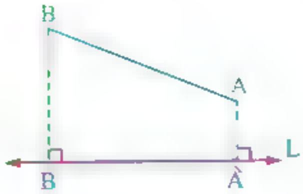
Generally

1. The projection of a point on a given straight line is the point of intersection of the perpendicular segment from this point to the straight line.
2. If the point lies on a given straight line, them its projection on it is the same point.

2 The projection of a line segment on a straight line

In the opposite figure :

The line segment $\overline{A'B'}$ is the projection of the line segment $\overline{AB}$ on the straight line L

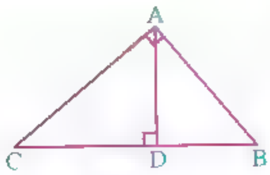


Generally

The projection of a line segment on a given straight line is the line segment whose two endpoints are the projections of the two endpoints of the main line segment on this straight line.

For example:

In the opposite figure :



If Δ ABC is right-angled at A and $\overline{AD} \perp \overline{BC}$, then :

- The projection of $\overline{AB}$ on $\overleftrightarrow{BC}$ is $\overline{DB}$
- The projection of $\overline{AC}$ on $\overleftrightarrow{BC}$ is $\overline{DC}$
- The projection of $\overline{BC}$ on $\overleftrightarrow{AC}$ is $\overline{AC}$
- The projection of $\overline{BC}$ on $\overleftrightarrow{AB}$ is $\overline{BA}$
- The projection of $\overline{AC}$ on $\overleftrightarrow{AD}$ is $\overline{AD}$
- The projection of $\overline{AD}$ on $\overleftrightarrow{BC}$ is the point D
- The projection of $\overline{AB}$ on $\overleftrightarrow{AD}$ is $\overline{AD}$

and notice that :

The length of the projection of a line segment on a given straight line $\leq$ the length of the line segment.

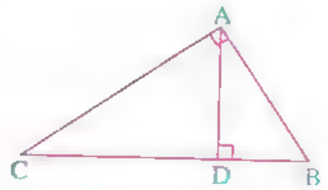
Euclidean theorem

In the right-angled triangle, the area of the square on a side of the right angle is equal to the area of the rectangle whose dimensions are the length of the projection of this side on the hypotenuse and the length of the hypotenuse.

i.e. If Δ ABC is right-angled at A , D $\in \overline{BC}$ where $\overline{AD} \perp \overline{BC}$

, then $(AB)^2 = BD \times BC$

$(AC)^2 = CD \times CB$



Notice that :

- BD is the length of the projection of $\overline{AB}$ on $\overleftrightarrow{BC}$
- CD is the length of the projection of $\overline{AC}$ on $\overleftrightarrow{BC}$

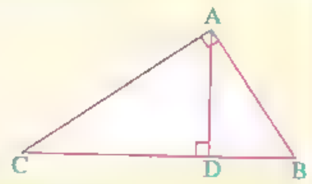
Corollary

If Δ ABC is right-angled at A , D $\in \overline{BC}$ such that : $\overline{AD} \perp \overline{BC}$, then $(AD)^2 = BD \times DC$

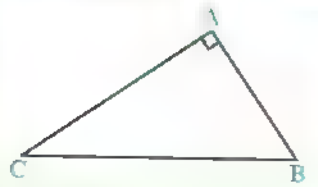
Remark

If Δ ABC is right-angled at A and D $\in \overline{BC}$

such that $\overline{AD} \perp \overline{BC}$, then $AD = \frac{AB \times AC}{BC}$



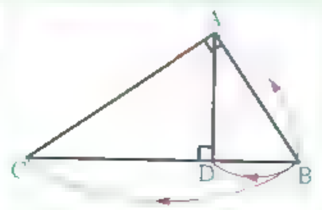
The following is a summary of the relations of Pythagoras' theorem and Euclidean theorem :



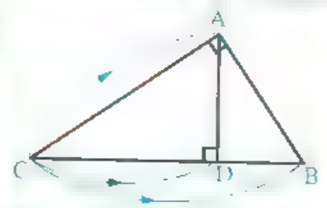
$(BC)^2 = (AB)^2 + (AC)^2$

$(AB)^2 = (BC)^2 - (AC)^2$

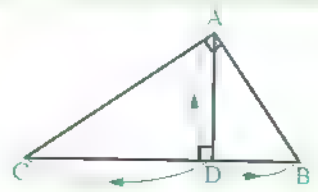
$(AC)^2 = (BC)^2 - (AB)^2$



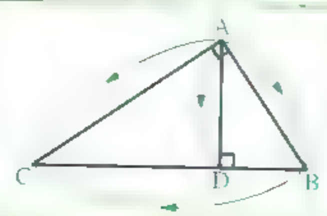
$(BA)^2 = BD \times BC$



$(CA)^2 = CD \times CB$



$(DA)^2 = DB \times DC$

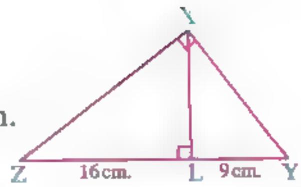


$AD = \frac{AB \times AC}{BC}$

Example **In the opposite figure :**

XYZ is a right-angled triangle at X , $\overline{XL} \perp \overline{YZ}$

such that : $L \in \overline{YZ}$, YL = 9 cm. and LZ = 16 cm.



Find : 1 The length of $\overline{XY}$

2 The length of $\overline{XZ}$

3 The length of $\overline{XL}$

Solution $\because$ Δ XYZ is right-angled at X , $\overline{XL} \perp \overline{YZ}$

$\therefore (XY)^2 = YL \times YZ$ (Euclidean theorem)

$\therefore (XY)^2 = 9 \times 25 = 225$

$\therefore$ XY = 15 cm. (First req.)

Similarly : $(XZ)^2 = ZL \times ZY$ (Euclidean theorem)

$\therefore (XZ)^2 = 16 \times 25 = 400$

$\therefore$ XZ = 20 cm. (Second req.)

, $\because (XL)^2 = LY \times LZ$ (Corollary)

$\therefore (XL)^2 = 9 \times 16 = 144$

$\therefore$ XL = 12 cm. (Third req.)

Classifying triangles according to their angles

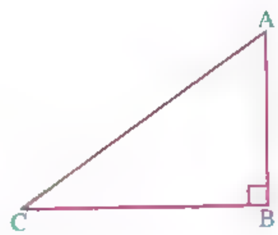
To determine the type of the triangle according to its angles in case of knowing the lengths of its three sides , you should compare between the square of the length of the longest side of the triangle and the sum of squares of the lengths of the other two sides , then this comparison will determine the type of the triangle as follows :

- Let ABC be a triangle in which $\overline{AC}$ is the longest side , then :

1 If $(AC)^2 = (AB)^2 + (BC)^2$

, then m ($\angle$ ABC) = 90°

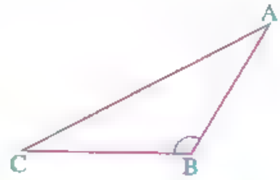
and ABC is a right-angled triangle at B



i.e. If the square of the length of the longest side equals the sum of the squares of the lengths of the other two sides , then the triangle is right-angled.

2 If $(AC)^2 > (AB)^2 + (BC)^2$, then $m(\angle ABC) > 90°$

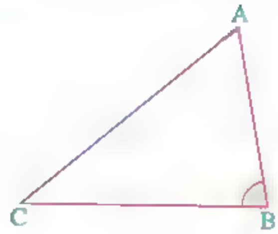
and ABC is an obtuse-angled triangle at B



i.e. If the square of the length of the longest side is greater than the sum of the squares of the lengths of the other two sides , then the triangle is obtuse angled.

3 If $(AC)^2 < (AB)^2 + (BC)^2$, then $m(\angle ABC) < 90°$

and ABC is an acute-angled triangle.



i.e. If the square of the length of the longest side is less than the sum of the squares of the lengths of the other two sides , then the triangle is acute angled.

Example **In each of the following, determine the type of the triangle ABC according to its angles if :**

1 AB = 4 cm. , BC = 5 cm. and AC = 7 cm.

2 AB = 11 cm. , BC = 8 cm. and AC = 9 cm.

Solution

1 $\because \overline{AC}$ is the longest side

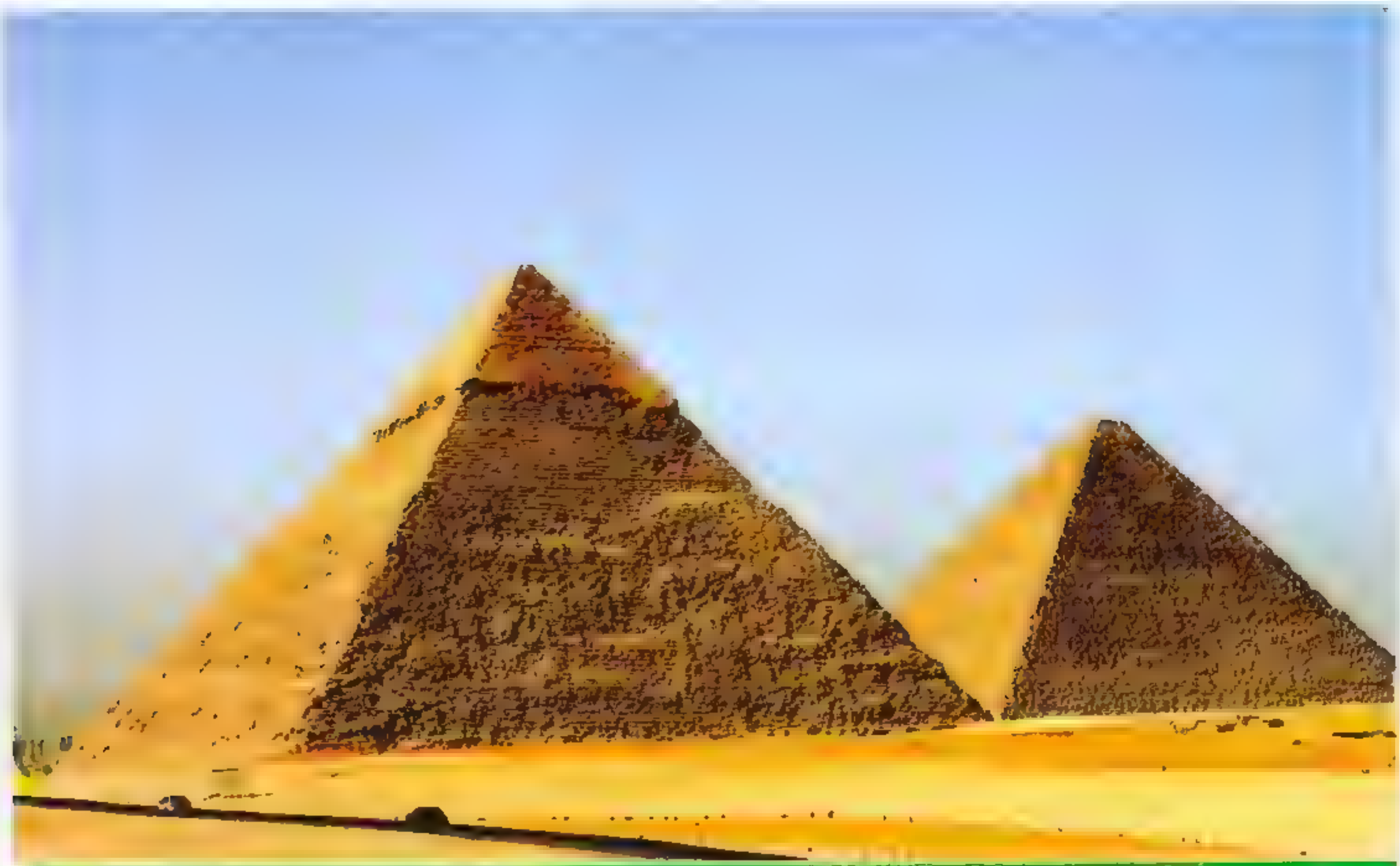
$, (AC)^2 = (7)^2 = 49 , (AB)^2 + (BC)^2 = (4)^2 + (5)^2 = 16 + 25 = 41$

$\therefore (AC)^2 > (AB)^2 + (BC)^2$ $\therefore$ ABC is an obtuse-angled triangle at B

2 $\because \overline{AB}$ is the longest side

$, (AB)^2 = (11)^2 = 121 , (BC)^2 + (AC)^2 = (8)^2 + (9)^2 = 64 + 81 = 145$

$\therefore (AB)^2 < (BC)^2 + (AC)^2$ $\therefore$ ABC is an acute-angled triangle.



UNIT

4 Trigonometry

Lessons of the unit :

1. The main trigonometrical ratios of the acute angle.

2. The main trigonometrical ratios of some angles.

- Use your smart phone or tablet to scan the QR Code and enjoy watching videos.

Unit Objectives :

By the end of this unit, student should be able to :

- recognize the main trigonometrical ratios of the acute angle.
- recognize the main trigonometrical ratios of the angles of measures 30° , 60° and 45°
- find the main trigonometrical ratios of a given angle.
- find the measure of the angle if one of its trigonometrical ratios is given.
- use the calculator to find the main trigonometrical ratios.

Enriching information

Trigonometry is one of mathematics branches and it is one of the general geometry branches , it concerneds studying the relations between the sides and angles of the triangle and the trigonometric ratios as the sine and cosine of the angle.

Ancient Egyptians were the first to use the trigonometric theorems and rules in building pyramids and temples.

Trigonometry has many applications in surveying roads and manufactoring motors , TV sets , football playgrounds , calculating geographic distances and astronomy discovering.

1 The main trigonometrical ratios of the acute angle

Prelude

- **You studied before the units of the degree measure of the angle and they are :**

  The degree which is denoted by 1° , the minute which is denoted by $1'$ and the second which is denoted by $1''$

For example:

The angle whose measure is 22 degrees , 36 minutes and 48 seconds is written as $22^\circ\, 36'\, 48''$

The relation between the degrees, the minutes and the seconds

- $1^\circ = 60'$

  i.e. $1^\circ = 60 \times 60 = 3600''$

- $1' = 60''$

Example 1

1. Write in degrees : $22^\circ\, 36'\, 48''$
2. Write in degrees , minutes and seconds : 45.18°

Solution

1. Convert the minutes into degrees , as the following :

   $36' = \frac{36}{60} = 0.6^\circ$

   Convert the seconds into degrees , as the following :

   $48'' = \frac{48}{3600} = 0.01\dot{3}^\circ$

   i.e. $22^\circ\, 36'\, 48'' = 22^\circ + 0.6^\circ + 0.01\dot{3}^\circ = 22.61\dot{3}^\circ$

Remember that

$0.00\dot{3}$ is read as the recurring decimal 0.003

Another solution by using the scientific calculator :

Press the keys in sequence from left as follows :

Then the result will be 22.61333333

2 Convert 0.18° into minutes as the following : $0.18 \times 60 = 10.8'$

Convert $0.8'$ into seconds as the following : $0.8 \times 60 = 48''$

i.e. $45.18^\circ = 45^\circ\ 10'\ 48''$

Another solution by using the scientific calculator :

Press the keys in sequence from left as follows :

Then the result will be $45^\circ\ 10'\ 48''$

Example 2

If the ratio between the measures of two complementary angles is 7 : 9 , find the degree measure of each of them.

Solution

Let the measures of the two angles be :

$7\,x$ and $9\,x$

$\therefore 7\,x + 9\,x = 90^\circ$

$\therefore 16\,x = 90^\circ$

$\therefore x = \frac{90^\circ}{16} = 5.625^\circ$

$\therefore$ The measure of the first angle

$= 5.625^\circ \times 7 = 39.375^\circ$

$= 39^\circ\ 22'\ 30''$

, the measure of the second angle $= 5.625^\circ \times 9 = 50.625^\circ = 50^\circ\ 37'\ 30''$

Remember that

- The sum of measures of two complementary angles = 90°
- The sum of measures of two supplementary angles = 180°
- The sum of measures of the interior angles of any triangle = 180°

TRY by yourself 1

If the ratio between the measures of two supplementary angles is 5 : 11 , find the degree measure of each of them.

Final answers of try by yourself questions are at the end of each lesson to check your answer.

The main trigonometrical ratios of the acute angle

WATCH VIDEO

The trigonometrical ratio of the acute angle

It is the ratio between two side lengths of the right-angled triangle that contains this angle.

There are three main trigonometrical ratios of the acute angle and they are :

1. **The sine of the angle :**
   abbreviated (sin) and equals $\dfrac{\text{the length of the opposite side to the angle}}{\text{the length of the hypotenuse}}$

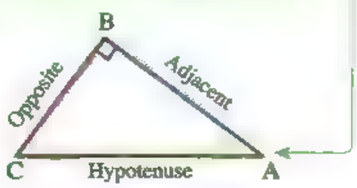
2. **The cosine of the angle :**
   abbreviated (cos) and equals $\dfrac{\text{the length of the adjacent side to the angle}}{\text{the length of the hypotenuse}}$

3. **The tangent of the angle :**
   abbreviated (tan) and equals $\dfrac{\text{the length of the opposite side to the angle}}{\text{the length of the adjacent side to the angle}}$

i.e.

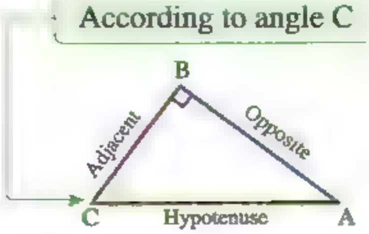
If Δ ABC is a right-angled triangle at B , then :

According to angle A



1. $\sin A = \dfrac{\text{Opposite}}{\text{Hypotenuse}} = \dfrac{BC}{AC}$
2. $\cos A = \dfrac{\text{Adjacent}}{\text{Hypotenuse}} = \dfrac{AB}{AC}$
3. $\tan A = \dfrac{\text{Opposite}}{\text{Adjacent}} = \dfrac{BC}{AB}$

According to angle C



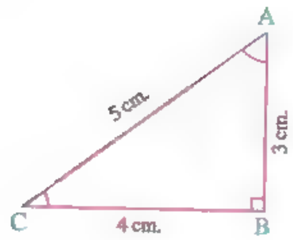
1. $\sin C = \dfrac{\text{Opposite}}{\text{Hypotenuse}} = \dfrac{AB}{AC}$
2. $\cos C = \dfrac{\text{Adjacent}}{\text{Hypotenuse}} = \dfrac{BC}{AC}$
3. $\tan C = \dfrac{\text{Opposite}}{\text{Adjacent}} = \dfrac{AB}{BC}$

For example:

In the opposite figure :

If Δ ABC is a right-angled triangle at B ,

AB = 3 cm. , BC = 4 cm. and AC = 5 cm. , then :



1. $\sin A = \frac{4}{5}$
2. $\cos A = \frac{3}{5}$
3. $\tan A = \frac{4}{3}$

1. $\sin C = \frac{3}{5}$
2. $\cos C = \frac{4}{5}$
3. $\tan C = \frac{3}{4}$

Example 3

In the opposite figure :

Δ ABC is right-angled at A where

AB = 9 cm. and AC = 12 cm.

1. **Find each of :** sin B , cos B , tan B , sin C , cos C and tan C
2. **Prove that :** sin B cos C + cos B sin C = 1

Solution

$\because$ In Δ ABC : m ($\angle$ A) = 90°

$\therefore (BC)^2 = (AB)^2 + (AC)^2$ (Pythagoras' theorem)

$\therefore (BC)^2 = 81 + 144 = 225$ $\therefore$ BC = 15 cm.

1. $\sin B = \frac{AC}{BC} = \frac{12}{15} = \frac{4}{5}$,

$\cos B = \frac{AB}{BC} = \frac{9}{15} = \frac{3}{5}$,

$\tan B = \frac{AC}{AB} = \frac{12}{9} = \frac{4}{3}$,

$\sin C = \frac{AB}{BC} = \frac{9}{15} = \frac{3}{5}$,

$\cos C = \frac{AC}{BC} = \frac{12}{15} = \frac{4}{5}$,

$\tan C = \frac{AB}{AC} = \frac{9}{12} = \frac{3}{4}$

Remember Pythagoras' theorem :

If ABC is a right-angled triangle at B , then :

- $(AC)^2 = (AB)^2 + (BC)^2$
- $(AB)^2 = (AC)^2 - (BC)^2$
- $(BC)^2 = (AC)^2 - (AB)^2$

2. $\sin B \cos C + \cos B \sin C = \frac{4}{5} \times \frac{4}{5} + \frac{3}{5} \times \frac{3}{5} = \frac{16}{25} + \frac{9}{25} = \frac{25}{25} = 1$

TRY 2

XYZ is a right-angled triangle at Y , XY = 4 cm. and XZ = 5 cm.

1. ***Find the value of :*** 2 sin X cos X
2. ***Prove that :*** sin X cos Z + cos X sin Z = 1

Remarks

In the previous example, note that :

1. $\sin B = \cos C = \frac{4}{5}$, $\sin C = \cos B = \frac{3}{5}$

   and by noticing : $m(\angle B) + m(\angle C) = 90°$ "Complementary angles"

– We can deduce that :

The sine of any acute angle equals the cosine of its complementary angle

i.e. If $m(\angle A) + m(\angle B) = 90°$

, then $\sin A = \cos B$ $\sin B = \cos A$

and vice versa

i.e. If $\angle A$ and $\angle B$ are acute angles and $\sin A = \cos B$

then $m(\angle A) + m(\angle B) = 90°$

2. $\frac{\sin B}{\cos B} = \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{3}$, $\tan B = \frac{4}{3}$ $\therefore$ $\boxed{\tan B = \frac{\sin B}{\cos B}}$

   , $\frac{\sin C}{\cos C} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$, $\tan C = \frac{3}{4}$ $\therefore$ $\boxed{\tan C = \frac{\sin C}{\cos C}}$

Generally : The tangent of the angle = $\frac{\text{The sine of the angle}}{\text{The cosine of the angle}}$

Example 4

Choose the correct answer from the given ones :

1 If $\sin 30° = \cos \theta$ where θ is the measure of an acute angle , then θ = ...

(a) 15° (b) 30° (c) 60° (d) 90°

2 If x and y are the measures of two complementary angles and $\cos x = \frac{4}{5}$, then $\sin y$ = ...

(a) $\frac{3}{5}$ (b) $\frac{4}{5}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

3 In Δ ABC , if m ($\angle$ A) = 60° and sin B = cos B , then m ($\angle$ C) = ...

(a) 30° (b) 75° (c) 90° (d) 105°

4 If Δ ABC is right-angled at B , then sin A+ 2 cos C = ...

(a) 2 sin C (b) 3 sin A (c) 2 sin A (d) 3 cos A

Solution

1 **(c) The reason :** $\because$ sin 30° = cos θ $\therefore$ 30° + θ = 90°

$\therefore \theta = 60°$

2 **(b) The reason :** $\because x$ and y are the measures of two complementary angles

$\therefore$ sin y = cos x $\therefore$ sin y = $\frac{4}{5}$

3 **(b) The reason :** $\because$ sin B = cos B $\therefore$ m ($\angle$ B) = 45°

$\therefore$ m ($\angle$ C) = 180° – (45° + 60°) = 75°

4 **(b) The reason :** $\because$ m ($\angle$ B) = 90° $\therefore$ m ($\angle$ A) + m ($\angle$ C) = 90°

$\therefore$ sin A = cos C

$\therefore$ sin A + 2 cos C = sin A + 2 sin A = 3 sin A

TRY yourself 3 **Choose the correct answer from the given ones :**

1 If m ($\angle$ A) = 75° , sin B = cos A where B is an acute angle , then m ($\angle$ B) = ...

(a) 15° (b) 45° (c) 75° (d) 105°

2 In Δ ABC , if m ($\angle$ B) = 90° , then cos A + sin C = ...

(a) 2 cos C (b) 2 cos A (c) 2 sin A (d) tan A

Example 5 ABC is a triangle in which : AB = AC = 10 cm. , BC = 12 cm. , $\overrightarrow{AD}$ is drawn perpendicular to $\overline{BC}$ to cut it at D

1 **Find the value of :** sin B + cos C

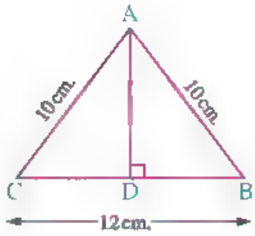
2 **Find the value of :** tan ($\angle$ CAD)

3 **Show that :** sin C + cos C > 1 **and find the value of :** $\sin^2 C + \cos^2 C$ **and deduce that :** $\sin^2 C + \cos^2 C$ < sin C + cos C

Solution $\because \overline{AD} \perp \overline{BC}$ and $AB = AC$

$\therefore$ D is the midpoint of $\overline{BC}$

$\therefore BD = DC = 6$ cm.



In Δ ADB :

$\because m(\angle ADB) = 90°$

$\therefore (AD)^2 = (AB)^2 - (BD)^2$ (Pythagoras' theorem)

$\therefore (AD)^2 = 100 - 36 = 64$ $\therefore AD = 8$ cm.

1 $\because \sin B = \frac{AD}{AB} = \frac{8}{10} = \frac{4}{5}$, $\cos C = \frac{CD}{AC} = \frac{6}{10} = \frac{3}{5}$

$\therefore \sin B + \cos C = \frac{4}{5} + \frac{3}{5} = \frac{7}{5}$

2 $\tan(\angle CAD) = \frac{CD}{AD} = \frac{6}{8} = \frac{3}{4}$

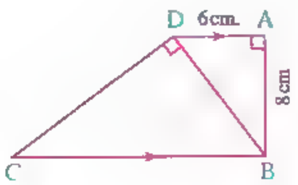
3 $\because \sin C = \frac{AD}{AC} = \frac{8}{10} = \frac{4}{5}$, $\cos C = \frac{3}{5}$

$\therefore \sin C + \cos C = \frac{4}{5} + \frac{3}{5} = \frac{7}{5}$ $\therefore \sin C + \cos C > 1$

, $\sin^2 C + \cos^2 C = \left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2 = \frac{16}{25} + \frac{9}{25} = 1$

$\therefore \sin^2 C + \cos^2 C < \sin C + \cos C$

Example 6 **In the opposite figure :**



ABCD is a quadrilateral in which :

$m(\angle A) = m(\angle BDC) = 90°$

, $\overline{AD} \parallel \overline{BC}$, $AD = 6$ cm. and $AB = 8$ cm.

Find the length of $\overline{DC}$

Solution In Δ ABD :

$\because m(\angle A) = 90°$

$\therefore (DB)^2 = (AB)^2 + (AD)^2 = 64 + 36 = 100$

$\therefore DB = 10$ cm.

, $\because \overline{AD} \parallel \overline{BC}$ and $\overleftrightarrow{BD}$ is a transversal

$\therefore$ m ($\angle$ ADB) = m ($\angle$ DBC) "Alternate angles"

$\therefore$ tan ($\angle$ ADB) = tan ($\angle$ DBC)

$\therefore \frac{AB}{AD} = \frac{DC}{BD}$ $\qquad \therefore \frac{8}{6} = \frac{DC}{10}$

$\therefore DC = \frac{10 \times 8}{6} = 13\frac{1}{3}$ cm. (The req.)

Notice that : *Also , you can solve this example by using the similarity.*

TRY 4

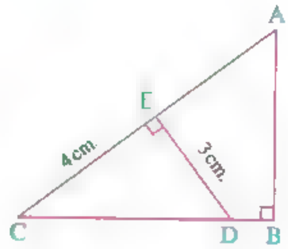
In the opposite figure :

ABC is a triangle in which :

m ($\angle$ B) = 90° , D $\in \overline{BC}$, E $\in \overline{AC}$

where $\overline{DE} \perp \overline{AC}$, DE = 3 cm. and EC = 4 cm.

Prove that : sin A cos C + sin C cos ($\angle$ EDC) = 1



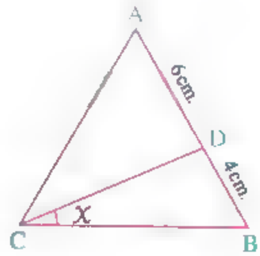
Example 7

In the opposite figure :

ABC is an equilateral triangle , D $\in \overline{AB}$

where : AD = 6 cm. , DB = 4 cm. ,

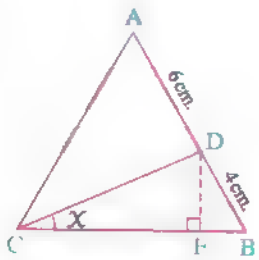
if k tan $x = \sqrt{3}$, **find the value of : k**



Solution

Construction : Draw $\overline{DE} \perp \overline{BC}$ and intersects it at E

Proof :



$\because$ ABC is an equilateral triangle

$\therefore$ m ($\angle$ B) = 60°

In Δ BDE :

$\because \overline{DE} \perp \overline{BC}$ $\qquad \therefore$ m ($\angle$ DEB) = 90°

$\therefore$ m ($\angle$ BDE) = 180° – (60° + 90°) = 30°

$\therefore BE = \frac{1}{2}$ BD = 2 cm.

$\because (DE)^2 = (DB)^2 - (BE)^2$ (Pythagoras' theorem)

$\therefore (DE)^2 = 16 - 4 = 12 \qquad \therefore DE = \sqrt{12} = 2\sqrt{3}$ cm.

, $\because BC = BA = 10$ cm. $\qquad \therefore EC = BC - BE = 10 - 2 = 8$ cm.

$\therefore \tan X = \frac{DE}{EC} = \frac{2\sqrt{3}}{8} = \frac{\sqrt{3}}{4} \qquad , \because k \tan X = \sqrt{3}$

$\therefore k \times \frac{\sqrt{3}}{4} = \sqrt{3} \qquad \therefore k = \sqrt{3} \times \frac{4}{\sqrt{3}} = 4$ (The req.)

TRY by yourself 5

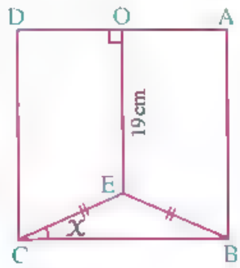
In the opposite figure :

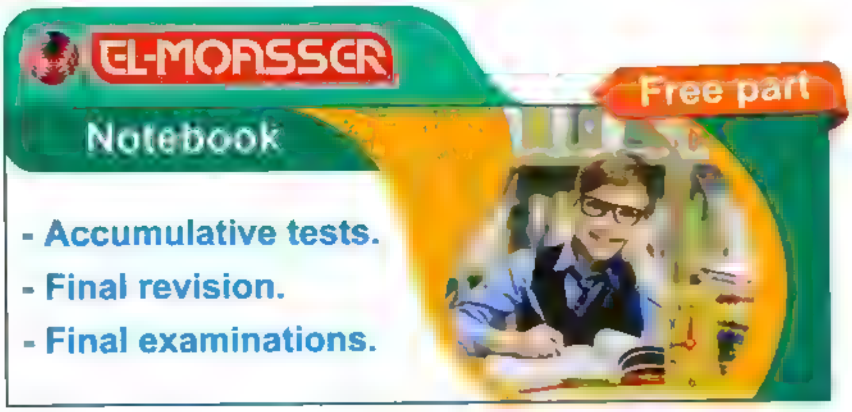
ABCD is a square of side length 24 cm. ,

E is a point inside it where : BE = CE , OE = 19 cm.

, $\overline{OE} \perp \overline{AD}$

If $k (\cos X - \sin X) = \frac{1}{13}$, ***find the value of :*** k





At the end of each lesson , you will find the final answers of try by yourself questions in the same form.

Answers of try by yourself

1 $56^\circ 15'$, $123^\circ 45'$

2 (1) $\frac{24}{25}$ (2) Prove by yourself

3 (1) (a) (2) (b)

4 Prove by yourself [Hint : DC = 5 cm. , m (∠ A) = m (∠ EDC)]

5 $\frac{1}{7}$

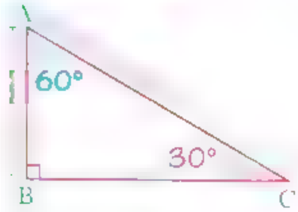
2 The main trigonometrical ratios of some angles

The main trigonometrical ratios of the angles measuring 30° and 60°

In the opposite figure :

ABC is a right-angled triangle at B in which : m (∠ A) = 60° and m (∠ C) = 30°

and it is called "thirty and sixty triangle".



And in it , the length of the side opposite to the angle of measure 30° equals half the length of the hypotenuse.

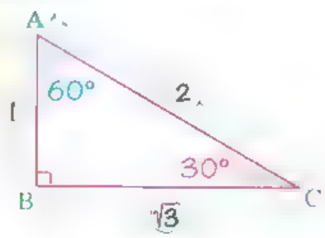
i.e. $AB = \frac{1}{2} AC$

Assume that : The length of $\overline{AB} = \ell$ length unit , then the length of $\overline{AC} = 2\,\ell$ length unit.

By applying Pythagoras' theorem to find the length of $\overline{BC}$ **, we find that :**

$BC = \sqrt{(AC)^2 - (AB)^2} = \sqrt{4\,\ell^2 - \ell^2} = \sqrt{3\,\ell^2} = \sqrt{3}\,\ell$ length unit.

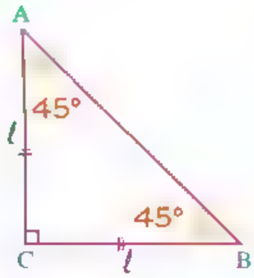
i.e. $AB : AC : BC = \ell : 2\,\ell : \sqrt{3}\,\ell = 1 : 2 : \sqrt{3}$



And from Δ ABC , we can find the main trigonometrical ratios of the angles measuring 30° and 60° as follows :

30° $\sin 30° = \frac{AB}{AC} = \frac{1}{2}$	$\cos 30° = \frac{BC}{AC} = \frac{\sqrt{3}}{2}$	$\tan 30° = \frac{AB}{BC} = \frac{1}{\sqrt{3}}$
60° $\sin 60° = \frac{BC}{AC} = \frac{\sqrt{3}}{2}$	$\cos 60° = \frac{AB}{AC} = \frac{1}{2}$	$\tan 60° = \frac{BC}{AB} = \sqrt{3}$

The main trigonometrical ratios of the angle measuring 45°



In the opposite figure :

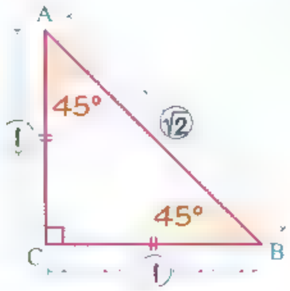
ABC is an isosceles triangle where AC = BC = ℓ length unit

and m (∠ C) = 90° ∴ m (∠ A) = m (∠ B) = 45°

By applying Pythagoras' theorem to find the length of $\overline{AB}$

, we find that : $AB = \sqrt{(AC)^2 + (BC)^2}$

$= \sqrt{\ell^2 + \ell^2} = \sqrt{2\ell^2} = \sqrt{2}\,\ell$ length unit.



i.e. AC : BC : AB = $\ell : \ell : \sqrt{2}\,\ell = 1 : 1 : \sqrt{2}$

From Δ ABC , we can find the main trigonometrical ratios of the angle measuring 45° as follows :

45° $\sin 45° = \frac{1}{\sqrt{2}}$	$\cos 45° = \frac{1}{\sqrt{2}}$	$\tan 45° = 1$

* **And the following table summarizes the main trigonometrical ratios of the angles whose measures are 30° , 60° and 45° :**

The measure of the angle / The trigonometrical ratio	30°	60°	45°
sin	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$
tan	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	1

Example 1 **Find the value of :** $\sin 30° \cos 60° + \cos^2 30° + 5 \tan 45° - 10 \cos^2 45°$

Solution

The expression $= \frac{1}{2} \times \frac{1}{2} + \left(\frac{\sqrt{3}}{2}\right)^2 + 5 \times 1 - 10 \times \left(\frac{1}{\sqrt{2}}\right)^2$

$= \frac{1}{4} + \frac{3}{4} + 5 - \frac{10}{2} = 1 + 5 - 5 = 1$

Example 2 **Prove that :** $\sin^2 60° + \sin^2 45° + \sin^2 30° = \cos^2 30° + \frac{1}{3} \tan^2 60° - \cos^2 60°$

Solution

L.H.S. $= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{3}{4} + \frac{1}{2} + \frac{1}{4} = \frac{3}{2}$

R.H.S. $= \left(\frac{\sqrt{3}}{2}\right)^2 + \frac{1}{3}\left(\sqrt{3}\right)^2 - \left(\frac{1}{2}\right)^2 = \frac{3}{4} + 1 - \frac{1}{4} = \frac{3}{2}$

∴ The two sides are equal.

TRY by yourself 1

1. *Find the value of :* (1) $\cos 60° + \sin 30° - \tan 45°$ (2) $\sin^2 30° + \sin^2 60°$
2. *Prove that :* $2 \sin 30° + 4 \cos 60° = \tan^2 60°$

Example 3 Find the value of X which satisfies :

1. $X \sin 30° \cos^2 45° = \cos^2 30°$
2. $2 \sin X = \tan^2 60° - 2 \tan 45°$ where X is the measure of an acute angle.

Solution

1. $\because X \sin 30° \cos^2 45° = \cos^2 30°$ $\quad \therefore X \times \frac{1}{2} \times \left(\frac{1}{\sqrt{2}}\right)^2 = \left(\frac{\sqrt{3}}{2}\right)^2$

$\therefore \frac{1}{4} X = \frac{3}{4}$ $\quad \therefore X = 3$

2. $\because 2 \sin X = \tan^2 60° - 2 \tan 45°$ $\quad \therefore 2 \sin X = (\sqrt{3})^2 - 2 \times 1 = 3 - 2 = 1$

$\therefore \sin X = \frac{1}{2}$ $\quad \therefore X = 30°$

TRY by yourself 2

Find the value of X which satisfies :

1. $X \cos 30° = \tan 60°$
2. $\tan X = \dfrac{2 \tan 30°}{1 - \tan^2 30°}$ where X is the measure of an acute angle.

Example 4 Choose the correct answer from the given ones :

1. If $\cos 4X = \frac{1}{2}$ where X is the measure of an acute angle , then $X =$ ....

(a) 15° (b) 30° (c) 45° (d) 60°

2. If $\tan (X + 10°) = \sqrt{3}$ where $(X + 10°)$ is the measure of an acute angle , then $X =$ ....

(a) 20° (b) 40° (c) 50° (d) 70°

3. If $\sin X = \frac{1}{2}$ where X is the measure of an acute angle , then $\sin 2X =$ ....

(a) 1 (b) $\frac{1}{4}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{3}}$

4 If $\cos(X + 15^\circ) = \frac{1}{2}$ where $(X + 15^\circ)$ is the measure of an acute angle , then $\sin(75^\circ - X) = \cdots\cdots\cdots$

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 1

5 If $4\cos 60^\circ \sin 30^\circ = \tan X$ where X is the measure of an acute angle , then $X = \cdots\cdots\cdots$

(a) 30° (b) 45° (c) 60° (d) 90°

Solution

1 **(a) The reason :** $\because \cos 4X = \frac{1}{2}$ $\therefore 4X = 60^\circ$

$\therefore X = \frac{60^\circ}{4} = 15^\circ$

2 **(c) The reason :** $\because \tan(X + 10^\circ) = \sqrt{3}$ $\therefore X + 10^\circ = 60^\circ$

$\therefore X = 60^\circ - 10^\circ = 50^\circ$

3 **(c) The reason :** $\because \sin X = \frac{1}{2}$ $\therefore X = 30^\circ$

$\therefore \sin 2X = \sin 60^\circ = \frac{\sqrt{3}}{2}$

4 **(a) The reason :** $\because \cos(X + 15^\circ) = \frac{1}{2}$ $\therefore X + 15^\circ = 60^\circ$

$\therefore X = 60^\circ - 15^\circ = 45^\circ$

$\therefore \sin(75^\circ - X) = \sin(75^\circ - 45^\circ) = \sin 30^\circ = \frac{1}{2}$

5 **(b) The reason :** $\because 4\cos 60^\circ \sin 30^\circ = 4 \times \frac{1}{2} \times \frac{1}{2} = 1$

$\therefore \tan X = 1$ $\therefore X = 45^\circ$

TRY by yourself 3

Choose the correct answer from the given ones :

1 $2\cos^2 30^\circ - 1 = \cdots\cdots\cdots$

(a) $\cos 60^\circ$ (b) $\sin 60^\circ$

(c) $2\sin 30^\circ$ (d) $\tan 60^\circ$

2 If $\tan(X + 15^\circ) = 1$ where $(X + 15^\circ)$ is the measure of an acute angle , then $X = \cdots\cdots\cdots$

(a) 15° (b) 30° (c) 45° (d) 60°

3 If $\left(\cos X , \frac{1}{2}\right) = \left(\frac{1}{2} , \sin y\right)$ where X and y are the measures of two acute angles , then $X + y = \cdots\cdots\cdots$

(a) 30° (b) 60° (c) 90° (d) 120°

Using the calculator

First: Finding the main trigonometrical ratios of a given angle

In the calculator , there are three keys : sin , cos , tan

1. The key sin means sine.
2. The key cos means cosine.
3. The key tan means tangent.

By using these keys we can find the main trigonometrical ratios of any angle if its measure is known.

Example 5 **By using the calculator , find the value of each of the following approximated to the nearest four decimals :**

1. $\sin 36^\circ$ 2. $\cos 72^\circ\, 35'$ 3. $\tan 50^\circ\, 46'\, 25''$

Solution Use the keys of the calculator as the following sequence from left :

1. sin 3 6 =

$\therefore \sin 36^\circ \simeq 0.5878$

2. cos 7 2 °''' 3 5 °''' =

$\therefore \cos 72^\circ\, 35' \simeq 0.2993$

3. tan 5 0 °''' 4 6 °''' 2 5 °''' =

$\therefore \tan 50^\circ\, 46'\, 25'' \simeq 1.2250$

TRY by yourself 4 ***By using the calculator, find the value of each of the following approximated to the nearest three decimals :***

1. $\sin 35^\circ\, 12'$ 2. $\tan 58^\circ\, 24'$

Second: Finding the measure of the angle if one of its trigonometrical ratios is given

If $\sin A = 0.6218$, then A is the measure of the angle whose sine is 0.6218

To find the measure of this angle , we can use the calculator as the following sequence from left :

SHIFT sin . 6 2 1 8 = °''' Then $A \simeq 38^\circ\ 26'\ 52''$

Example 6

Find A in each of the following , where A is the measure of an acute angle :

1 $\sin A = 0.8$ 2 $\cos A = 0.7152$ 3 $\tan A = 1.5156$

Solution

Use the keys of the calculator as the following sequence from left :

1 SHIFT sin . 8 = °'''

$\therefore A \simeq 53^\circ\ 7'\ 48''$

2 SHIFT cos . 7 1 5 2 = °'''

$\therefore A \simeq 44^\circ\ 20'\ 25''$

3 SHIFT tan 1 . 5 1 5 6 = °'''

$\therefore A \simeq 56^\circ\ 34'\ 59''$

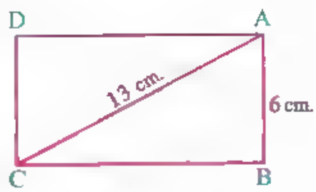
TRY by yourself 5

Using the calculator , find A in each of the following where A is the measure of an acute angle :

1 $\sin A = 0.3945$ 2 $\cos A = 0.3824$

Example 7

In the opposite figure :



ABCD is a rectangle in which :

AB = 6 cm. and AC = 13 cm. **Find :**

1 m ($\angle$ ACB)

2 The area of the rectangle ABCD to the nearest one decimal digit.

Solution

$\because$ ABCD is a rectangle.

$\therefore m(\angle B) = 90^\circ$

In Δ ABC :

$\sin(\angle ACB) = \frac{AB}{AC} = \frac{6}{13}$

And by using the calculator :

$\therefore m(\angle ACB) \simeq 27^\circ\ 29'\ 11''$ (First req.)

, $\because \cos(\angle ACB) = \frac{BC}{AC}$

$\therefore \cos 27^\circ\ 29'\ 11'' = \frac{BC}{13}$

$\therefore BC = 13 \times \cos 27^\circ\ 29'\ 11''$

$\therefore$ The area of the rectangle ABCD = AB × BC

$= 6 \times 13 \times \cos 27^\circ\ 29'\ 11'' \simeq 69.2\ cm^2$

(Second req.)

Notice that :

Also , you can find the length of $\overline{BC}$ by using Pythagoras' theorem in Δ ABC

TRY by yourself 6

In the opposite figure :

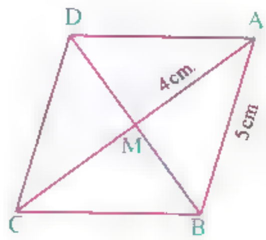
ABCD is a rhombus , whose diagonals intersect at M

If AB = 5 cm. and AM = 4 cm.

, ***find :***

1 m (∠ BAD)

2 The area of the rhombus ABCD



Answers *of try by yourself*

1 1 (1) zero (2) 1 2 Prove by yourself.

2 1 2 2 60°

3 1 (a) 2 (b) 3 (c)

4 1 0.576 2 1.625

5 1 $23^\circ\ 14'\ 5''$ (approximately) 2 $67^\circ\ 31'\ 3''$ (approximately)

6 1 $73^\circ\ 44'\ 23''$ (approximately) 2 $24\ cm^2$

UNIT 5

Analytical geometry

Lessons of the unit :

1. Distance between two points.
2. The two coordinates of the midpoint of a line segment.
3. The slope of the straight line.
4. The equation of the straight line given its slope and the intercepted part of y-axis.

- Use your smart phone or tablet to scan the QR Code and enjoy watching videos.

Unit Objectives :

By the end of this unit, student should be able to :

- find the distance between two points in the coordinates plane.
- find the two coordinates of the midpoint of a line segment.
- recognize the slope of the straight line.
- find the slope of the straight line given the measure of the positive angle which this straight line makes with the positive direction of the x-axis.
- recognize the relation between the two slopes of the two parallel straight lines.
- recognize the relation between the two slopes of the two perpendicular straight lines.
- find the slope of the straight line and the length of the intercepted part from y-axis given the equation of the straight line.
- find the equation of the straight line given its slope and the length of the intercepted part from y-axis.
- use the slope of the straight line for solving some life problems.

LESSON 1

Distance between two points

Let M (x_1 , y_1) and N (x_2 , y_2) be two points in the same coordinates plane.

From the geometry of the figure we find that :

NL = NB – LB = $y_2 - y_1$

Generally NL = $|y_2 - y_1|$

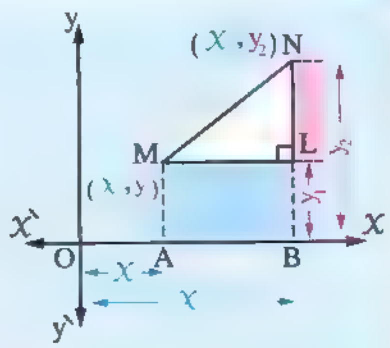
Similarly LM = BO – AO = $x_2 - x_1$

Generally LM = $|x_2 - x_1|$

$\because$ Δ NLM is right-angled at L

$\therefore$ $(MN)^2 = (LM)^2 + (NL)^2$

$\therefore$ $(MN)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$



$$\therefore MN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

i.e.

The distance between the two points M and N equals $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

and we know that :

$(x_2 - x_1)^2 = (x_1 - x_2)^2$, and similarly : $(y_2 - y_1)^2 = (y_1 - y_2)^2$, therefore :

The distance between the two points M and N equals also $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Generally :

The distance between two points = $\sqrt{\text{square of the difference between } x\text{-coordinates} + \text{square of the difference between y-coordinates}}$

For example : If A (3 , 6) and B (– 1 , 4) , then

the length of $\overline{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-1-3)^2 + (4-6)^2} = \sqrt{(-4)^2 + (-2)^2}$

$= \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$ length unit.

you can find the length of $\overline{AB}$ as follows : the length of $\overline{AB}$

$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{(3-(-1))^2 + (6-4)^2} = \sqrt{4^2 + 2^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$ length unit.

Example 1

Choose the correct answer from the given ones :

1 The distance between the two points (6 , 0) and (0 , 8) equals ..... length unit.

(a) 12 (b) 10 (c) 8 (d) 6

2 The distance between the point A $(\sqrt{2}, 4)$ and the origin point equals .... length unit.

(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $3\sqrt{2}$ (d) $4\sqrt{2}$

3 The distance between the point (– 7 , – 3) and y-axis equals .... length unit.

(a) – 7 (b) – 3 (c) 7 (d) 3

4 ABCD is a rectangle in which A (– 1 , – 3) and C (2 , 1) , then the length of $\overline{BD}$ = ... length unit.

(a) 25 (b) 5 (c) $\sqrt{7}$ (d) $\sqrt{5}$

Solution

1 **(b) The reason :** The required distance $= \sqrt{(0-6)^2 + (8-0)^2}$

$= \sqrt{(-6)^2 + (8)^2} = \sqrt{36+64}$

$= \sqrt{100} = 10$ length unit.

2 **(c) The reason :** The distance between any point (x , y) and the origin point (0 , 0) equals $\sqrt{x^2 + y^2}$

$\therefore$ The required distance $= \sqrt{\left(\sqrt{2}\right)^2 + (4)^2}$

$= \sqrt{2+16} = \sqrt{18} = \sqrt{9 \times 2}$

$= 3\sqrt{2}$ length unit.

3 **(c) The reason :** The distance between the point (– 7 , – 3) and $\overleftrightarrow{yy}$ equals | – 7 | because the distance is a positive number.

$\therefore$ The required distance = 7 length unit.

4 **(b) The reason :** The length of $\overline{BD}$ = the length of $\overline{AC}$ because the rectangle diagonals are equal in length.

$\therefore$ The length of $\overline{BD} = \sqrt{(2+1)^2+(1+3)^2}$

$= \sqrt{3^2+4^2} = \sqrt{9+16}$

$= \sqrt{25} = 5$ length unit.

Example 2 If the distance between the two points $(a , 5)$ and $(3a - 1 , 1)$ equals 5 length units , **find the value of :** a

Solution

$\because \sqrt{(3a-1-a)^2+(1-5)^2} = 5$

$\therefore \sqrt{(2a-1)^2+(-4)^2} = 5$ "Squaring the two sides"

$\therefore (2a-1)^2 + 16 = 25$

$\therefore (2a-1)^2 = 9$ "Taking the square root of the two sides"

$\therefore 2a - 1 = \pm 3$

$\therefore 2a - 1 = 3$ thus , $2a = 4$ $\therefore \boxed{a = 2}$

or $2a - 1 = -3$ thus , $2a = -2$ $\therefore \boxed{a = -1}$

TRY by yourself 1 If A (2 , 5) and B (– 1 , 1) , ***find the length of :*** $\overline{AB}$

Example 3 If ABC is a triangle where A (0 , 0) , B (3 , 4) and C (– 4 , 3) , find the perimeter of Δ ABC

Solution

$\because$ The perimeter of $\Delta ABC = AB + BC + CA$

$, AB = \sqrt{3^2+4^2}$

$= \sqrt{9+16} = \sqrt{25} = 5$ length unit.

$, BC = \sqrt{(-4-3)^2+(3-4)^2}$

$= \sqrt{(-7)^2+(-1)^2}$

$= \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}$ length unit.

$, CA = \sqrt{(-4)^2+(3)^2} = \sqrt{16+9} = \sqrt{25} = 5$ length unit.

$\therefore$ The perimeter of $\Delta ABC = 5 + 5\sqrt{2} + 5 = \left(10 + 5\sqrt{2}\right)$ length unit.

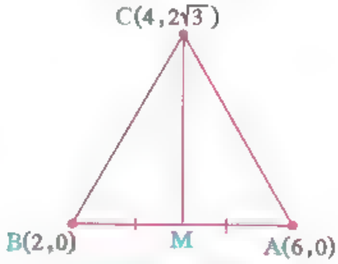
Example 4 **Prove that :** ΔABC is an equilateral triangle where : A (6 , 0) , B (2 , 0) and C $\left(4 , 2\sqrt{3}\right)$, then find its area.

Solution $\because AB = \sqrt{(6-2)^2 + (0-0)^2} = \sqrt{16+0} = \sqrt{16} = 4$ length unit.

, $BC = \sqrt{(2-4)^2 + (0-2\sqrt{3})^2} = \sqrt{4+12} = \sqrt{16} = 4$ length unit.

and $AC = \sqrt{(6-4)^2 + (0-2\sqrt{3})^2}$

$= \sqrt{4+12} = \sqrt{16} = 4$ length unit.



Illustrative drawing

$\therefore AB = BC = AC$ $\therefore \Delta ABC$ is equilateral

Let M be the midpoint of the base $\overline{AB}$

$\therefore \overline{CM} \perp \overline{AB}$

$\therefore$ By using Pythagoras' theorem , we find that :

$\therefore$ The height $MC = \sqrt{(AC)^2 - (AM)^2} = \sqrt{4^2 - 2^2} = \sqrt{12} = 2\sqrt{3}$ length unit

$\therefore$ The area of $\Delta ABC = \frac{1}{2} \times AB \times MC = \frac{1}{2} \times 4 \times 2\sqrt{3} = 4\sqrt{3}$ square unit.

TRY yourself 2 ***Prove that :*** ΔABC is an isosceles triangle where : A (3 , 3) , B (5 , 9) and C (– 1 , 7)

! Remark 1

To prove that three given points are collinear (i.e. they lie on one straight line) we can find the distance between each two of these points , then prove that the greatest distance equals the sum of the two other distances.

Example 5 **Prove that :** The points A (– 2 , 7) , B (– 3 , 4) and C (1 , 16) are collinear.

Solution $\because AB = \sqrt{(-2+3)^2 + (7-4)^2} = \sqrt{1+9} = \sqrt{10}$ length unit.

, $BC = \sqrt{(-3-1)^2 + (4-16)^2} = \sqrt{16+144} = \sqrt{160} = 4\sqrt{10}$ length unit.

and $AC = \sqrt{(-2-1)^2 + (7-16)^2} = \sqrt{9+81} = \sqrt{90} = 3\sqrt{10}$ length unit.

$\therefore BC = AB + AC$ $\therefore$ A , B and C are collinear.

Remark 2

- To prove that the points A , B and C are the vertices of a triangle , we can find AB , BC and AC , then prove that the sum of the smaller two lengths is greater than the third length.
- To determine the type of the triangle ABC according to its angle measures (where $\overline{AC}$ is the longest side of the triangle ABC)
  , we compare between $(AC)^2$ and $(AB)^2 + (BC)^2$ as the following :
  1. If $(AC)^2 > (AB)^2 + (BC)^2$, then the triangle is obtuse-angled at B
  2. If $(AC)^2 = (AB)^2 + (BC)^2$, then the triangle is right-angled at B
  3. If $(AC)^2 < (AB)^2 + (BC)^2$, then the triangle is acute-angled.

Example 6 **Prove that :** The triangle whose vertices are A (3 , 2) , B (– 4 , 1) and C (2 , – 1) is right-angled , then find its area.

Solution $\because AB = \sqrt{(3+4)^2+(2-1)^2}$

$= \sqrt{49+1} = \sqrt{50}$ length unit.

, $BC = \sqrt{(-4-2)^2+(1+1)^2}$

$= \sqrt{36+4} = \sqrt{40}$ length unit.

and $AC = \sqrt{(3-2)^2+(2+1)^2}$

$= \sqrt{1+9} = \sqrt{10}$ length unit.

$\because (AC)^2 + (BC)^2 = 10 + 40 = 50$

, $(AB)^2 = 50$

$\therefore (AC)^2 + (BC)^2 = (AB)^2$

$\therefore \Delta ABC$ is right-angled at C

$\therefore$ The area of the triangle ABC $= \frac{1}{2} AC \times BC$

$= \frac{1}{2} \times \sqrt{10} \times \sqrt{40}$

$= \frac{1}{2} \times \sqrt{10} \times 2\sqrt{10} = 10$ square unit.

TRY by yourself 3 If A (– 1 , – 1) , B (2 , 3) and C (6 , 0)

, ***prove that :*** Δ ABC is right-angled at B , then find its area.

Remark 4

If ABCD is a quadrilateral :

1. To prove that ABCD is a parallelogram , we prove that : AB = CD , BC = AD
2. To prove that ABCD is a rhombus , we prove that : AB = BC = CD = DA
3. To prove that ABCD is a rectangle , we prove that : AB = CD , BC = AD , AC = BD
4. To prove that ABCD is a square , we prove that : AB = BC = CD = DA , AC = BD

Example 7

If A (3 , – 2) , B (– 5 , 0) , C (0 , – 7) and D (8 , – 9) ,
prove that : ABCD is a parallelogram.

Solution

$\because$ AB $=\sqrt{(3+5)^2+(-2-0)^2}=\sqrt{64+4}$

$=\sqrt{68}$ length unit.

, BC $=\sqrt{(-5-0)^2+(0+7)^2}=\sqrt{25+49}$

$=\sqrt{74}$ length unit.

, CD $=\sqrt{(0-8)^2+(-7+9)^2}=\sqrt{64+4}$

$=\sqrt{68}$ length unit.

and DA $=\sqrt{(8-3)^2+(-9+2)^2}=\sqrt{25+49}=\sqrt{74}$ length unit.

$\therefore$ AB = CD , BC = DA $\qquad\therefore$ ABCD is a parallelogram.

Example 8

Prove that : The points A (– 1 , 4) , B (1 , 1) , C (– 1 , – 2) and D (– 3 , 1) are the vertices of a rhombus and graph it , then find its area.

Solution

$\because$ AB $=\sqrt{(-1-1)^2+(4-1)^2}$

$=\sqrt{4+9}=\sqrt{13}$ length unit.

, BC $=\sqrt{(1+1)^2+(1+2)^2}$

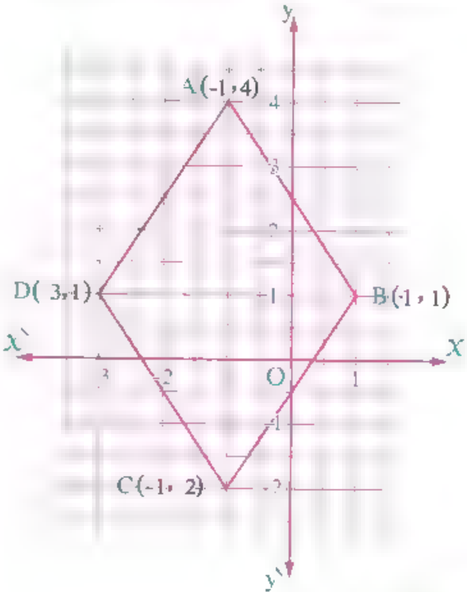
$=\sqrt{4+9}=\sqrt{13}$ length unit.

, CD $=\sqrt{(-1+3)^2+(-2-1)^2}$

$=\sqrt{4+9}=\sqrt{13}$ length unit.

and DA $=\sqrt{(-3+1)^2+(1-4)^2}$

$=\sqrt{4+9}=\sqrt{13}$ length unit.



$\therefore$ AB = BC = CD = DA

$\therefore$ The quadrilateral ABCD is a rhombus.

$\because$ AC $=\sqrt{(-1+1)^2+(4+2)^2}=\sqrt{0+36}=\sqrt{36}=6$ length unit.

, BD $=\sqrt{(1+3)^2+(1-1)^2}=\sqrt{16+0}=\sqrt{16}=4$ length unit.

$\therefore$ The area of the rhombus ABCD $=\frac{1}{2}\times 6\times 4=12$ square unit.

TRY by yourself 4

Prove that : The points A (– 1 , 3) , B (5 , 1) , C (6 , 4) and D (0 , 6) are the vertices of a rectangle , then find its area.

Remark 4

- The axis of symmetry of a line segment is the straight line that is perpendicular to it at its midpoint.
- Any point on the axis of symmetry of a line segment is at equal distances from its terminals.

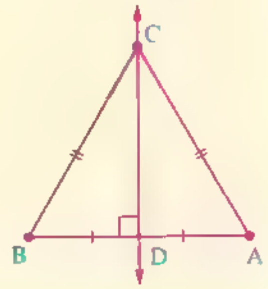
The converse is true , i.e. If a point is at equal distances from the two terminals of a line segment , then this point lies on the axis of this line segment.

For example:

In the opposite figure :

If CA = CB

, then C $\in$ the axis of symmetry of $\overline{AB}$



Example 9 If A (1 , – 1) and B (1 , 3)

, **prove that :** The point C (– 1 , 1) lies on the axis of symmetry of $\overline{AB}$

Solution

$\because$ CA $=\sqrt{(-1-1)^2+(1+1)^2}=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$ length unit.

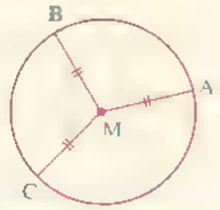
, CB $=\sqrt{(-1-1)^2+(1-3)^2}=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$ length unit.

$\therefore$ CA = CB

$\therefore$ The point C lies on the axis of symmetry of $\overline{AB}$

Remark 5

- If $A \in$ the circle M , then the radius length of this circle (r) = MA
- To prove that : Three points as A , B and C lie on the same circle of centre M we prove that : MA = MB = MC
- Remember that :
  - The circumference of the circle = $2 \pi r$
  - The area of the circle = πr^2



Example 10

Choose the correct answer from the given ones :

1. The diameter length of the circle of centre A (– 2 , 3) and passing through B (2 , – 1) equals .... length unit.

(a) $8\sqrt{2}$ (b) $4\sqrt{2}$ (c) 5 (d) 4

2. A circle is of centre (3 , – 4) and its radius length is 5 length unit. Which of the following points belongs to this circle ?

(a) (– 3 , 4) (b) (0 , 0) (c) (5 , 0) (d) (0 , 4)

Solution

1. **(a) The reason :** r = the length of $\overline{AB} = \sqrt{(2+2)^2+(-1-3)^2}$

$$= \sqrt{(4)^2+(-4)^2} = \sqrt{32}$$

$= 4\sqrt{2}$ length unit.

$\therefore$ The diameter length $= 2r = 2 \times 4\sqrt{2}$

$= 8\sqrt{2}$ length unit.

2. **(b) The reason :** The right answer is the point whose distance from the centre of the circle equals the radius length of the circle. Finding the distance between each point and the centre of the circle (3 , – 4) , you find that (0 , 0) is the right answer because

$\sqrt{(3-0)^2+(-4-0)^2} = \sqrt{9+16} = \sqrt{25} = 5$ length unit = r

Example 11 **Prove that :** The points A $(-6 , 2)$, B $(0 , 8)$ and C $(-8 , 4)$ lie on the circle whose centre is M $(-4 , 6)$ and find its area where $\pi \approx 3.14$

Solution

$\because$ MA $= \sqrt{(-6+4)^2+(2-6)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$ length units.

, MB $= \sqrt{(0+4)^2+(8-6)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$ length units.

and MC $= \sqrt{(-8+4)^2+(4-6)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$ length units.

$\therefore$ MA = MB = MC

$\therefore$ The points A , B and C lie on the circle M whose radius length $r = 2\sqrt{5}$ length units.

$\therefore$ The area of the circle M $= \pi r^2 \approx 3.14 \times \left(2\sqrt{5}\right)^2 \approx 62.8$ square units.

TRY by yourself 5

Prove that : The points A $(-2 , 0)$, B $(5 , 1)$ and C $(6 , -6)$ lie on the circle whose centre is M $(2 , -3)$ and find the circumference of the circle in terms of π

Answers *of try by yourself*

1. 5 length units.
2. Prove by yourself [Hint : find AB , BC and CA]
3. Prove by yourself [Hint : prove that $(AC)^2 = (AB)^2 + (BC)^2$, the area = 12.5 square units.]
4. Prove by yourself [Hint : prove that : AB = CD , BC = AD , AC = BD , the area = 20 square units.]
5. Prove by yourself [Hint : prove that : MA = MB = MC , the circumference = 10 π length units.]

2 The two coordinates of the midpoint of a line segment

If A (x_1 , y_1) and B (x_2 , y_2) are two points in a coordinates plane and M (x , y) is the midpoint of $\overline{AB}$

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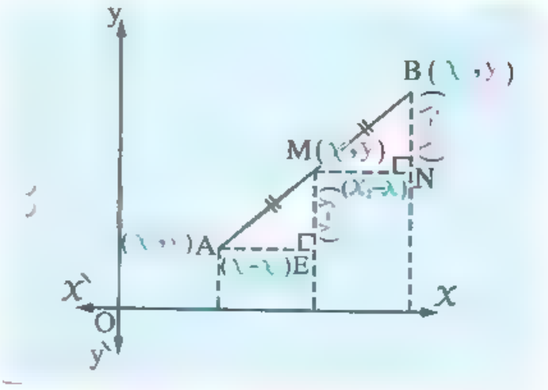
From the opposite figure :



Δ AEM and Δ MNB are congruent

$\therefore$ AE = MN , EM = NB

$\therefore x - x_1 = x_2 - x$, $y - y_1 = y_2 - y$

$\therefore 2x = x_1 + x_2$, $2y = y_1 + y_2$

$\therefore x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$

$$\therefore M = \left(\frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2}\right)$$

For example:

If X (3 , – 2) , Y (– 1 , – 4) and M is the midpoint of $\overline{XY}$, then :

$M = \left(\frac{3 + (-1)}{2} , \frac{-2 + (-4)}{2}\right) = (1 , -3)$

Example 1 If C (10 , – 4) is the midpoint of $\overline{AB}$ where A (4 , – 2) , find the point B

Solution Let B (x , y)

Notice that : If (a , b) = (c , d) , then a = c , b = d

$\because$ C is the midpoint of $\overline{AB}$

$\therefore (10 , -4) = \left(\frac{x+4}{2} , \frac{y+(-2)}{2}\right)$

$\therefore \frac{x+4}{2} = 10$ $\quad \therefore x + 4 = 20$ $\quad \therefore x = 16$

, $\frac{y-2}{2} = -4$ $\quad \therefore y - 2 = -8$ $\quad \therefore y = -6$ $\quad \therefore B = (16 , -6)$

TRY by yourself 1

If C is the midpoint of $\overline{AB}$, then find the value of each of x and y in each of the following :

1. A (2 , 5) , B (– 2 , – 3) and C (x , y)
2. A (x , 4) , B (– 1 , – 6) and C (– 2 , y)

! Remark

If $\overline{AB}$ is a diameter in a circle of centre M , then M is the midpoint of $\overline{AB}$

Example 2 If $\overline{AB}$ is a diameter in the circle M where A (4 , – 1) and B (– 2 , 7) , find the point M , then find the circumference and the area of the circle.

Solution $\because$ $\overline{AB}$ is a diameter in the circle M $\quad \therefore$ M is the midpoint of $\overline{AB}$

$\therefore$ The point M $= \left(\frac{4+(-2)}{2} , \frac{-1+7}{2}\right) = (1 , 3)$

, $\because r = AM = \sqrt{(1-4)^2 + (3+1)^2} = \sqrt{9+16} = \sqrt{25} = 5$ length units.

$\therefore$ The circumference of the circle $= 2\pi r = 2\pi \times 5 = 10\pi$ length units.

, the area of the circle $= \pi r^2 = \pi \times 5^2 = 25\pi$ square units.

Another method to calculate the radius length of the circle :

$\because AB = \sqrt{(-2-4)^2 + (7+1)^2} = \sqrt{36+64} = \sqrt{100} = 10$ length units.

, $\because$ $\overline{AB}$ is a diameter $\quad \therefore r = \frac{1}{2} AB = 5$ length units.

, then complete the solution to find the circumference and the area of the circle.

TRY by yourself 2

If $\overline{AB}$ is a diameter in the circle M where A (4 , 1) and B (– 6 , 3) , then find the point M

Example 3 **Prove that :** The quadrilateral ABCD is a parallelogram where A $(4 , 3)$, B $(0 , 2)$, C $(-2 , -3)$ and D $(2 , -2)$

Solution $\because$ The two diagonals of the quadrilateral are $\overline{AC}$ and $\overline{BD}$

, the midpoint of $\overline{AC} = \left(\frac{4+(-2)}{2} , \frac{3+(-3)}{2}\right) = (1 , 0)$

and the midpoint of $\overline{BD} = \left(\frac{0+2}{2} , \frac{2+(-2)}{2}\right) = (1 , 0)$

$\therefore$ The midpoint of $\overline{AC}$ is the same midpoint of $\overline{BD}$

$\therefore$ The two diagonals bisect each other.

$\therefore$ ABCD is a parallelogram.

Notice that : You can solve this example by using the distance between two points as the previous.

Example 4 **Prove that :** The points A $(5 , 1)$, B $(1 , -3)$ and C $(-5 , 3)$ are the vertices of a right-angled triangle at B , then find the point D that makes the figure ABCD a rectangle.

Solution $\because$ AB $= \sqrt{(1-5)^2 + (-3-1)^2} = \sqrt{16+16} = \sqrt{32}$ length unit.

, BC $= \sqrt{(-5-1)^2 + (3+3)^2} = \sqrt{36+36} = \sqrt{72}$ length unit.

, AC $= \sqrt{(-5-5)^2 + (3-1)^2} = \sqrt{100+4} = \sqrt{104}$ length unit.

$\therefore (AB)^2 + (BC)^2 = 32 + 72 = 104 = (AC)^2$

$\therefore \Delta$ ABC is a right-angled triangle at B

Let D (x , y) such that the figure ABCD is a rectangle.

$\therefore \overline{AC}$ and $\overline{BD}$ bisect each other.

$\therefore$ The midpoint of $\overline{AC}$ = the midpoint of $\overline{BD}$

, $\because$ the midpoint of $\overline{AC} = \left(\frac{5-5}{2} , \frac{1+3}{2}\right) = (0 , 2)$

, the midpoint of $\overline{BD} = \left(\frac{x+1}{2} , \frac{y-3}{2}\right)$

$\therefore \left(\frac{x+1}{2} , \frac{y-3}{2}\right) = (0 , 2)$

$\therefore \frac{x+1}{2}=0 \qquad \therefore x+1=0 \qquad \therefore x=-1$

$, \frac{y-3}{2}=2 \qquad \therefore y-3=4 \qquad \therefore y=7$

$\therefore D=(-1\ ,7)$

Example 5

Prove that : The triangle whose vertices are A $(-1\ ,4)$, B $(3\ ,1)$ and C $(-5\ ,1)$ is an isosceles triangle , then find its area.

Solution

$\because AB=\sqrt{(3+1)^2+(1-4)^2}=\sqrt{16+9}$

$=5$ length unit.

$, BC=\sqrt{(3+5)^2+(1-1)^2}=\sqrt{64}=8$ length unit.

$, AC=\sqrt{(-5+1)^2+(1-4)^2}=\sqrt{16+9}$

$=5$ length unit.

$\therefore AB=AC$

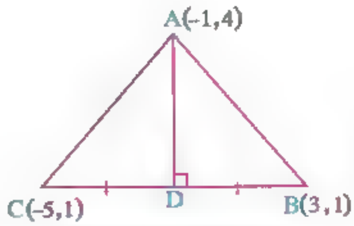
$\therefore \Delta ABC$ is an isosceles triangle.

Let D $(x\ ,y)$ be the midpoint of $\overline{BC}$

$\therefore D=\left(\frac{3-5}{2}\ ,\frac{1+1}{2}\right)=(-1\ ,1)$

$, \because$ D is the midpoint of $\overline{BC}$

$\therefore \overline{AD}\perp\overline{BC}$



Illustrative drawing

$, \because AD=\sqrt{(-1+1)^2+(1-4)^2}=\sqrt{9}=3$ length unit.

$, BC=8$ length unit

$\therefore$ The area of $\Delta ABC=\frac{1}{2}BC\times AD=\frac{1}{2}\times 8\times 3=12$ square unit.

TRY by yourself 3

If C is the midpoint of $\overline{AB}$ where A $(2\ ,3)$, B $(4\ ,-7)$ and C is the midpoint of $\overline{DE}$ where D $(-3\ ,5)$, find the point E

Answers of try by yourself

1 [1] $x=0$, $y=1$ [2] $x=-3$, $y=-1$

2 $(-1\ ,2)$

3 $(9\ ,-9)$

3 The slope of the straight line

You studied before the slope of the straight line given two points on it.

If A and B are two points in the coordinates plane where A (x_1, y_1) and B (x_2, y_2)

, then : The slope of the straight line $\overleftrightarrow{AB} = \dfrac{y_2 - y_1}{x_2 - x_1}$

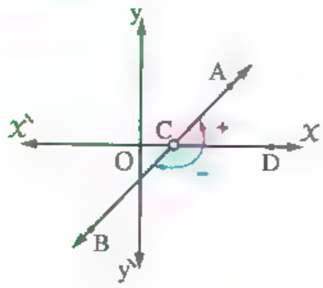
In this lesson , you will learn :

- How to find the slope of the straight line given the measure of the positive angle which this straight line makes with the positive direction of the x-axis.
- The relation between the slopes of two parallel straight lines.
- The relation between the slopes of two perpendicular straight lines.

And before studying these topics , you will study the positive and negative measures of an angle.

The positive measure and the negative measure of an angle

In the opposite figure :



If $\overleftrightarrow{AB}$ intersects the x-axis at the point C , then $\overleftrightarrow{AB}$ makes two angles with the positive direction of the x-axis.

- One of them is positive (i.e. it has a positive measure) taken from the positive direction of the x-axis to the straight line in the direction of anticlockwise and it is $\angle$ DCA

- The another one is negative (i.e. it has a negative measure) taken from the positive direction of the X-axis to the straight line in the direction of clockwise and it is $\angle DCB$

The slope of the straight line

Definition

The slope of the straight line is the tangent of the positive angle which this straight line makes with the positive direction of the X-axis.

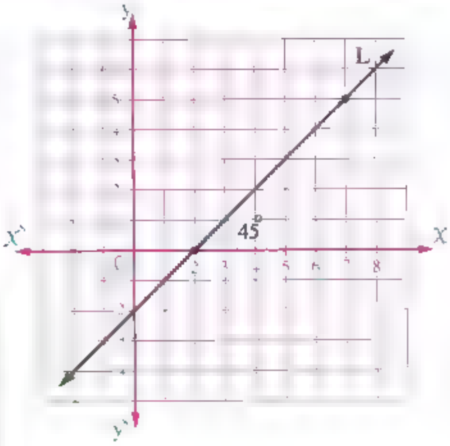
i.e. The slope of the straight line = $\tan \theta$ where θ is the measure of the positive angle which the straight line makes with the positive direction of the X-axis.

For example:

In the opposite figure :

The straight line L makes an angle of measure 45° with the positive direction of the X-axis , then :

the slope of the straight line L = $\tan 45° = 1$



Notice that :

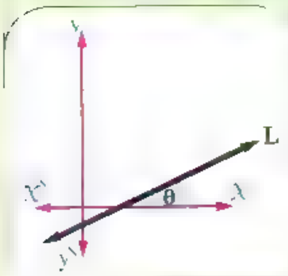
The straight line passes through the two points (2 , 0) and (7 , 5) , then : the slope of the straight line

$$L = \frac{y_2 - y_1}{X_2 - X_1} = \frac{5-0}{7-2} = \frac{5}{5} = 1$$

Remark

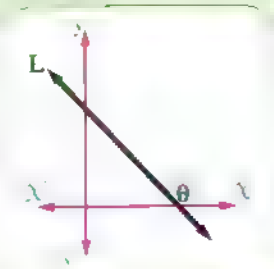
The angle which the straight line L makes with the positive direction of the X-axis takes one of the following cases :

1. **Acute angle**



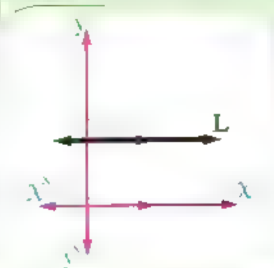
The slope is positive

2. **Obtuse angle**



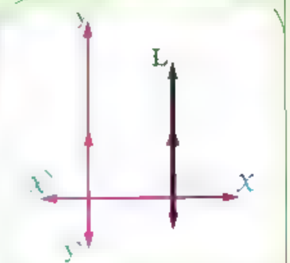
The slope is negative

3. **Zero angle**



The slope is zero

4. **Right angle**



The slope is undefined

Example 1 Find the slope of the straight line which makes a positive angle with the positive direction of X-axis where the measure of the angle is :

1 45° 2 $124^\circ\ 15'\ 12''$

Solution

1 The slope of the straight line = $\tan 45^\circ = 1$ Start → tan 4 5 =

2 The slope of the straight line = $\tan 124^\circ\ 15'\ 12'' \approx -1.4685$

Start → tan 1 2 4 °''' 1 5 °''' 1 2 °''' =

Example 2 Find the measure of the positive angle (θ) which the straight line makes with the positive direction of X-axis if the slope of the straight line is :

1 1.486 2 $-\frac{1}{\sqrt{3}}$

Solution

1 $\because m = \tan\theta$ $\therefore \tan\theta = 1.486$

$\because$ The slope is positive $\therefore \angle\theta$ is an acute angle.

Start → SHIFT tan 1 . 4 8 6 = °'''

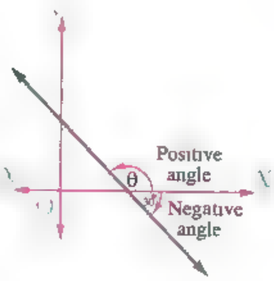
$\therefore m(\angle\theta) \approx 56^\circ\ 3'\ 41''$

2 $\because m = \tan\theta$ $\therefore \tan\theta = -\frac{1}{\sqrt{3}}$

, $\because$ The slope is negative $\therefore \angle\theta$ is an obtuse angle.

By using the calculator as follows :

Start → SHIFT tan (-) 1 ÷ √ 3 =



We will find the calculator gives the result -30°

Where the calculator is programmed to get the acute angle only either negative or positive.

But the required is the positive angle , so we find $m(\angle\theta)$ by finding the supplementary of the angle of measure 30°

Then : $m(\angle\theta) = 180^\circ - 30^\circ = 150^\circ$

Example 3 Find the measure of the positive angle (θ) which the straight line L makes with the positive direction of X-axis if the straight line (L) passes through the two points :

1 $\left(-2 , \sqrt{3}\right) , \left(1 , 4\sqrt{3}\right)$ **2** $(-2 , 3) , (-3 , 4)$

Solution

1 ∵ The straight line L passes through the two points $\left(-2 , \sqrt{3}\right) , \left(1 , 4\sqrt{3}\right)$

∴ The slope of the straight line L

$$= \frac{4\sqrt{3}-\sqrt{3}}{1-(-2)} = \frac{3\sqrt{3}}{3} = \sqrt{3}$$

Notice that : The slope is positive, then the angle is acute.

∴ m (∠ θ) = 60°

2 ∵ The straight line passes through the two points (- 2 , 3) and (– 3 , 4)

∴ The slope of the straight line L

$$= \frac{4-3}{-3-(-2)} = -1$$

Notice that : The slope is negative , then the angle is obtuse.

By using the calculator as follows :

We will find that , the calculator gives the result – 45° (a negative acute angle)

We will find the positive obtuse angle as follows :

m (∠ θ) = 180° – 45° = 135°

TRY by yourself 1

1 Find the slope of the straight line which makes a positive angle with the positive direction of X-axis with measure :

(1) 30° (2) $54^\circ\ 30'\ 6''$ (3) 120°

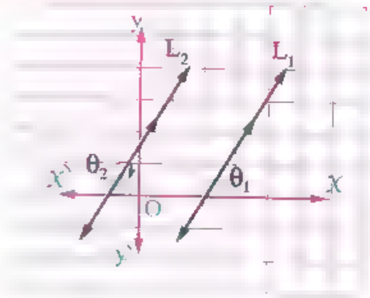
2 Find the measure of the positive angle which the straight line makes with the positive direction of X-axis if the slope of the straight line = 6.2

3 Find the measure of the positive angle (θ) which the straight line L makes with the positive direction of X-axis if the straight line L passes through the two points (4 , –1) and (5 , –3)

The relation between the two slopes of the two parallel straight lines

In the opposite figure :

If L_1 and L_2 are two parallel straight lines of slopes m_1 and m_2 respectively and make two positive angles with the positive direction of X-axis of measures θ_1 and θ_2 respectively , then



$\because L_1 // L_2$ $\quad\quad$ $\therefore \theta_1 = \theta_2$ corresponding angles

$\therefore \tan \theta_1 = \tan \theta_2$ $\quad\quad$ $\therefore m_1 = m_2$

thus we deduce the following :

If $\boxed{L_1 // L_2}$, then $\boxed{m_1 = m_2}$

i.e. If two straight lines are parallel , then their slopes are equal.

Also , we can deduce the opposite :

If $\boxed{m_1 = m_2}$, then $\boxed{L_1 // L_2}$

i.e. If the two straight lines have equal slopes , then the two straight lines are parallel.

Example 4 **Prove that :** The straight line which passes through the two points (2 , 3) and (–1 , 6) is parallel to the straight line which makes with the positive direction of X-axis a positive angle of measure 135°

Solution The slope of the first straight line $m_1 = \frac{6-3}{-1-2} = \frac{3}{-3} = -1$

, the slope of the second straight line $m_2 = \tan 135° = -1$

$\therefore m_1 = m_2$ $\quad\quad$ $\therefore$ The two straight lines are parallel.

Example 5 If A (–1 , 2) , B (2 , 3) , C (– 4 , 1) and D (X , 2) are four points in the Cartesian coordinates plane and $\overleftrightarrow{AB} // \overleftrightarrow{CD}$, **find the value of :** X

Solution

$\because \overleftrightarrow{AB} // \overleftrightarrow{CD}$

$\therefore$ The slope of the straight line passes through A (– 1 , 2) and B (2 , 3) is equal to the slope of the straight line passes through C (– 4 , 1) and D (X , 2)

$\therefore \frac{3-2}{2-(-1)} = \frac{2-1}{X-(-4)}$ $\quad\quad$ $\therefore \frac{1}{3} = \frac{1}{X+4}$

$\therefore X + 4 = 3$ $\quad\quad$ $\therefore X = -1$

Example 6 In the Cartesian coordinates plane , prove that the points A (–1 , 6) , B (3 , – 4) and C (2 , –1.5) are collinear.

Solution $\because$ The slope of $\overleftrightarrow{AB} = \frac{-4-6}{3-(-1)} = \frac{-10}{4} = -\frac{5}{2}$

, the slope of $\overleftrightarrow{BC} = \frac{-1.5-(-4)}{2-3} = \frac{2.5}{-1} = -2\frac{1}{2} = -\frac{5}{2}$

$\therefore$ The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$ $\therefore \overleftrightarrow{AB} \parallel \overleftrightarrow{BC}$

, $\because$ B is a common point between $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$

$\therefore$ A , B and C are collinear.

Notice that :

If the slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$, then A , B and C are collinear points.

TRY by yourself 2

1. ***Prove that :*** The straight line L_1 passing through the two points (1 , 5) and (– 2 , – 1) is parallel to the straight line L_2 that passes through the two points (0 , – 1) and (5 , 9)

2. If the straight line $\overleftrightarrow{AB}$ // the X-axis where A (5 , – 4) and B (– 2 , y) , ***find the value of :*** y

The relation between the slopes of the two perpendicular (orthogonal) straight lines

If L_1 and L_2 are two straight lines of slopes m_1 and m_2 respectively and $L_1 \perp L_2$, then $m_1 \times m_2 = -1$, unless one of them is parallel to one of the coordinate axes.

i.e. The product of the slopes of the perpendicular straight lines = – 1

and vice versa : If L_1 and L_2 are two straight lines of slopes m_1 and m_2 respectively and $m_1 \times m_2 = -1$, then $L_1 \perp L_2$

i.e. If the product of the two slopes of two straight lines equals –1 , then the two straight lines are perpendicular (orthogonal)

Example 7 **Prove that :** The straight line L_1 which passes through the two points (–1 , 4) and (3 , 7) is perpendicular to the straight line L_2 which passes through the two points (1 , 1) and (4 , –3)

Solution $\because$ The slope of $L_1 = \frac{7-4}{3-(-1)} = \frac{3}{4}$, the slope of $L_2 = \frac{-3-1}{4-1} = -\frac{4}{3}$

, $\because$ The slope of L_1 × the slope of $L_2 = \frac{3}{4} \times -\frac{4}{3} = -1$ $\therefore L_1 \perp L_2$

Example 8 In the Cartesian coordinates plane, if the points A (1 , 7) , B (2 , 4) and C (5 , y) represent the vertices of a right-angled triangle at B ,
find the value of : y

Solution $\because$ The slope of $\overleftrightarrow{AB} = \frac{4-7}{2-1} = -3$, the slope of $\overleftrightarrow{BC} = \frac{y-4}{5-2} = \frac{y-4}{3}$,

$\because \overleftrightarrow{AB} \perp \overleftrightarrow{BC}$ $\quad \therefore$ The slope of $\overleftrightarrow{AB} \times$ the slope of $\overleftrightarrow{BC} = -1$

$\therefore -3 \times \frac{y-4}{3} = -1$ $\quad \therefore y - 4 = 1$ $\quad \therefore y = 5$

! Remark

If $L_1 \perp L_2$, the slope of L_1 is m_1 and the slope of L_2 is m_2 where $m_1 \in \mathbb{R}^*$, $m_2 \in \mathbb{R}^*$
, then $m_2 = \frac{-1}{m_1}$, $m_1 = \frac{-1}{m_2}$

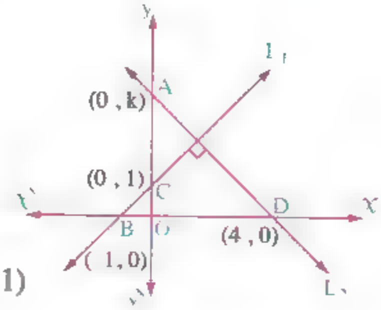
For example:

- If the slope of the straight line L is 2 , then the slope of the perpendicular to it $= -\frac{1}{2}$
- If the slope of the straight line L is $-\frac{2}{3}$, then the slope of the perpendicular to it $= \frac{3}{2}$

Example 9 **In the opposite figure :**

If $L_1 \perp L_2$

Find : The value of k



Solution $\because$ The straight line L_1 passes through the two points B (−1 , 0) and C (0 , 1)

$\therefore$ The slope of $L_1 = \frac{1-0}{0-(-1)} = 1$

, $\because$ the straight line L_2 passes through the two points A (0 , k) and D (4 , 0)

$\therefore$ The slope of $L_2 = \frac{0-k}{4-0} = -\frac{k}{4}$ (1)

, $\because L_1 \perp L_2$, the slope of $L_1 = 1$ $\quad \therefore$ The slope of $L_2 = -1$ (2)

From (1) and (2) : $\therefore -\frac{k}{4} = -1$ $\quad \therefore k = 4$

TRY by yourself 3

1. If A (− 2 , 5) , B (1 , 2) and C (3 , 4) are three points in a Cartesian coordinates plane , ***prove that :*** $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$
2. ***Prove that :*** The straight line which passes through the two points (7 , − 1) and (5 , −3) is perpendicular to the straight line which makes with the positive direction of X-axis an angle of measure 135°

Remarks to solve the problems on quadrilateral

To prove that a quadrilateral is a trapezium , we prove that :

Two opposite sides are parallel and the other two sides are not parallel.

To prove that a quadrilateral is a parallelogram , we prove only one of the following properties :

1. Each two opposite sides are parallel.
2. Each two opposite sides are equal in length.
3. Two opposite sides are parallel and equal in length.
4. The two diagonals bisect each other.

To prove that a quadrilateral is a rectangle, rhombus or square, we prove at first that the quadrilateral is a parallelogram, then :

- **To prove that the parallelogram is a rectangle, we prove only one of the following two properties :**
  1. Two adjacent sides are perpendicular.
  2. The two diagonals are equal in length.
- **To prove that the parallelogram is a rhombus, we prove only one of the following two properties :**
  1. Two adjacent sides are equal in length.
  2. The two diagonals are perpendicular.
- **To prove that the parallelogram is a square, we prove only one of the following properties :**
  1. Two adjacent sides are perpendicular and equal in length.
  2. Two adjacent sides are perpendicular and its diagonals are perpendicular.
  3. Two diagonals are equal in length and perpendicular.
  4. Two adjacent sides are equal in length and its two diagonals are equal in length.

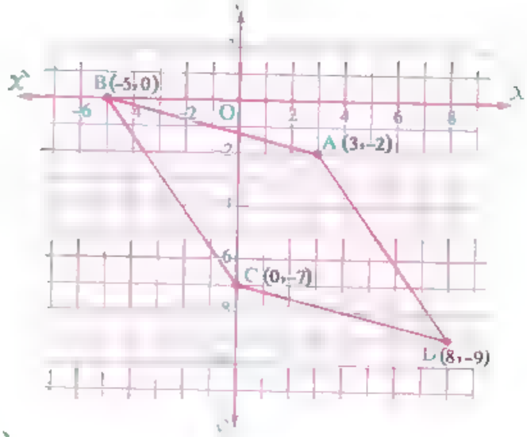
Example 10 On a Cartesian coordinates plane , represent the points A $(3 , -2)$, B $(-5 , 0)$, C $(0 , -7)$ and D $(8 , -9)$, then prove that the quadrilateral ABCD is a parallelogram.

Solution $\because$ The slope of $\overleftrightarrow{AB} = \dfrac{0-(-2)}{-5-3}$

$= \dfrac{2}{-8} = -\dfrac{1}{4}$

, the slope of $\overleftrightarrow{CD} = \dfrac{-9-(-7)}{8-0}$

$= \dfrac{-2}{8} = -\dfrac{1}{4}$



$\therefore$ The slope of $\overleftrightarrow{AB}$

= the slope of $\overleftrightarrow{CD}$

$\therefore \overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ (1)

$\because$ The slope of $\overleftrightarrow{AD} = \dfrac{-9-(-2)}{8-3} = \dfrac{-7}{5}$, the slope of $\overleftrightarrow{BC} = \dfrac{-7-0}{0-(-5)} = \dfrac{-7}{5}$

$\therefore$ The slope of $\overleftrightarrow{AD}$ = the slope of $\overleftrightarrow{BC}$

$\therefore \overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$ (2)

From (1) and (2) : $\therefore$ The quadrilateral ABCD is a parallelogram.

Example 11 **Prove that :** The points A $(2 , -2)$, B $(8 , 4)$, C $(5 , 7)$ and D $(-1 , 1)$ are vertices of the rectangle ABCD

Solution $\because$ The slope of $\overleftrightarrow{AB} = \dfrac{-2-4}{2-8} = \dfrac{-6}{-6} = 1$

, the slope of $\overleftrightarrow{CD} = \dfrac{7-1}{5-(-1)} = \dfrac{6}{6} = 1$

$\therefore$ The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{CD}$ $\therefore \overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ (1)

$\because$ The slope of $\overleftrightarrow{AD} = \dfrac{-2-1}{2-(-1)} = \dfrac{-3}{3} = -1$

, the slope of $\overleftrightarrow{BC} = \dfrac{4-7}{8-5} = \dfrac{-3}{3} = -1$

$\therefore$ The slope of $\overleftrightarrow{AD}$ = the slope of $\overleftrightarrow{BC}$ $\therefore \overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$ (2)

From (1) and (2) we deduce that the quadrilateral ABCD is a parallelogram.

$\because$ The slope of $\overleftrightarrow{AB} \times$ the slope of $\overleftrightarrow{BC} = 1 \times -1 = -1$

$\therefore \overleftrightarrow{AB} \perp \overleftrightarrow{BC}$ $\therefore$ The quadrilateral ABCD is a rectangle.

Example 12 On a Cartesian coordinates plane , represent the points A (– 3 , – 3) , B (3 , 1) , C (1 , 5) and D (– 2 , 3) , then prove that the quadrilateral ABCD is a trapezium.

Solution

$\because$ The slope of $\overleftrightarrow{CD} = \frac{5-3}{1-(-2)} = \frac{2}{3}$

, the slope of $\overleftrightarrow{AB} = \frac{1-(-3)}{3-(-3)} = \frac{4}{6} = \frac{2}{3}$

$\therefore$ The slope of $\overleftrightarrow{CD}$ = the slope of $\overleftrightarrow{AB}$

$\therefore \overleftrightarrow{CD} \parallel \overleftrightarrow{AB}$ (1)

The slope of $\overleftrightarrow{BC} = \frac{5-1}{1-3} = -2$

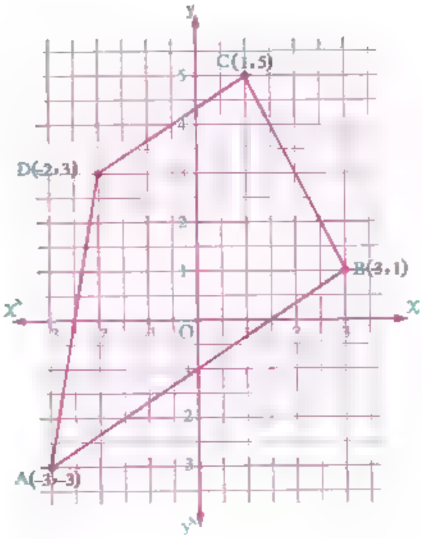
, the slope of $\overleftrightarrow{AD} = \frac{3-(-3)}{-2-(-3)} = 6$

$\therefore$ The slope of $\overleftrightarrow{BC} \neq$ the slope of $\overleftrightarrow{AD}$

$\therefore \overleftrightarrow{BC}$ is not parallel to $\overleftrightarrow{AD}$ (2)

From (1) and (2) :

$\therefore$ The quadrilateral ABCD is a trapezium.



Answers of try by yourself

1 | 1 | (1) 0.58 (approximately) (2) 1.4 (approximately) (3) - 1.73 (approximately)

2 | 80° 50′ 16″ (approximately)

3 | 116° 33′ 54″ (approximately)

2 | 1 | Prove by yourself [Hint : The slope of L_1 = The slope of L_2]

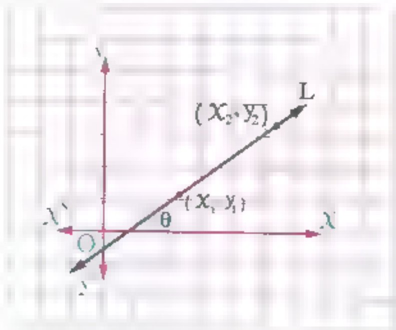
2 | $y = -4$

3 | 1 | Prove by yourself [Hint : The slope of $\overleftrightarrow{AB}$ × The slope of $\overleftrightarrow{BC} = -1$]

2 | Prove by yourself [Hint : The product of the slopes of the two straight lines = – 1]

4 The equation of the straight line given its slope and the intercepted part of y-axis

We studied before that the relation : a x + b y + c = 0 where a ≠ 0 , b ≠ 0 together is a linear relation represented graphically by a straight line and we can find its slope (m) by one of the following methods :



1 $m = \dfrac{\text{vertical change}}{\text{horizontal change}} = \dfrac{y_2 - y_1}{x_2 - x_1}$

Where (x_1 , y_1) and (x_2 , y_2) are two points on the straight line

2 $m = \tan \theta$

Where θ is the measure of the positive angle which the straight line makes with the positive direction of the x-axis.

- **We will continue our study about this subject by studying how :**
  - To find the slope of the straight line and the length of the intercepted part from y-axis if we know the equation of the straight line.
  - To find the equation of the straight line if we know its slope and the length of the intercepted part from the y-axis.

First Finding the slope of the straight line and the length of the intercepted part from y-axis

Prelude example

Represent graphically the relation : $2x - y + 3 = 0$ and from the graph , find the slope of the straight line which represents the relation and the intercepted part of the y-axis by the straight line.

Solution

To graph the straight line which represents the relation , find two points of the points of the straight line at least , to facilitate that , put one of the variables x or y in a side of the equation

$\because 2x - y + 3 = 0$ $\therefore -y = -2x - 3$ $\therefore y = 2x + 3$

At $x = 0$ $\therefore y = 0 + 3 = 3$

$\therefore$ (0 , 3) is one of the points of the straight line.

At $x = -1$ $\therefore y = -2 + 3 = 1$

$\therefore$ (– 1 , 1) is one of the points of the straight line.

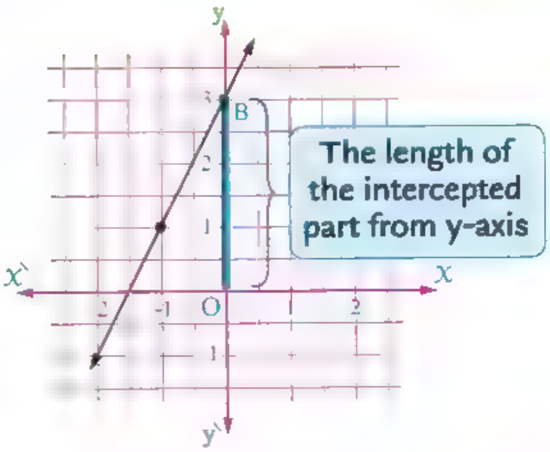
i.e. The straight line passes through the two points (0 , 3) and (– 1 , 1)

$$\therefore \text{The slope of the straight line} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 3}{-1 - 0} = \frac{-2}{-1} = 2$$

- From the graph, we find that :

  OB = 3 length units.

i.e. The straight line intercepts from the positive part from y-axis 3 length units



Observing the graph of the straight line : $y = 2x + 3$

we find that :

- The slope of the straight line = the coefficient of $x = 2$
- The length of the intercepted part from y-axis = | absolute term | = | 3 | = 3 length units.

The slope of the straight line

$$y = 2x + 3$$

The length of the intercepted part from y-axis

i.e.

If the equation of a straight line is in the form : $y = m x + c$, then :

- The slope of the straight line $= m$
- The length of the intercepted part from y-axis $= |c|$ and it passes through the point $(0 , c)$

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Example 1 Find the slope of the straight line : $2x + 5y - 15 = 0$, then find the intercepted part of y-axis.

Solution Write the equation of the straight line in the form : $y = m x + c$

$\therefore 5y = -2x + 15$ $\qquad \therefore y = \frac{-2}{5}x + 3$

$\therefore$ The slope of the straight line $= \frac{-2}{5}$ and the intercepted part of the positive part of y-axis is of length $= 3$ length units.

Remark

In the previous example , observing the equation in the form : $2x + 5y - 15 = 0$, we find that :

- The slope of the straight line $= \frac{-\text{coefficient of } x}{\text{coefficient of } y} = \frac{-2}{5}$
- The straight line cuts y-axis at the point $\left(0 , \frac{-\text{absolute term}}{\text{coefficient of } y}\right)$ i.e. $(0 , 3)$

i.e. The straight line intercepts a part of y-axis of length $= \left|\frac{-\text{absolute term}}{\text{coefficient of } y}\right|$

$= |3| = 3$ length units.

i.e.

If the equation of a straight line is in the form : $a x + b y + c = 0$, then

- The slope of the straight line $= \frac{-\text{coefficient of } x}{\text{coefficient of } y} = \frac{-a}{b}$
- The straight line cuts y-axis at the point $\left(0 , \frac{-c}{b}\right)$

i.e. The length of the intercepted part from y-axis $= \left|\frac{-c}{b}\right|$

For example:

- The straight line whose equation is : $x - 2y + 3 = 0$

  Its slope $= \frac{-1}{-2} = \frac{1}{2}$ and cuts y-axis at the point $\left(0 , \frac{3}{2}\right)$

  i.e. The straight line intercepts a part of length $\frac{3}{2}$ length unit from the positive part of y-axis.

- The straight line whose equation is : $3x + y + 4 = 0$

  Its slope $= -3$ and cuts y-axis at the point $(0 , -4)$

  i.e. The straight line intercepts a part of length 4 length units from the negative part of y-axis.

Example 2 If the straight line that passes through the two points $(-1 , 7)$ and $(9 , 3)$ is perpendicular to the straight line whose equation is : $x + k y - 13 = 0$, **find the value of :** k

Solution Let the slope of the straight line that passes through the two points $(-1 , 7)$ and $(9 , 3)$ be m_1

$$\therefore m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-7}{9-(-1)} = \frac{-4}{10} = \frac{-2}{5}$$

Let the slope of the straight line whose equation is : $x + k y - 13 = 0$ be m_2

$$\therefore m_2 = \frac{-a}{b} = \frac{-1}{k}$$

$\because$ The two straight lines are perpendicular

$\therefore m_1 \times m_2 = -1$ $\qquad \therefore \frac{-2}{5} \times \frac{-1}{k} = -1$

$\therefore \frac{2}{5k} = -1$ $\qquad \therefore -5k = 2$ $\qquad \therefore k = \frac{-2}{5}$

TRY by yourself 1

1. If the two straight lines : $3y + x - 7 = 0$ and $y = kx + 5$ are perpendicular , then find the value of : k

2. Find the measure of the positive angle which is made by the straight line whose equation is : $3x - 3y + 5 = 0$ with the positive direction of x-axis.

3. Find the length of the intercepted part from y-axis by the straight line whose equation is : $2y = 3x + 12$

Second: Finding the equation of the straight line given its slope and the length of intercepted part of y-axis

The straight line whose slope = m and cuts y-axis at the point (0 , c) its equation is in the form :

$$\boxed{y = m x + c}$$

Example 3 Find the equation of the straight line :

1. Whose slope $= -\frac{3}{4}$ and intercepts from the positive part of y-axis 3 length units.
2. Whose slope = 2 and intercepts from the negative part of y-axis 7 length units.

Solution $y = m x + c$

1. $\because m = -\frac{3}{4}$, $c = 3$ $\quad$ $\therefore$ The equation is : $y = -\frac{3}{4} x + 3$
2. $\because m = 2$, $c = -7$ $\quad$ $\therefore$ The equation is : $y = 2 x - 7$

Example 4 Find the equation of the straight line which makes with the positive direction of x-axis a positive angle of measure 135° and intercepts from the positive part of y-axis a part of length 7 length units.

Solution $\because$ The slope $= \tan 135° = -1$

$\therefore$ The equation of the straight line is : $y = -x + 7$

! Remarks

1. The equation of the straight line which passes through the origin point O (0 , 0) is $\boxed{y = m x}$, where m is the slope of the straight line.
2. The equation of x-axis is $\boxed{y = 0}$
3. The equation of y-axis is $\boxed{x = 0}$
4. The equation of the straight line which is parallel to x-axis and passes through the point $(0 , \ell)$ is $\boxed{y = \ell}$
5. The equation of the straight line which is parallel to y-axis and passes through the point (k , 0) is $x = k$

Example 5 Find the equation of the straight line which passes through the two points $(1 , -1)$ and $(2 , 2)$

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Solution Let the equation of the straight line be in the form $y = m\,x + c$

$\because$ The slope $(m) = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2 - (-1)}{2 - 1} = 3$

$\therefore$ The equation of the straight line is in the form : $y = 3\,x + c$

$\because$ $(1 , -1)$ belongs to the straight line.

$\therefore -1 = 3 \times 1 + c$ $\qquad \therefore c = -1 - 3 = -4$

$\therefore$ The equation of the straight line is : $y = 3\,x - 4$

Example 6 Find the equation of the straight line which passes through the point $(1 , 2)$ and parallels the straight line $2\,x + 3\,y - 6 = 0$

Solution $\because$ The slope of the given straight line $= \dfrac{-\text{coefficient of } x}{\text{coefficient of } y} = \dfrac{-2}{3}$

$\therefore$ The slope of the required straight line $= \dfrac{-2}{3}$

$\therefore$ The equation of the required straight line is : $y = -\dfrac{2}{3}\,x + c$

$\because$ The straight line passes through the point $(1 , 2)$

$\therefore 2 = -\dfrac{2}{3} \times 1 + c$ $\qquad \therefore c = \dfrac{8}{3}$

$\therefore$ The equation of the required straight line is : $y = -\dfrac{2}{3}\,x + \dfrac{8}{3}$

Example 7 Find the equation of the straight line which passes through the point $(2 , 3)$ and perpendicular to the straight line passing through the two points $A\,(3 , -4)$ and $B\,(5 , -3)$

Solution $\because$ The slope of the straight line which passes through the two points $(3 , -4)$ and $(5 , -3)$ equals $\dfrac{-3 - (-4)}{5 - 3} = \dfrac{1}{2}$

$\therefore$ The slope of the required straight line $= -2$

$\therefore$ The equation of the required straight line is $y = -2\,x + c$

$\because$ The straight line passes through the point $(2 , 3)$

$\therefore$ It satisfies the equation.

$\therefore 3 = -2 \times 2 + c$ $\qquad \therefore c = 7$

$\therefore$ The equation of the required straight line is : $y = -2\,x + 7$

TRY by yourself 2

1. Find the equation of the straight line which intercepts from the positive part of y-axis 5 length units and it is parallel to the straight line passing through the two points $(-2, 3)$ and $(-1, -6)$

2. Find the equation of the straight line which passes through the point $(3, 4)$ and perpendicular to $\overleftrightarrow{AB}$, where $A(2, -3)$ and $B(5, 4)$

Example 8 ABC is a triangle whose vertices are $A(1, 2)$, $B(-2, 3)$, $C(-4, -3)$, $\overline{AD}$ is a median of it, find the equation of $\overleftrightarrow{AD}$

Solution $\because$ $\overline{AD}$ is a median of ΔABC

$\therefore$ D is the midpoint of $\overline{BC}$

$\therefore D = \left(\frac{-2+(-4)}{2}, \frac{3+(-3)}{2}\right) = (-3, 0)$

$\therefore$ The slope of $\overleftrightarrow{AD} = \frac{2-0}{1-(-3)} = \frac{1}{2}$

$\therefore$ The equation of $\overleftrightarrow{AD}$ is : $y = \frac{1}{2}x + c$

$\because$ $\overleftrightarrow{AD}$ passes through the point $A = (1, 2)$

$\therefore$ It satisfies its equation

$\therefore 2 = \frac{1}{2} \times 1 + c$ $\qquad\qquad \therefore c = \frac{3}{2}$

$\therefore$ The equation of $\overleftrightarrow{AD}$ is : $y = \frac{1}{2}x + \frac{3}{2}$

TRY by yourself 3

ABC is a triangle whose vertices are $A(-1, 5)$, $B(4, -2)$ and $C(-3, 0)$

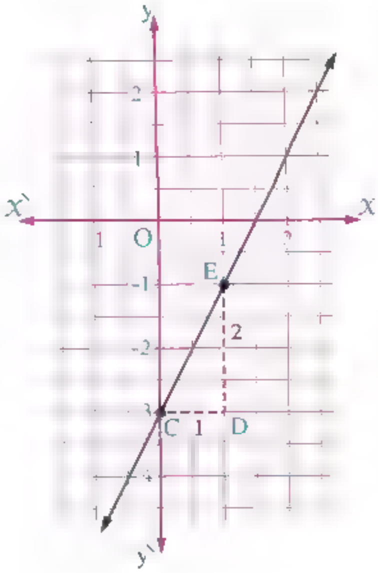
Find the equation of the straight line passing through A and perpendicular to $\overline{BC}$

Example 9 Using the slope and the intercepted part of y-axis, represent graphically the straight line whose equation is $y = 2x - 3$

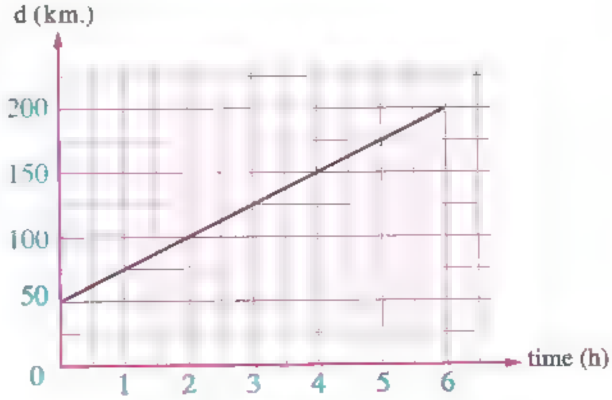
Solution The slope of the straight line $= 2 = \frac{2}{1} = \frac{\text{vertical change}}{\text{horizontal change}}$

and the straight line passes through the point $C(0, -3)$

From the point C , we move horizontally towards the right one unit (the horizontal change (+1)) to reach the point D , then we move vertically towards up two units (the vertical change (+2)) to reach the point E, then $\overleftrightarrow{CE}$ is the graph of the equation of the straight line $y = 2x - 3$



Example 10 The opposite graph represents the motion of a car moving with a uniform velocity where the distance (d) is measured in km. and the time (t) in hours , **find :**



1. The distance (d) at the beginning of the motion.
2. The velocity of the car.
3. The equation of the straight line representing the motion of the car.

Solution

1. The distance (d) at the beginning of the motion = 50 kilometres.
2. The velocity of the car = the slope of the straight line passing through the two points (0 , 50) and (6 , 200) $= \frac{200 - 50}{6 - 0} = \frac{150}{6}$
   $= 25$ km./hr.
3. The equation of the straight line is : $d = m\,t + c$

   i.e. $\boxed{d = 25\,t + 50}$

Example 11 Find the equation of the straight line which intercepts from the coordinate axes (X-axis and y-axis) two positive parts with lengths 3 and 4 length units respectively, then find the area of the triangle included between the straight line and the two axes.

Solution

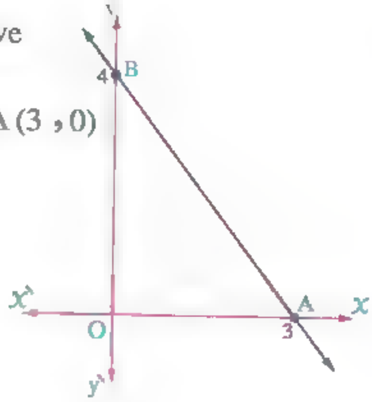
$\because$ The straight line intercepts from the positive part of X-axis 3 length units.

$\therefore$ The straight line passes through the point A (3 , 0)

$\because$ The straight line intercepts from the positive part of y-axis 4 length units.

$\therefore$ The straight line passes through the point B (0 , 4)

$\therefore$ The straight line passes through the two points A (3 , 0) and B (0 , 4)



Let the equation of the required straight line be $y = m X + c$

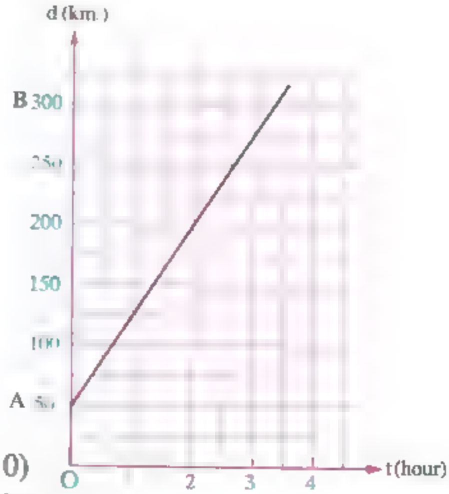
, where the slope $(m) = \frac{4-0}{0-3} = -\frac{4}{3}$ $\qquad \therefore y = -\frac{4}{3} X + c$

, $\because c = 4$ $\qquad \therefore$ The equation is : $y = -\frac{4}{3} X + 4$

, the area of Δ ABO $= \frac{1}{2} \times AO \times BO = \frac{1}{2} \times 3 \times 4 = 6$ square units.

TRY by yourself 4

A person moved between the cities A and B using his car with a uniform velocity and the opposite graph represents the relation between the distance (d) in kilometres and the time (t) in hours.



Answer the following :

1. What is the uniform velocity of the car ?
2. Find the equation of the straight line representing the motion of the car.
3. Find the distance between the car and O (0 , 0) after 3 hours from the beginning of the motion.

Answers *of try by yourself*

1 [1] 3 [2] 45° [3] 6 length units

2 [1] $y = -9 X + 5$ [2] $y = -\frac{3}{7} X + \frac{37}{7}$

3 $y = \frac{7}{2} X + \frac{17}{2}$

4 [1] 75 km./hr. [2] $d = 75 t + 50$ [3] 275 km.

EL-MOASSER

By a group of supervisors

EXERCISES

3rd PREP.

FIRST TERM

Maths

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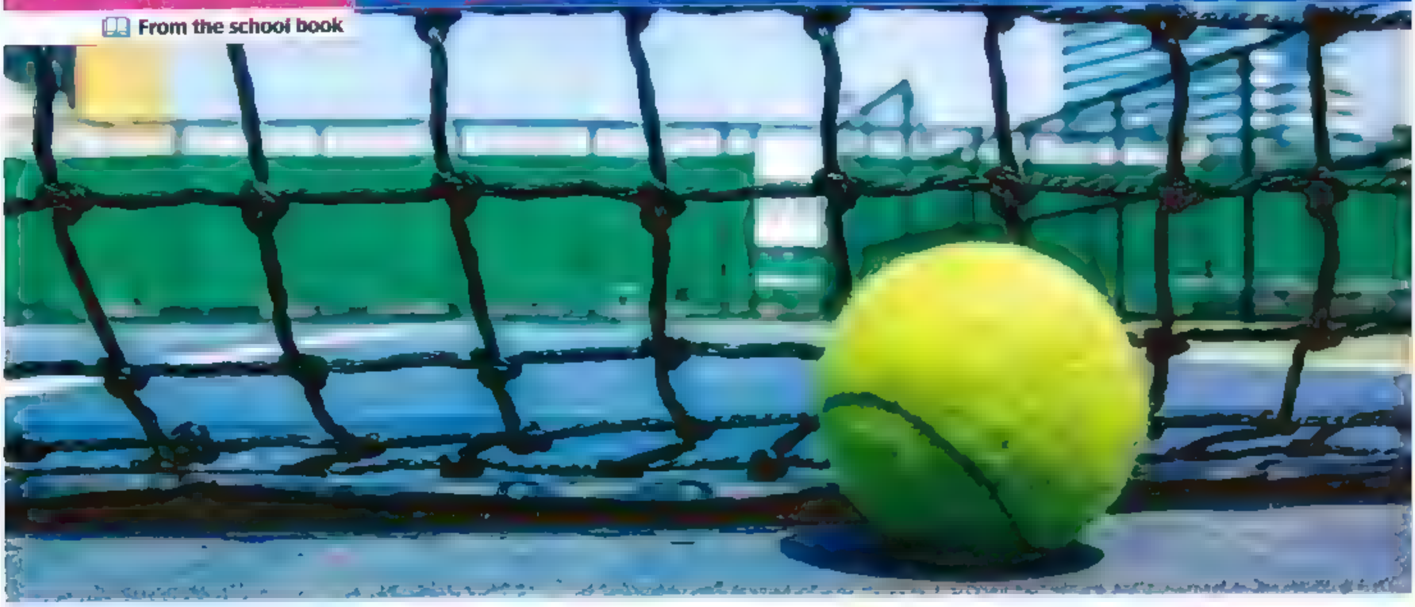
UNIT 1

Cartesian product

Interactive test

From the school book



Remember Understand Apply Problem Solving

First Problems on the equality of two orderd pairs

1 Find the values of a and b in each of the following if :

1. $(a , b) = (-5 , 9)$
2. $(a , b) = \left(\sqrt{25} , \sqrt[3]{27}\right)$
3. $(a - 2 , b + 1) = (2 , -3)$
4. $(6 , b - 3) = (2 - a , -1)$
5. $(a - 7 , 26) = (-2 , b^3 - 1)$
6. $(a , b) = (2 - a , 2b - 3)$
7. $(a^5 , b^2 - 1) = \left(32 , \sqrt[3]{27}\right)$
8. $(a , 7) = (b^2 , b)$
9. $(2a , 7) = (2b + 1 , a)$
10. $(3 , b) = (5a - 1 , 4a)$

2 Choose the correct answer from those given :

1. If $(x - 1 , 11) = (8 , y + 3)$, then $\sqrt{x + 2y} =$ .... *(Port Said 19)*
   (a) 5 (b) ± 5 (c) $\sqrt{17}$ (d) 25
2. If $(x + 2 , y) = (2 , 3)$, then $x^5 y + 1 =$ .... *(El-Sharkia 20)*
   (a) 3 (b) 2 (c) zero (d) 1
3. If $\left(3^x , \sqrt{y}\right) = (1 , 4)$, then $x + y =$ .... *(El-Gharbia 18)*
   (a) 2 (b) 3 (c) 16 (d) 17
4. If $(a + 2 , 63) = (-1 , b^3 - 1)$, then $\sqrt{a^2 + b^2} =$ ....
   (a) 25 (b) 7 (c) 5 (d) ± 5
5. If $(x - 3 , 2^y) = (2 , 16)$, then $(y , x) =$ ....
   (a) (1 , 4) (b) (5 , 4) (c) (4 , 1) (d) (4 , 5)

Second Problems on the Cartesian product of two finite sets

3 If $X = \{1, 2\}$, $Y = \{3, 4, 5\}$, **find $X \times Y$ and represent it by :**

1 The arrow diagram. 2 The Cartesian diagram.

4 If $X = \{3, 4, 8\}$, **find X^2 and represent it :**

1 By an arrow diagram. 2 By a Cartesian diagram.

5 If $X = \{1, 2, 3\}$, $Y = \{4\}$, **find :**

1 $X \times Y$ 2 $Y \times X$ 3 Y^2 4 $n(X^2)$

6 If $X = \{2, -1\}$, $Y = \{4, 0\}$, $Z = \{4, 5, -2\}$, **find :**

1 $X \times Y$ 2 $Y \times Z$ 3 X^2
4 $n(X \times Z)$ 5 $n(Y^2)$ 6 $n(Z^2)$

7 Choose the correct answer from those given :

1 If A and B are two sets , then the set $\{(x, y) : x \in A , y \in B\}$ expresses .... *(El-Dakahlia 16)*

(a) $n(A \times B)$ (b) $A \times B$ (c) $n(B \times A)$ (d) $B \times A$

2 If $X = \{1, 2\}$, then $X \times \varnothing =$ ....

(a) X (b) $\varnothing$ (c) $\{0\}$ (d) $\{(1, 0), (2, 0)\}$

3 If $X = \{2\}$, $Y = \{3\}$, then $X \times Y =$ .... *(Giza 17)*

(a) 6 (b) $\{6\}$ (c) $(2, 3)$ (d) $\{(2, 3)\}$

4 If $X = \{3\}$, then $X^2 =$ .... *(Cairo 13 – El-Sharkia 17)*

(a) 9 (b) $(3, 3)$ (c) $\{9\}$ (d) $\{(3, 3)\}$

5 If $X = \{3\}$, then $n(X^2) =$ .... *(Qena 20)*

(a) 1 (b) 9 (c) $\{3, 3\}$ (d) 3

6 If $X = \{1, 2\}$ and $Y = \{3, 4\}$, then $(3, 4) \in$ .... *(Qena 11 – Suez 19)*

(a) $X \times Y$ (b) $Y \times X$ (c) X^2 (d) Y^2

7 If $n(X) = 2$, $Y = \{1, 2\}$, then $n(X \times Y) =$ .... *(Giza 15)*

(a) 4 (b) 3 (c) 5 (d) 6

8 If $n(X) = 3$, $n(X \times Y) = 12$, then $n(Y) =$ ....

(Damietta 15 – Port Said 17 – Cairo 18 – El-Menia 19 – Port Said 20)

(a) 4 (b) 9 (c) 15 (d) 36

9 If $n(X^2) = 9$, then $n(X) = \dots\dots$

(a) 2 (b) 3 (c) 9 (d) 81

10 If $n(X^2) = 4$, $n(X \times Y) = 6$, then $n(Y^2) = \dots\dots$ *(Giza 20)*

(a) 4 (b) 9 (c) 16 (d) 12

11 If X is a non-empty set , $n(X) = n(X \times Y)$, then $n(Y) = \dots\dots$ *(Damietta 18)*

(a) 1 (b) 2 (c) 3 (d) 4

12 If $a \in X^2$, where $X = \{x : 5 < x < 7 , x \in \mathbb{N}\}$, then $a = \dots\dots$ *(El Sharkia 20)*

(a) 36 (b) $\{36\}$ (c) $(6 , 6)$ (d) $[5 , 7]$

13 If $(3 , 5) \in \{3 , 6\} \times \{x , 8\}$, then $x = \dots\dots$

(Qena 15 – Kafr El-Sheikh 18 – Port Said 19 – Alex. 20)

(a) 8 (b) 6 (c) 5 (d) 3

14 If $\{2\} \times \{x , y\} = \{(2 , 4) , (2 , 3)\}$, then $x - y = \dots\dots$

(El-Sharkia 15 – Kafr El Sheikh 20)

(a) 1 (b) -1 (c) ± 1 (d) 0

---

8 If $X \times Y = \{(2 , 6) , (2 , 9) , (3 , 6) , (3 , 9) , (5 , 6) , (5 , 9)\}$, **find :** X and Y

9 If $X \times Y = \{(1 , 1) , (1 , 3) , (1 , 5)\}$, **find :**

1 X and Y 2 $Y \times X$ 3 Y^2

(Giza 16 – Souhag 19 – El-Kalyoubia 20)

10 If $X^2 = \{(1 , 1) , (1 , 2) , (2 , 1) , (2 , 2)\}$, **find :** X

11 If $Y \times X = \{(1 , 3) , (2 , 3) , (3 , 3)\}$, **find :** X^2

12 If $X = \{1 , 2 , 3 , 4\}$, $Y = \{3 , 4 , 5\}$, **represent X and Y by Venn diagram , then find :**

1 $(X \cap Y) \times Y$ 2 $(X - Y) \times Y$ 3 $(Y - X) \times X$

13 If $X = \{3 , 4\}$, $Y = \{4 , 5\}$ and $Z = \{6 , 5\}$, **then find :**

1 $X \times (Y \cap Z)$ 2 $(X - Y) \times Z$ 3 $(X - Y) \times (Y - Z)$

(El-Dakahlia 13 – El-Monofia 18 – El-Menia 19)

14 If $X = \{1\}$, $Y = \{2 , 3\}$, $Z = \{2 , 5 , 6\}$

, represent each of X , Y and Z by Venn diagram , then find :

First : 1 $X \times Y$ 2 $Y \times Z$ 3 $X \times Z$ 4 Y^2

Second : $(X \times Y) \cup (Y \times Z)$

Third : $X \times (Y \cap Z)$

Fourth : $(X \times Y) \cap (X \times Z)$

Fifth : $(Z - Y) \times (X \cup Y)$

Third Problems on the Cartesian product of two infinite sets

15 **Identify the following points on a perpendicular graphical net of the Cartesian product $\mathbb{R} \times \mathbb{R}$:**

A $(4 , 5)$, B $(6 , -3)$, C $(-2 , 7)$, D $(-1 , 6)$, E $(-4 , -5)$

, M $(0 , 6)$, K $(9 , 0)$

Then mention the quadrant that each point is located on the perpendicular graphical net or the axis it belongs to.

---

16 **Choose the correct answer from those given :**

1 Which of the following points lies on the second quadrant ?

(a) $(3 , 2)$ (b) $(-4 , 5)$ (c) $(-3 , -2)$ (d) $(2 , -3)$

2 If the point $(a - b , 5)$ lies on the y-axis , then .... *(Giza 18)*

(a) $a = b$ (b) $a + b = 0$ (c) $a \neq b$ (d) $a - b = 5$

3 If the point $(5 , b - 7)$ is located on the X-axis ,

then b = ... *(Alex. 11 – North Sinai 16 – Qena 17 – Cairo 18 – El-Kalyoubia 20)*

(a) 2 (b) 5 (c) 7 (d) 12

4 If the point $(X , 7)$ lies on the y-axis , then $5 X + 1 =$ .... *(El-Beheira 17)*

(a) zero (b) 1 (c) 5 (d) 6

5 If $\left(X + 1 , \sqrt[3]{27}\right) = (-1 , y)$, then the point (X , y) lies in the .... quadrant. *(El-Fayoum 20)*

(a) first (b) second (c) third (d) fourth

6 If $b < 3$, then the point $(5 , b - 3)$ lies in the .... quadrant. *(Cairo 16)*

(a) first (b) second (c) third (d) fourth

7 If $X \in \mathbb{R}$, then the point $\left(-X , \sqrt[3]{X}\right)$ lies in the .... quadrant. *(El-Monofia 20)*

(a) first (b) second (c) third (d) fourth

8 If the point (a , b) lies in the fourth quadrant , then $a \times b$ .... zero.

(a) = (b) > (c) < (d) ≥

9 If the point $(2 a , 3 b) \in \overleftrightarrow{XX'}$, then $\frac{b}{a} =$ .... (where $a \neq 0$)

(a) zero (b) $\frac{2}{3}$ (c) 2 (d) 3

10 If $(|X| , 4) = (3 , y^2)$ and the point (X , y) lies in the second quadrant ,

then $X + y =$ .... *(El-Sharkia 14)*

(a) 7 (b) 1 (c) -1 (d) -7

11 If a < zero , b > zero , then the point which lies in the second quadrant is .... *(El-Fayoum 18)*

(a) (a , b) (b) (– a , b) (c) (a , – b) (d) (– a , – b)

12 If the point (X – 2 , X – 4) lies in the fourth quadrant , then X = ... where $X \in \mathbb{Z}$

(a) zero (b) 2 (c) 3 (d) 4

13 If the point (X – 4 , 2 – X) where $X \in \mathbb{Z}$ is located in the third quadrant , then X = ... *(Alex. 15 – Kafr El-Sheikh 16 – El-Monofia 17 – Port Said 19 – El-Beheira 20)*

(a) 2 (b) 3 (c) 4 (d) 6

14 If the point (k^2 – 4 , k) lies on the negative part of y-axis , then k = ... *(El-Sharkia 18)*

(a) ± 2 (b) 4 (c) – 2 (d) 2

17 If A (– 2 , 0) , B (– 2 , 3) , C (2 , 3) , identify on the perpendicular square net $\mathbb{R}^2$ the points A , B , C and find the area of Δ ABC « 6 square units »

Fourth Problems on the Cartesian product of two intervals

18 If X = [– 2 , 3] , find the location which represents X × X

Show which of the following points belongs to the Cartesian product of X × X

A (1 , 2) , B (3 , – 1) , C (– 1 , 4) and D (– 2 , 0)

19 If X = [– 2 , 3] , Y = [0 , 4] , **find the region which represents each of :**

1 X × Y 2 Y × X 3 Y^2

For excellent pupils

20 **Choose the correct answer from those given :**

1 If (X – Y) × Y = {(1 , 2) , (1 , 3)} , n (X × Y) = 6 , then X = ... *(El Sharkia 13)*

(a) {1} (b) {1 , 2} (c) {1 , 3 , 6} (d) {1 , 3 , 2}

2 If X – Y = {7} , Y – X = {2 , 4} , X ⋂ Y = {6} , then (X × Y) ⋂ (Y × X) = ...

(a) {(6 , 6)} (b) {(7 , 2) , (7 , 4)}

(c) {(2 , 7) , (4 , 7)} (d) {(7 , 6)}

3 $\{3\} \times [0, 2]$ is represented graphically in the figure ....

(a) (b) (c) (d)

21 If $X \subset Y$, $X \times Y = \{(a, 1), (a, 2), (a, 3), (2, 1), (2, 2), (2, 3)\}$

, **find the values of :** a

22 If $X \subset Y$, $n(X \times Y) = 6$, $4 \in X$ and $(1, 7) \in X \times Y$,

then find X , Y and $X \times Y$ *(Damietta 17)*

EXERCISE **2**

Relation - Function (mapping)

interactive test

From the school book

● Remember ● Understand ○ Apply Problem Solving

First Problems on relation and function from a set to another set

1 **Choose the correct answer from those given :**

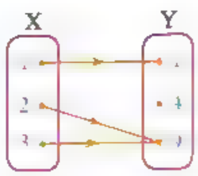
1. If f is a function from the set X to the set Y , then X is called ....
   (a) the range of the function f (b) the domain of the function f
   (c) the codomain of the function f (d) the rule of the function f

2. If f is a function from the set X to the set Y , then Y is called ....
   (a) the domain of the function. (b) the codomain of the function.
   (c) the range of the function. (d) the rule of the function.

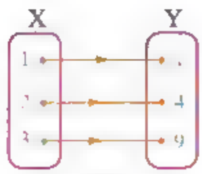
3. If the relation $R = \{(4 , 3) , (1 , 3) , (2 , 5)\}$, then R represents a function where its range is .... *(El Kalyoubia 17)*
   (a) $\{1 , 2 , 4\}$ (b) $\{4 , 1 , 2 , 3 , 5\}$ (c) $\{3 , 5\}$ (d) $\mathbb{N}$

4. If R is a function from X to Y where $X = \{2 , 4 , 5\}$, $Y = \{6 , 7\}$ and $R = \{(2 , 6) , (a , 6) , (5 , 6)\}$, then a = ...
   (a) 4 (b) 5 (c) 12 (d) 6

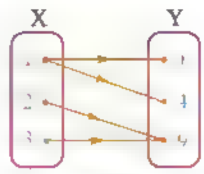
2 **Which of the following relations represents a function from X to Y ?**
If the relation represents a function , then find the function range :



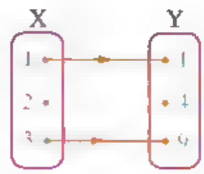
R_1



R_2

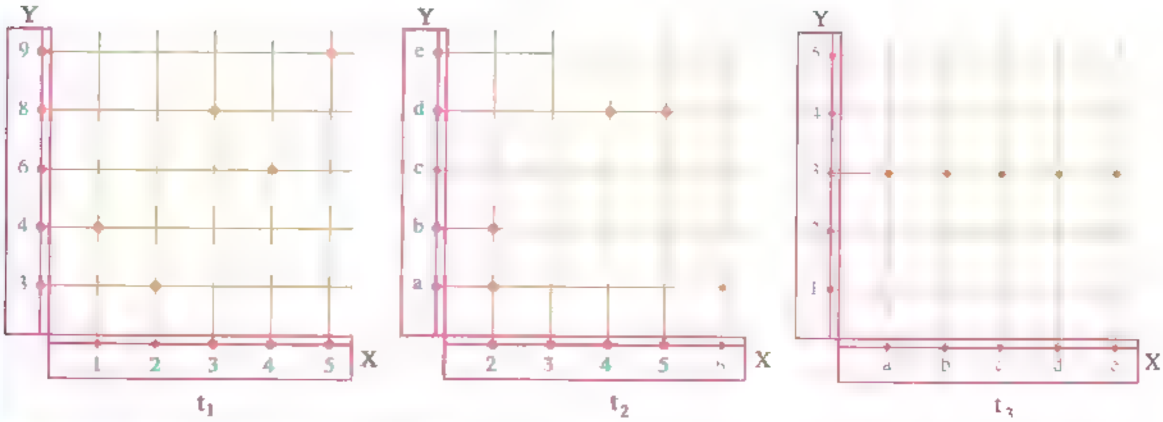


R_3



R_4

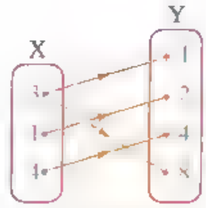
3 **Show which of the following Cartesian diagrams represents a function, then mention the set of each function and its range :**



4 If $X = \{a , b , c\}$, $Y = \{2 , 4 , 6 , 8 , 10\}$, which of the following relations is a function from X to Y and which is not with giving reasons , if the relation is a function , state its range :

1. $R_1 = \{(a , 2) , (b , 4)\}$
2. $R_2 = \{(a , 2) , (b , 4) , (b , 6) , (c , 8)\}$
3. $R_3 = \{(a , 2) , (b , 8) , (c , 10)\}$

5 The opposite arrow diagram represents a relation R from the set X to the set Y , where : $X = \{-3 , 1 , 4\}$, $Y = \{1 , 2 , 4 , 8\}$



1. Write R
2. Is R a function ? Why ?
3. Find the value of X if $(X , 2) \in R$

(Souhag 16 – Beni Suef 17)

6 If $X = \{1 , 2 , 3\}$, $Y = \{1 , 3 , 6 , 9 , 12\}$ and R is a relation from X to Y , where "a R b" means "$a = \frac{1}{3}$ b" for each $a \in X$, $b \in Y$

Write R and show that it is a function and write its range. *(El-Monofia 15 Souhag 17 – Matrouh 19)*

7 If $X = \{4 , 6 , 8 , 10\}$, $Y = \{2 , 3 , 4 , 5\}$ and R is a relation from X to Y , where "a R b" means "a = 2 b" for each $a \in X$, $b \in Y$

Write R and represent it by an arrow diagram. *(Aswan 11)*

8 If $X = \{1 , 3 , 4 , 5\}$, $Y = \{1 , 2 , 3 , 4 , 5 , 6\}$ and R is a relation from X to Y , where "a R b" means "a + b = 7" for each $a \in X$, $b \in Y$

Write R and represent it by an arrow diagram and also by a Cartesian diagram.

(El-Menia 11 Beni Suef 15 - Port Said 17)

9 If $X = \{0 , 1 , 4 , 7\}$, $Y = \{1 , 3 , 5 , 6\}$ and R is a relation from X to Y where "a R b" means "a + b < 8" for each $a \in X$, $b \in Y$ Write R and represent it by an arrow diagram.

Is R a function ? And why ? *(El-Kalyoubia 11 – Alex. 18)*

10 If $X = \{2, 4, 5, 7\}$, $Y = \{4, 5, 6, 7, 9\}$ and R is a relation from X to Y where "a R b" means "$a \leq b$" for each $a \in X$ and $b \in Y$

Write R and represent it by an arrow diagram and also by a Cartesian diagram.

11 If $X = \{1, 2, 3, 4\}$, $Y = \{y : y \in \mathbb{N}$, y is an even number $\leq 10\}$ where $\mathbb{N}$ is the set of natural number and R is a relation from X to Y where "a R b" means "$a = \frac{1}{2}$ b" for each $a \in X$, $b \in Y$

1. Write R and represent it by an arrow diagram.
2. Show that R is a function from X to Y and find its range. *(El-Monofia 17)*

12 If $X = \{1, 2, 3\}$, $Y = \{2, 3, 7\}$ and R is a relation from X to Y , where "a R b" means "$a + b =$ a prime number" for each $a \in X$, $b \in Y$

1. Write R and represent it by an arrow diagram. Is R a function ?
2. If 2 a R 3 , then find the value of a

13 If $X = \{-1, 0, 1, 2, 3\}$, $Y = \{0, 1, 4, 6, 9\}$ and R is a relation from X to Y , where "a R b" means "$a^2 = b$" for each $a \in X$, $b \in Y$

1. Write R and represent it by a Cartesian diagram.
2. Is R a function ? And why ? *(Red Sea 16 – Qena 18)*

14 If $X = \{-2, -1, 1, 2\}$, $Y = \left\{\frac{1}{8}, \frac{1}{3}, 1, 3, 8\right\}$ and R is a relation from X to Y , where "a R b" means "$a^3 = b$" for each $a \in X$, $b \in Y$

Write R and represent it by an arrow diagram and also by a Cartesian diagram.

15 If $X = \{-2, -1, 0, 1, 2\}$ and $Y = \left\{4, 2, \frac{3}{2}, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}\right\}$ and R is a relation from X to Y where "a R b" means "$b = 2^a$" for each $a \in X$, $b \in Y$ Write R and represent it by an arrow diagram. Prove that R represents a function and mention its range.

16 If $X = \{2, 5, 8\}$ and $Y = \{10, 16, 24, 30\}$ and R is a relation from X to Y where "a R b" means "a is a factor of b" for each $a \in X$, $b \in Y$

Write R and represent it by an arrow diagram and by a Cartesian diagram. Is R a function ? And why ?

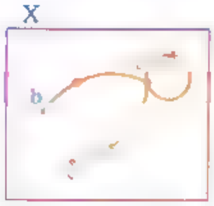
17 If $X = \{2, 3, 4\}$, $Y = \{6, 8, 10, 11, 15\}$ and R is a relation from X to Y , where "a R b" means "a divides b" for each $a \in X$, $b \in Y$

Write the relation R

Second | Problems on relation and function from a set to itself

18 **Choose the correct answer from those given :**

1 The opposite diagram represents a function on X , its range is ....

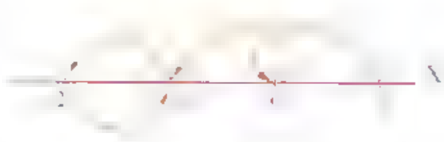


(Cairo 11)

(a) $\{a\}$ (b) $\{a , b , c\}$

(c) $\{a , b\}$ (d) $\{b , c\}$

2 The opposite figure represents a function on X , its range is ....



(a) $\{1 , 0 , -1 , -2\}$ (b) $\{1 , 0 , -1\}$

(c) $\{0 , -1 , -2\}$ (d) $\{1 , -1 , -2\}$

19 If $X = \{1 , 2 , 3 , 4\}$, which of the following arrow diagrams represents a function on the set X ?

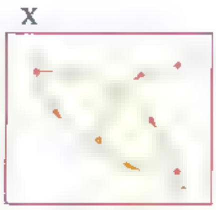


Fig. (1)

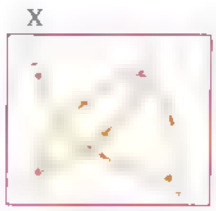


Fig. (2)

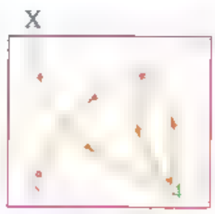


Fig. (3)

20 If $X = \{6 , 4 , 2 , 0 , -2 , -4 , -6\}$ and R is a relation on X where "a R b" means "a is the additive inverse of b" for each $a \in X$, $b \in X$

Write R and represent it by an arrow diagram and show with reason if R is a function or not , and if R is a function , mention its range.

21 If $X = \left\{0 , 1 , 2 , \frac{1}{2}\right\}$ and R is a relation on X where "a R b" means "a is the multiplicative inverse of b" for each $a \in X$, $b \in X$ Write R and represent it by an arrow diagram and show if R is a function or not.

22 If $X = \{1 , 2 , 3 , 6 , 11\}$ and R is a relation on X where "a R b" means "$a + 2b$ = an odd number" for each $a \in X$, $b \in X$

Write R and represent it by an arrow diagram. Is R a function ? And why ?

23 If $X = \{x : x \in \mathbb{N} , 1 \le x \le 3\}$ and R is a relation on X where "a R b" means "a + b is divisible by 3" for each $a \in X$, $b \in X$

Write R and represent it by an arrow diagram , then mention if R is a function or not. And if R is a function , mention its range. *(Luxor 16)*

24 If $X = \{1, 2, 4, 6, 10\}$ and R is a relation on X where "a R b" means "a is a multiple of b" for each $a \in X, b \in X$

Write R and represent it by an arrow diagram and also by a Cartesian diagram.

Is R a function ? And why ?

---

25 If $X = \{-2, -1, 0, 1, 2\}$ and R is a relation on X where "a R b" means "$b = |a|$" for each $a \in X$ and $b \in X$ write R and represent it by an arrow diagram, and show whether R is a function or not.

---

26 If $X = \{-2, 2, 5\}$, $Y = \{3, 7, \ell\}$ and R is a function from X to Y where "a R b" means "$b = a^2 - 1$" for each $a \in X$ and $b \in Y$

1 Find the value of ℓ

2 Represent R by an arrow diagram.

---

27 If $X = \{0, 4, 16\}$, $Y = \{0, 2, 4\}$, show which of the following relations represents a function from X to Y :

1 R_1 where "a R_1 b" means "$a = b^2$" for each $a \in X, b \in Y$

2 R_2 where "a R_2 b" means "$a = \sqrt{b}$" for each $a \in X, b \in Y$

3 R_3 where "a R_3 b" means "$\frac{1}{2} a = b$" for each $a \in X, b \in Y$

---

28 If R is a relation on the set of natural numbers ($\mathbb{N}$) where "a R b" means "$a \times b = 12$" for each $a \in \mathbb{N}, b \in \mathbb{N}$:

1 If x R 4 , then find the value of x

2 If y R 3 y , then find the value of y

---

29 If R is a relation on the set of the positive real numbers ($\mathbb{R}_+$) where "x R y" means "$y^2 = 2x$" for each $x \in \mathbb{R}_+, y \in \mathbb{R}_+$, and each of the following ordered pairs belongs to R :

$(a, 2)$, $\left(\frac{2}{9}, b\right)$, $(c, 3)$ and $\left(\frac{9}{32}, d\right)$ Find the value of each of : a , b , c and d

---

30 If $X = \{1, 0, -1\}$, R_1 is the relation of the additive inverse on X and R_2 is the relation of the multiplicative inverse on X

Find $R = R_1 \cap R_2$ Is R a function on X ?

31 If $X = \{1, 2, 3\}$, $Y = \{13, 31, 65, 23\}$ and R is a relation from X to Y where "a R b" means "a is a digit of the number b" for each $a \in X$, $b \in Y$

1 Write R and represent it by an arrow diagram.

2 Show which of the following is true, giving reasons : 2 R 65, 1 R 31, 3 R 13

3 Write by listing method : $M = \{(y, 23) : (y, 23) \in R\}$

32 If $A = \{-1, 1, 2\}$, $B = \{d : d \in \mathbb{N}\}$ and R is a relation from A to B where "x R y" means "$y = 2x + 3$" for each $x \in A$, $y \in B$

Write R and represent it by an arrow diagram.

33 If $X = \{1, 2, 3\}$, $Y = \{3, 4, 5\}$, show with reasons which of the following represents a relation from X to Y :

1 $L = \{(1, 3), (3, 3), (5, 3)\}$

2 $M = \{(2, 4), (1, 3), (3, 3), (3, 4)\}$

34 If $X = \{1, 3, 5\}$ and R is a function on X where $R = \{(a, 3), (b, 1), (1, 5)\}$

, find : 1 The range of the function.

2 The numerical value of the expression : a + b *(El Kalyoubia 20, Aswan 20)*

For excellent pupils

35 If $X = \{-2, -1, 0, 1, 2\}$, $Y = [0, 4[$ and R is a relation from X to Y where "a R b" means "$a^2 = b$" for each $a \in X$, $b \in Y$

Write R and mention whether R is a function from X to Y or not. Give reasons.

36 If f is a function from X to Y where "a R b" means "a divides b" for each $a \in X$, $b \in Y$, $X \cup Y = \{2, 3, 5, 11, 14, 9, 35\}$, $n(X) = 3$ and $n(X \times Y) = 12$

Find each of X and Y and write R of the function f and find its range.

37 If f is a function from X to Y where "a R b" means "a is a multiple of b" for each $a \in X$, $b \in Y$, $n(X) = 4$, $n(Y) = 2$ and $X \cup Y = \{4, 8, 9, 27\}$

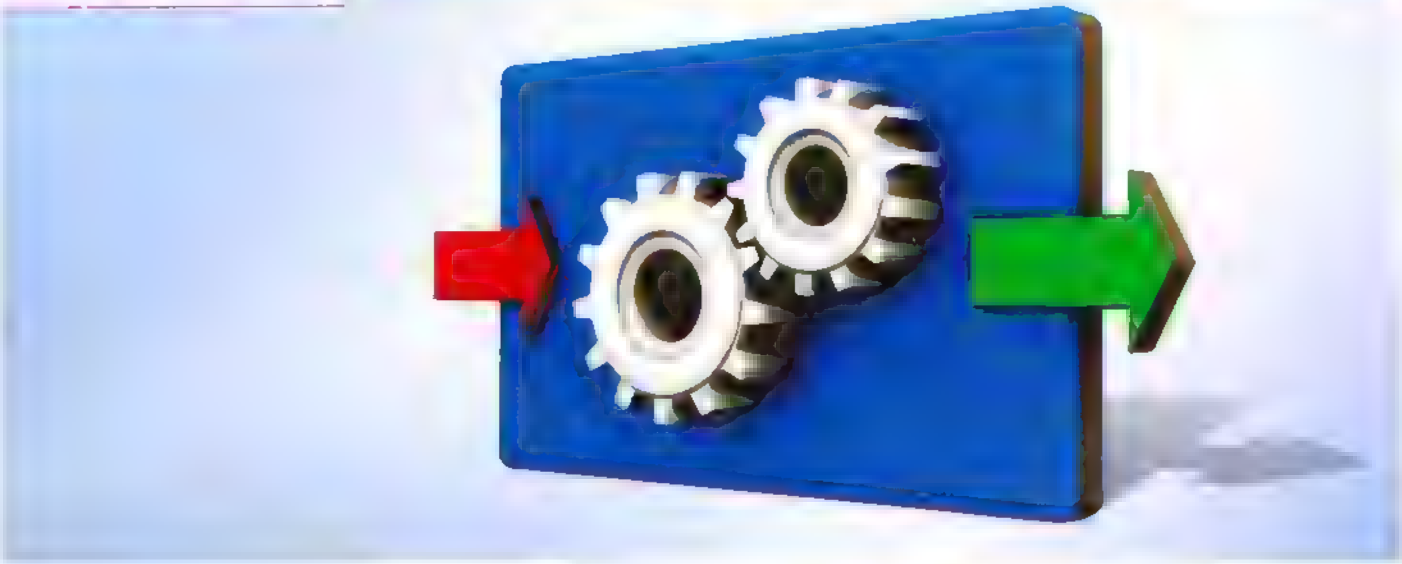
Find each of X and Y and write R of the function f and find its range.

EXERCISE **3**

The symbolic representation of the function - Polynomial functions

Interactive test

From the school book



● Remember ● Understand ○ Apply ♣ Problem Solving

1 Choose the correct answer from those given :

1 The set of images of the elements of the domain of the function is called .... *(Damietta 15 – Matrouh 16)*

(a) the rule. (b) the domain. (c) the range. (d) the codomain.

2 If the function $f : X \longrightarrow Y$, then the range of the function $f \subset$ .... *(Cairo 17)*

(a) $X \times Y$ (b) X (c) $Y \times X$ (d) Y

3 Which of the following functions is polynomial ?

(a) $f : f(x) = x(x^2 + x^{-2} - 4)$ (b) $f : f(x) = x^3 + x^2 + 3$

(c) $f : f(x) = x^2 + \sqrt{x} + 8$ (d) $f : f(x) = \sqrt[3]{x} + 8$

4 All the following functions are polynomials except ....

(a) $f : f(x) = 2x - 5$ (b) $f : f(x) = 3$

(c) $f : f(x) = x\left(x + \frac{1}{x} - 2\right)$ (d) $f : f(x) = \frac{x}{2} - 7$

5 The function f where $f(x) = x^4 - 2x^3 + 7$ is a polynomial function of the .... degree. *(Suez 15 – South Sinai 19)*

(a) first (b) second (c) third (d) fourth

6 The function $f : f(x) = x(x - 2x^2)$ is a polynomial function of the .... degree.

(a) first (b) second (c) third (d) fourth

7 The function $f : f(x) = x^2 - (x^2 - 3x)$ is a polynomial function of the .... degree.

(a) first (b) second (c) third (d) fourth

(Port Said 16)

8 The function $f : f(x) = x^2 (x-3)^2$ is a polynomial function of the .... degree.

(a) first (b) second (c) third (d) fourth

9 The function $f : f(x) = (x-5)^3$ is a polynomial function of the .... degree. (Qena 11)

(a) zero (b) second (c) third (d) fourth

10 If $f(x) = x^2 - x + 3$, then $f(-2) =$ ....

(a) -2 (b) -1 (c) 5 (d) 9

11 If $f(x) = x^2 - \sqrt{2}\, x$, then $f(\sqrt{2}) =$ .... (El-Dakahlia 11)

(a) 4 (b) 2 (c) 6 (d) zero

12 If the function $f : \mathbb{Z} \longrightarrow \mathbb{Z}$ where $f(x) = x^2$, then $f(2) + f(-2) =$ ....

(a) 0 (b) 4 (c) 8 (d) -8

13 If $f(x) = k\,x + 8$, $f(2) = 0$, then $k =$ .... (El-Sharkia 15 – El-Dakahlia 20)

(a) 8 (b) 6 (c) 4 (d) -4

14 If $f(x) = x - 5$ and $\frac{1}{2} f(a) = 3$, then $a =$ ....

(a) 2 (b) 8 (c) 11 (d) 16

15 If $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = x^{k-2} + 3$, $f(2) = 11$, then $k =$ .... (El-Sharkia 20)

(a) 5 (b) 3 (c) 2 (d) -3

16 If $(-1, 0) \in$ the set of the function f where $f(x) = m\,x + 2$, then $m =$ ....

(a) 0 (b) -1 (c) 2 (d) -2

17 If $(3, y) \in$ the set of the function f where $f(x) = x + 2$, then $y =$ ......

(a) 5 (b) 3 (c) 2 (d) 1

18 If $(a, a) \in$ the set of the function f where $f(x) = 2\,x + 3$, then $a =$ ....

(a) 2 (b) 3 (c) -3 (d) -2

19 If $f(x + 3) = x - 3$, then $f(7) =$ .... (El-Dakahlia 19)

(a) 4 (b) 1 (c) 7 (d) 10

20 If $X = \{2, 4, 6\}$, $n(Y) = 4$ and the function $f : X \longrightarrow Y$, $f(x) = x^2 - 1$, then Y may be ....

(a) $\{3, 7, 13\}$ (b) $\{3, 15, 25, 45\}$

(c) $\{3, 15, 35\}$ (d) $\{3, 15, 25, 35\}$

21 If $f(x) = n\,x^2 + 2\,x^n - 3$, then the possible values of n such that f is a function of the second degree is .... (El-Dakahlia 16)

(a) $\{2, 3\}$ (b) $\{1, -1\}$ (c) $\{2, 1, 0\}$ (d) $\{2, 1\}$

2 If $f : \mathbb{R} \longrightarrow \mathbb{R}$, mention the degree of f , then find $f(-2)$, $f(0)$, $f\left(\frac{1}{2}\right)$ when :

1 $f(X) = 3 - 2X$ 2 $f(X) = X^2 - 4$

3 If $f(X) = 2X^2 - 5X + 2$, **then prove that :** $f(2) = f\left(\frac{1}{2}\right)$ *(Luxor 14)*

4 If $f(X) = 2X - 1$, **then prove that :** $f(2) - 3f(1) =$ zero *(El-Gharbia 11)*

5 If $f(X) = X^2 - 3X$, $g(X) = X - 3$ *(El-Menia 17 – Alex.18 – Qena 19 – Port Said 20)*

1 **Find :** $f\left(\sqrt{2}\right) + 3g\left(\sqrt{2}\right)$ 2 **Prove that :** $f(3) = g(3) = 0$

6 If $f(X) = X^2 - 2X - 5$, **then prove that :** $f\left(1 + \sqrt{6}\right) = f\left(1 - \sqrt{6}\right) = 0$

7 The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = aX^2 + bX + 5$, a = zero and b is a real number not equal to zero.

1 Find the degree of the function f

2 If $f(3) = 11$, then find the value of b *(El-Menia 18)* « 2 »

8 If $f(X) = 5X - b$ and $h(X) = X - 2b$ and $f(1) + h(3) = -7$

, **then find :** $f(3) + h(1)$ « 1 »

9 If the function $f : \mathbb{Z} \longrightarrow \mathbb{N}$ where $f(X) = (X - 3)^2$ and the function $t : \mathbb{Z} \longrightarrow \mathbb{N}$ where $t(X) = X - 3$, then find the value of X which makes : $f(X) = t(X)$ « 3 or 4 »

10 If f is a function on X where $X = \{3, 4, 5, 6\}$

and $f(3) = 3$, $f(4) = 5$, $f(5) = 5$, $f(6) = 5$

1 Represent f by an arrow diagram.

2 Write the set of f and mention its range. *(Ismailia 15)*

11 If $X = \{0, 1, 3\}$, $Y = \{1, 2, 3, 4, 5, 7\}$ and the function $f : X \longrightarrow Y$ where $f(X) = 5 - X$

1 Find the range of f

2 Draw a Cartesian diagram for the function f *(New Valley 17)*

12 If the function $t : \mathbb{N} \longrightarrow \mathbb{N}$ where $\mathbb{N}$ is the set of natural numbers , $t : X \longmapsto 2X + 3$

1 **Find :** $t(0)$, $t(1)$, $t(2)$, $t(3)$, $t(4)$, $t(5)$

2 Represent five elements of the elements of t on a part of the square net of the Cartesian product $\mathbb{N} \times \mathbb{N}$

3 What is the range of t ?

13 If the function $f : \mathbb{Z} \longrightarrow \mathbb{Z}$ where $\mathbb{Z}$ is the set of integers , $f(X) = X^2 - 2X - 3$

1. **Find :** $f(4)$, $f(3)$, $f(2)$, $f(1)$, $f(0)$, $f(-1)$, $f(-2)$
2. Draw a part of the perpendicular square net of the Cartesian product $\mathbb{Z} \times \mathbb{Z}$ and represent on it seven elements of the elements of f
3. If $f(X) = 5$, find the value of X « 4 or - 2 »

14 If $f(X) = a X + b$, $f(a) = b$, **find the value of :** $a b^2 + 5$ *(El-Sharkia 19)* « »

15 If the set of the function $f = \{(1 , 3) , (2 , 5) , (3 , 7) , (4 , 9) , (5 , 11)\}$, write :

1. The domain of the function f
2. The range of the function f
3. The rule of the function f *(Damietta 16 – North Sinai 17 – Luxor 19)*

For excellent pupils

16 If $f(X) = 2X^2 + bX + c$ and $f(X) = 0$, when $X \in \{0 , 3\}$, find the value of each of b and c « 6 , 0 »

4

The study of some polynomial functions

Interactive test

From the school book

Remember Apply Problem Solving

First Problems on the linear function and the constant function

1 **Choose the correct answer from those given :**

1. If $f(X) = 7$, then $f(-3) =$ .... *(Giza 17)*

(a) 7 (b) -7 (c) 21 (d) -21

2. If $f(X) = 2$, then $3 f\left(\sqrt{2}\right) =$ ....

(a) $f\left(3\sqrt{2}\right)$ (b) 6 (c) 3 (d) 2

3. If $f(X) = 2$, then $f(3) - f(1) =$ .... *(El-Dakahlia 13)*

(a) $f(2)$ (b) 2 (c) zero (d) 10

4. If $f(X) = 5$, then $\frac{f(5)}{f(10)} =$ ....

(a) 5 (b) $\frac{1}{2}$ (c) 1 (d) 10

5. If f is a function such that $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(X) = 3$, then $\frac{f(6)}{f(\text{zero})} =$ .... *(El Dakahlia 17)*

(a) 6 (b) 1 (c) 3 (d) undefined.

6. If $f(X) = 3$, then $\frac{2 f(3)}{3 f(2)} =$ .... *(Alex. 05)*

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) 1 (d) $\frac{32}{23}$

7 If $f(X) = -7$, then $f(X + 7) = \cdots$

(a) -7 (b) 0 (c) 7 (d) 14

8 If $f(2X) = 4$, then $f(-X) = \cdots\cdots$ *(El-Dakahlia 09)*

(a) -2 (b) -4 (c) 4 (d) 2

9 The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = 5$ is represented by a straight line intersecting the y-axis at the point $\cdots\cdots$

(a) $(5 , 0)$ (b) $(0 , 5)$ (c) $(-5 , 0)$ (d) $(0 , -5)$

10 The linear function defined by the rule $y = 2X - 1$ is represented by a straight line intersecting the y-axis at the point $\cdots\cdots$ *(Matrouh 20)*

(a) $(0 , 1)$ (b) $(0 , -1)$ (c) $(1 , 0)$ (d) $(-1 , 0)$

11 The linear function defined by the rule $f(X) = 3X + 6$ is represented by a straight line intersecting the X-axis at the point $\cdots\cdots$

(a) $(0 , -2)$ (b) $(-2 , 0)$ (c) $(0 , -6)$ (d) $(-6 , 0)$

12 The function f where $f(X) = 3X$ is represented graphically by a straight line which passes through the point $\cdots\cdots$ *(Beni Suef 17)*

(a) $(3 , 3)$ (b) $(3 , 0)$ (c) $(0 , 0)$ (d) $(0 , 3)$

13 If the straight line which represents the function $f : f(X) = 2X - a$ passes through the origin point , then a = $\cdots\cdots$ *(El-Fayoum 17)*

(a) -2 (b) 2 (c) zero (d) 3

14 If the point $(a , 3)$ lies on the straight line representing the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = 4X - 5$, then a = $\cdots\cdots$ *(New Valley 20)*

(a) 2 (b) 3 (c) 4 (d) 5

15 If the point $(a , 4)$ is one of the points of the function $g : \mathbb{R} \longrightarrow \mathbb{R}$ where $g(X) = 2X + b$, then $6a + 3b = \cdots\cdots$ *(El Dakahlia 17)*

(a) 12 (b) 9 (c) 6 (d) 3

2 **Represent the following functions graphically , where $X \in \mathbb{R}$:**

1 $f : f(X) = 5$

2 $f : f(X) = -4$

3 $f : f(X) = 0$

4 $f : f(X) = 2\frac{1}{2}$

3 ○ **Represent each of the following linear functions graphically and find the points of intersection of the straight line which represents each of them with the coordinate axes , where $X \in \mathbb{R}$:**

1. $f : f(X) = X$
2. $f : f(X) = -X$
3. 📖 $f : f(X) = 3X$
4. 📖 $f : f(X) = -2X$
5. 📖 $f : f(X) = X + 2$
6. 📖 $f : f(X) = 2 - X$
7. 📖 $f : f(X) = 3X - 1$
8. 📖 $f : f(X) = -2X + 3$
9. $f : f(X) = \frac{1}{2}X$
10. $f : f(X) = 5 - \frac{1}{2}X$

---

4 ○ If the straight line which represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = 6X - a$ intersects the y-axis at the point (b , 2) , find the value of each of a , b *(Aswan 20)* « – 2 , 0 »

---

5 ○ If the function $f : f(X) = 3X - 6$ is represented by a straight line passing through the point (a , 2 a) , find the value of a , then find the intersection point of the straight line with the y-axis. *(El-Gharbia 20)* « 6 , (0 , – 6) »

---

6 ○ If $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = 2X + a$ and $f(3) = 9$, **find :**

1. The value of a
2. The coordinates of the intersection point of the straight line representing the function with the X-axis. *(Giza 20)* « 3 , $(-\frac{3}{2}, 0)$ »

---

7 ○ If the straight line representing the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = aX + b$ cuts a positive part of the y-axis of length 3 units and passes through the point (1 , 5) **, find the value of each of :** a , b *(Kafr El-Sheikh 20)* « 2 , 3 »

---

8 ○ If the straight line which represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(X) = aX + b$ intersects the X-axis at the point (3 , 0) and intersects the y-axis at the point (0 , – 3) , then find the values of the two constants a and b and find the value of $f(1)$ *(El-Sharkia 17)* « 1 , – 3 , – 2 »

---

9 ○ If X = {2 , 3 , 6} , Y = {3 , 4 , 5 , 6 , 7 , 8} and r : X ⟶ Y where $r(X) = 9 - X$

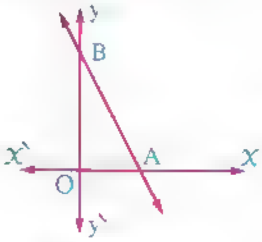
1. Find the set of images of the elements of the set X by the function r
2. Is r a linear function ? "state the reason" *(El-Dakahlia 14)*

---

10 ♣ The opposite figure represents the function f where $f(X) = 4 - 2X$

Find :

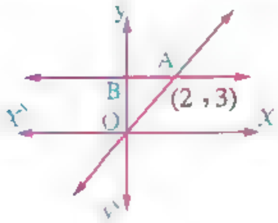
1. The coordinates of A , B
2. The area of Δ AOB



(Ismailia 16 – Luxor 19)

11 **In the opposite figure :**

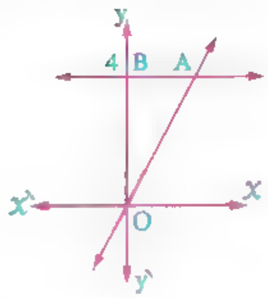
The constant function f is represented graphically by the straight line $\overleftrightarrow{BA}$ and the linear function g is represented graphically by the straight line $\overleftrightarrow{OA}$ where A = (2 , 3)



1 Write the rule of the function f and the rule of the function g

2 Find the value of : f (– 10) + g (6) *(El-Sharkia 14)* « 12 »

12 The opposite figure shows the straight line $\overleftrightarrow{AB}$ which represents the function $f : f(x) = 4$, if $\overleftrightarrow{AO}$ represents the linear function g : g $(x) = n\,x + k$ and the area of the triangle ABO equals 4 square units , then find the values of n and k , where O is the origin point. *(El-Dakahlia 17)* « 2 , 0 »



13 While Karim was reading a book , he found that after 3 hours , 50 pages remained and after 6 hours , 20 pages remained. If the relation between the time (t) and the number of remained pages (b) is a linear relation :

1 Represent graphically the relation between t and b , then find the algebraic relation between the two variables.

2 What is the time that should be taken to finish the book ?

3 What is the number of pages remaining when Karim began to read ? *(Ismailia 20)*

Second Problems on the quadratic function

14 **Choose the correct answer from those given :**

1 If the point (3 , 2) is the vertex of the curve of the quadratic function f , then the equation of the line of symmetry is ....

(a) $x = 3$ (b) $x = 2$ (c) y = 3 (d) y = – 3

2 The vertex of the curve of the function $f : f(x) = 2x^2 - 4x + 5$ is ....

(a) (– 1 , 11) (b) (1 , 3) (c) (2 , 5) (d) (3 , 11)

3 The equation of the axis of symmetry of the curve of the function $f : f(x) = x^2$ is ....

(a) $x = 1$ (b) $x = 0$ (c) y = 1 (d) y = 0

4 The equation of the axis of symmetry of the curve of the function $f : f(X) = (X-2)^2$ is ……

(a) $X = 0$ (b) $X = 2$ (c) $X = -2$ (d) $X = -4$

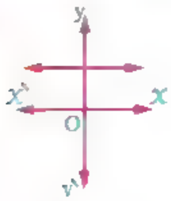
5 If the curve of the function f such that $f(X) = X^2 + c$ passes through the point $(0 , 2)$, then $c =$ ………

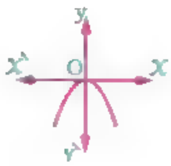
(a) -4 (b) -2 (c) 2 (d) 4

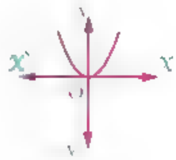
6 If $(-2 , y)$ belongs to the curve of the function $f : f(X) = X^2 + 1$, then $y =$ ………

(a) -3 (b) -1 (c) 3 (d) 5

7 The graph of the function $f : f(X) = X^2 - 2X + 1$ is the graph number …… *(Giza 08)*

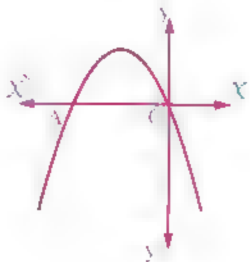

(a)


(b)


(c)


(d)

8 The opposite figure represents the curve of a quadratic function , $A(-4 , 0)$, then the equation of the axis of symmetry is $X =$ ……… *(El-Dakahlia 19)*



(a) 1 (b) -1

(c) -2 (d) 0

9 The maximum value of the function $f : f(X) = -2X^2 + 4X + 3$ is ………

(a) -1 (b) 1 (c) 3 (d) 5

10 If $f(X) = X^2$, $X \in [-2 , 2]$, then $f(X) \in$ ……… *(El-Dakahlia 08)*

(a) $]0 , 4]$ (b) $]0 , 4[$ (c) $[0 , 4]$ (d) $[-4 , 4[$

15 Represent each of the following functions graphically and from the graph , deduce the coordinates of the vertex of the curve , the equation of the line of symmetry and the maximum or minimum value of the function , where $X \in \mathbb{R}$:

1 $f : f(X) = 2X^2$ taking $X \in [-2 , 2]$

2 $f : f(X) = X^2 + 1$ taking $X \in [-3 , 3]$ *(Beni Suef 14 – El-Fayoum 16)*

3 $f : f(X) = X^2 - 2$ taking $X \in [-3 , 3]$ *(El Beheira 17 – Port Said 18 – Damietta 20)*

4 $f : f(X) = 2 - X^2$ taking $X \in [-3 , 3]$ *(Alex. 18 – El-Gharbia 19 – Souhag 20)*

5 $f : f(x) = x^2 - 2x$ taking $x \in [-2, 4]$ (Qena [illegible] – Cairo 18 – Kafr El-Sheikh 20)

6 $f : f(x) = x^2 + 2x + 1$ taking $x \in [-4, 2]$ (El-Sharkia 17 – El-Gharbia 18)

7 $f : f(x) = (x-2)^2$ taking $x \in [-1, 5]$ (Kafr El-Sheikh 19 – El-Gharbia 20)

8 $f : f(x) = x(x-2) - 3$ taking $x \in [-2, 4]$ (El-Dakahlia 17)

9 $f : f(x) = 3 - 2x - x^2$ taking $x \in [-4, 2]$

10 $f : f(x) = 4x + 3 - 2x^2$ taking $x \in [-2, 3]$

11 $f : f(x) = x^2 - 4x + 5$ taking $x \in [0, 5]$

12 $f : f(x) = 1 - 3x + x^2$ taking $x \in [-1, 4]$

16 If the curve of the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = m\ x^2$ intersects the x-axis at the point $(-2, b)$, **find the value of :** $m^b + 2m$ (El-Sharkia 15) « 9 »

17 If $f(x) = a + x^2$, $\ell(x) = c$ are two polynomial functions where $3f(2) + 3\ell(x) = 6$, find the numerical value of : $2f(0) + 2\ell(7)$ where a and c are constants.

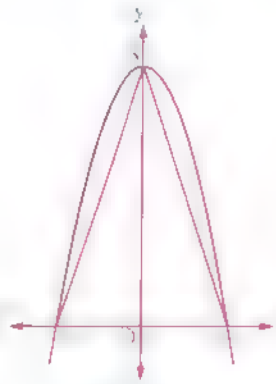
(El-Dakahlia 19) « – 4 »

18 The opposite figure represents the curve of the function f where $f(x) = 9 - x^2$

Find :

1 The coordinates of A and C

2 The area of the triangle ABC



(Kafr El-Sheikh 18)

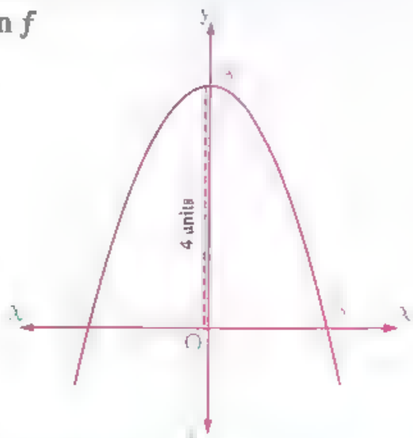
19 The opposite figure represents the curve of the function f where $f(x) = m - x^2$,

if OA = 4 units

, **find :** 1 The value of m

2 The coordinates of B and C

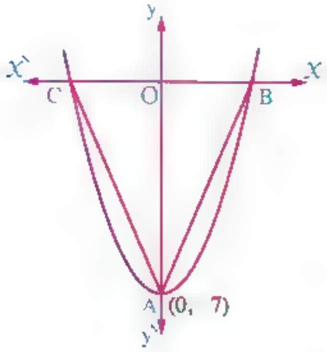
3 The area of the triangle with vertices A, B and C



(North Sinai 16 – Luxor 18 – Giza 20)

20 The opposite figure represents the curve of the function $f : f(X) = \ell X^2 - 7$, the area of the triangle ABC = 21 square units, A (0 , – 7)

Find the coordinates of the point B, then find the value of ℓ



(El-Dakahlia 18)

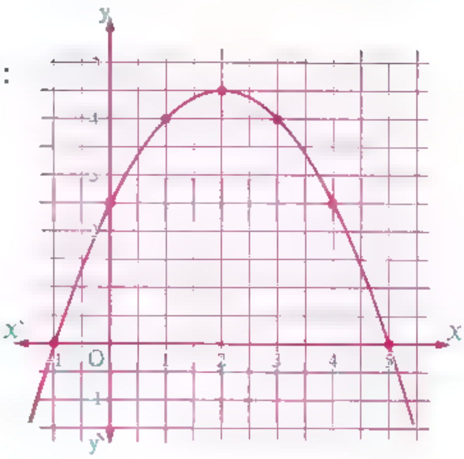
21 **In the opposite figure :**

The curve represents a function of the second degree f :

1. Write the domain of f

Use the graph to find :

2. The range of the function f
3. The equation of the line of symmetry of the curve of function f
4. The maximum value of f
5. The value of $f(1)$
6. If $f(X) = a(X - 2)^2 + k$, then find the numerical value of : a + k



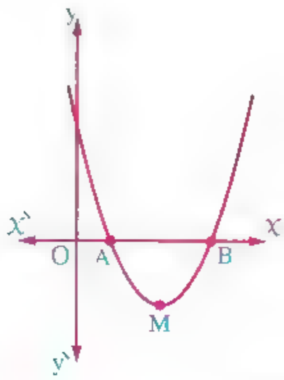
(El-Dakahlia 16)

For excellent pupils

22 **In the opposite figure :**

If the curve of the function f intersects the X-axis at the two points : A (1 , 0) , B (4 , 0) and M is the point of the vertex of the curve and $f(-2) + f(7) = 8$

, **find :** $f(-2)$



« 4 »

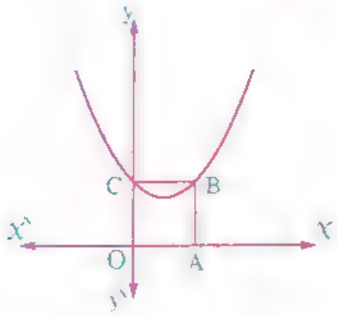
23 **In the opposite figure :**

The drawn curve represents the quadratic function

$f : f(X) = X^2 - (k-2)X - k + 4$

If ABCO is a square

, **find the value of :** k



(El-Dakahlia 19) « 3 »

24 **In the opposite figure :**

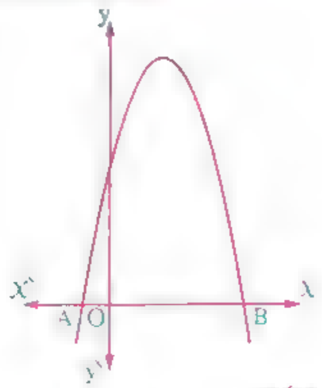
The curve represents the function

$f : f(X) = -X^2 + 4X + k - 1$ and intersects

the X-axis at the two points A and B

If OB = 5 OA

, **find the value of :** k



« 6 »

Summary of Unit 1

The Cartesian product :

- For any two finite non empty sets X and Y

  The Cartesian product of $X \times Y$

  is the set of all ordered pairs whose first projection of each belongs to X

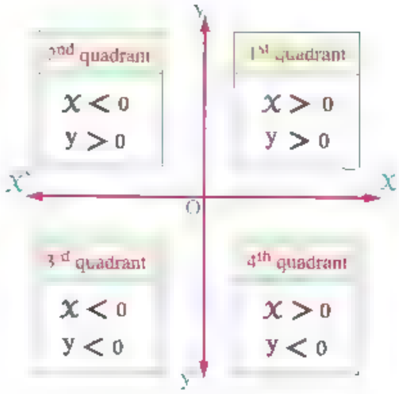
  and the second projection of each belongs to Y

  i.e. $X \times Y = \{(a , b) : a \in X , b \in Y\}$

  Note that : $X \times Y \neq Y \times X$ where $X \neq Y$

- For any set X : $X \times \varnothing = \varnothing \times X = \varnothing$

- The two axes $\overleftrightarrow{XX'}$ and $\overleftrightarrow{yy'}$ divide the plane into four quadrants as shown in the opposite figure and you can determine the quadrant in which any point lies by knowing the signs of its two coordinates.



- If the x-coordinate of the point $= 0$

  , then the point lies on y-axis.

- If the y-coordinate of the point $= 0$

  , then the point lies on x-axis.

The relation :

- The relation from the set X to the set Y is a connection joining some or all the elements of X with some or all the elements of Y

- If R is a relation from the set X to the set Y , then :
  - R is a set of ordered pairs such that the first projection belongs to X and the second projection belongs to Y
  - $R \subset X \times Y$

- If R is a relation from X to X , then R is a relation on X and $R \subset X \times X$

The function :

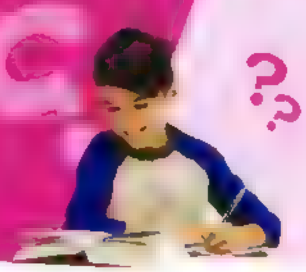
- A relation from X to Y is said to be a function if one of the following cases is satisfied :
  1. Each element of the set X appears only once as a first projection in one of the ordered pairs of the relation.
  2. Each element of the set X has one and only one arrow going out of it to an element of Y in the arrow diagram which represents the relation.
  3. Each vertical line has only one point lying on it of the points which represent the relation , in the Cartesian diagram which represents the relation.
- If f is a function from the set X to the set Y written as $f : X \longrightarrow Y$, then :
  1. X is called the domain of the function f
  2. Y is called the codomain of the function f
  3. The set of images of the elements of the set X by the function f is called the range of the function f which is a subset of the codomain Y

The polynomial functions :

- The polynomial function is a function whose rule is a term or an algebraic expression satisfying the following conditions :
  1. Each of the domain and the codomain of the function is the set of real numbers $\mathbb{R}$
  2. The power (the index) of the variable x in any of its terms is a natural number

     Notice that : the degree of the function is the highest power of the variable x
- **The constant function :** The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = b$, $b \in \mathbb{R}$ is called a constant function and is represented by a straight line parallel to x-axis and intersects y-axis at the point $(0 , b)$
- **The linear function :** The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = a x + b$, $a \in \mathbb{R} - \{0\}$, $b \in \mathbb{R}$ is called a linear function (function of the first degree) and is represented by a straight line intersects y-axis at $(0 , b)$ and x-axis at $\left(\frac{-b}{a} , 0\right)$
- **The quadratic function :** The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = a x^2 + b x + c$ where a , b and c are real numbers , $a \neq 0$ is called a quadratic function and it is a polynomial function of the second degree and it is represented by a curve whose vertex is $\left(\frac{-b}{2a} , f\left(\frac{-b}{2a}\right)\right)$

Exams on Unit One



Model 1

Answer the following questions :

1 **Choose the correct answer from those given :**

1. If $X \times Y = \{(2, 3), (2, 5)\}$, then $n(X) = \dots\dots$

   (a) 1 (b) 2 (c) 3 (d) 5

2. If $b < 3$, then the point $(6, b - 3)$ lies in the .... quadrant.

   (a) first (b) second (c) third (d) fourth

3. If the point $(x, 3)$ lies on y-axis, then $7x - 1 = \dots\dots$

   (a) 20 (b) -1 (c) 6 (d) 8

4. If $f(x) = 4x + b$, $\frac{1}{3} f(3) = 5$, then $b = \dots\dots$

   (a) -57 (b) 3 (c) 4 (d) -3

5. The function f where $f(x) = 3x$ is represented graphically by a straight line which passes through the point ....

   (a) $(3, 3)$ (b) $(3, 0)$ (c) $(0, 0)$ (d) $(0, 3)$

6. The maximum value of the function $f : f(x) = -2x^2 + 4x + 3$ is ....

   (a) 5 (b) 1 (c) 3 (d) -1

2 **[a]** If $X = \{1, 2, 3\}$, $Y = \{0, 1, 2, 3, 4\}$ and R is a relation from X to Y where "a R b" means "$b - a = 1$" for each $a \in X$, $b \in Y$, write R and show that it is a function and write its range.

[b] If $X = \{2, 3\}$, $Y = \{3, 5\}$ and $Z = \{7, 5\}$, **then find :**

1. $X \times (Y \cap Z)$ 2. $(X - Y) \times Z$ 3. $(X - Y) \times (Y - Z)$

3 **[a]** If the set of the function $f = \{(3, 9), (5, 15), (7, 21)\}$, **write :**

1. The domain of the function f
2. The range of the function f
3. The rule of the function f

[b] Represent graphically the function $f : f(x) = -2x + 3$ and find the two intersection points of the straight line which represents the function with the coordinates axes, where $x \in \mathbb{R}$

4 [a] If $X = \{-1, 1, 0, 2\}$ and R is a relation on X where "a R b" means "$b = a^2$" for each $(a, b) \in X^2$

1. Write R and represent it by an arrow diagram.
2. Is R a function ? And why ?

[b] Represent graphically the function $f : f(x) = (x-2)^2 + 1$, taking $x \in [0, 4]$ and from the graph deduce :

1. The coordinates of the vertex of the curve.
2. The equation of the line of symmetry.
3. The maximum or minimum value of the function.

5 [a] If f and g are two functions where $f(x) = -2x + 3$ and $g(x) = -7$

1. Find the degree of the function f
2. Calculate the value of : $f(0) + g(0)$

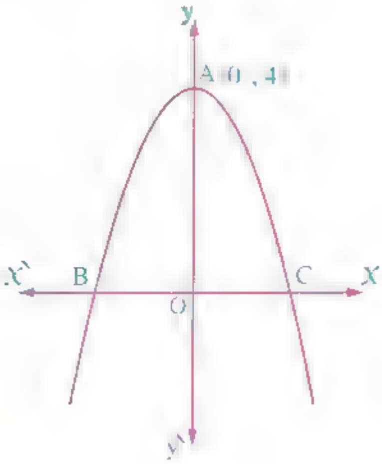
[b] The opposite figure represents the curve of the quadratic function f where :

$f(x) = 4 - kx^2$, (k is constant $\neq 0$)

, A (0 , 4) is the vertex of the curve

, O is the origin point , B and C belong to x-axis

The area of the triangle with vertices A , B and C equals 8 square units.



Find :

1. The equation of the symmetry axis, and the maximum value of the function f
2. The coordinates of B
3. The value of k

Model 2

Answer the following questions :

1 **Choose the correct answer from those given :**

1. If the point (a , 3 – a) is located on the x-axis , then a = ⋯

   (a) zero (b) 3 (c) – 3 (d) 5

2. If $f(x) = x^3$, then $f(2) + f(-2) = \cdots$

   (a) 16 (b) zero (c) – 16 (d) 4

3 If $X \times Y = \{(2, 3)\}$, then $X^2 = \cdots\cdots$.

(a) $\{(4, 9)\}$ (b) $\{(4, 3)\}$ (c) $\{(2, 2)\}$ (d) $\{(2, 9)\}$

4 If $f : f(X) = 5$ is represented by a straight line parallel to the X-axis, then it passes through the point $\cdots\cdots$

(a) $(0, 5)$ (b) $(5, 0)$ (c) $(5, -5)$ (d) $(0, 0)$

5 If R is a function from X to Y where $X = \{3, 5, 7\}$, $Y = \{4, 9\}$ and $R = \{(3, 4), (b, 9), (5, 9)\}$, then $b = \cdots\cdots$

(a) 3 (b) 5 (c) 7 (d) 9

6 The function $f : f(X) = (X - 5)^3$ is a polynomial function of the $\cdots\cdots$ degree.

(a) first (b) second (c) third (d) fourth

2 [a] If $X = \{1, 4, 7\}$, $Y = \{-1, 1, 4, 7\}$ and R is a relation from X to Y where "a R b" means "$a + |b| = 8$" for each $a \in X$, $b \in Y$

1 Write R and represent it by an arrow diagram.

2 Show if R is a function or not, with reason.

[b] If $X \times Y = \{(1, 1), (1, 5), (1, 3), (4, 1), (4, 5), (4, 3)\}$

, **find :** 1 $Y \times X$ 2 X and X^2

3 [a] Represent graphically the function $f : f(X) = X^2 + 2X - 4$, taking $X \in [-4, 2]$ and from the graph deduce :

1 The coordinates of the vertex of the curve.

2 The equation of the line of symmetry.

3 The maximum or minimum value of the function.

[b] If the straight line which represents the function f where $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(X) = 2X - 3k$ intersects the X-axis at the point $(6, m - 2)$

, **find the value of each of :** m and k

4 [a] If $(X - 2, 9) = (5, X + y)$, **find the value of :** $\sqrt{3X + 2y}$

[b] If $X = \{1, 2\}$, $Y = \{2, 5\}$ and $Z = \{4, 5\}$

, **find :** 1 $(X - Y) \times Z$ 2 $n(X \times Y) + n(Z^2)$

5 [a] If $X = \{4, 5, 7\}$ and R is a function on X, $R = \{(a, 5), (b, 5), (4, 7)\}$

, **find :** 1 The numerical value of the expression : $3a + 3b$

2 The range of the function.

[b] If $f : f(X) = 2X^2 - 5X + 2$

, **find the value of :** $f(2) - f\left(\frac{1}{2}\right)$

A Research Project

On Unit One

Project aims:

- Graphing quadratic function.
- Associating mathematics with computer science.

Do a research project on the following topic :

"The computer has become one of the important tools in different sciences, one of which is math".

Discuss the following points using available resources :

- Write a short note on the Visual Basic language.
- Use one of the Dynamic Mathematics Software (DMS) like GeoGebra (www.geogebra.org) to :

1. Graph the function $f : f(x) = x^2$
2. Graph the function $r : r(x) = (x - 1)^2$, on the same figure.
3. Graph the function $n : n(x) = (x + 1)^2$, on the same figure.
4. Compare the curve of function r with that of function f , then compare the curve of function n with that of function f , and write what do you observe.
5. Describe how $g : g(x) = (x - 3)^2$ and $k : k(x) = (x + 4)^2$ will appear after graphing them.

UNIT 2

Ratio, proportion, direct variation and inverse variation

Exercises of the unit :

5. Ratio and proportion.
6. Follow properties of proportion.
7. Continued proportion.
8. Direct variation and inverse variation.

- Unit Exams.
- A research project on unit two

Scan the QR code to solve an interactive test on each lesson

Unit 5

Ratio and proportion

Interactive test

From the school book

Remember   Understand   Apply   Problem Solving

1 Choose the correct answer from those given :

1. If a , b , 2 and 3 are proportional , then $\frac{a}{b}$ = ... *(Matrouh 19)*
   (a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$
2. The fourth proportional for the numbers 4 , 8 and 8 is .... *(North Sinai 19)*
   (a) 4   (b) 8   (c) 12   (d) 16
3. The third proportional for the numbers 4 , 12 , ... , 48 is .... *(Kafr El Sheikh 19)*
   (a) 7   (b) 32   (c) 16   (d) 36
4. If X , 3 , 4 and 6 are proportional , then X = ... *(Cairo 20)*
   (a) 0   (b) 1   (c) 2   (d) 3
5. The second proportional for the numbers 2 , ... , 8 , 12 is .... *(El Menia 18)*
   (a) 4   (b) 6   (c) 3   (d) 2
6. If 2 , 3 , 6 and $X - 1$ are proportional , then X = ... *(El-Monofia 18)*
   (a) 18   (b) 9   (c) 20   (d) 10
7. If 3 , a – 1 , a + 1 and 5 are proportional , then a = ...
   (a) 3   (b) 4   (c) ± 3   (d) ± 4
8. If $7X = 3y$, then $\frac{X}{y}$ = ...
   (a) $\frac{7}{3}$ (b) $\frac{3}{10}$ (c) $\frac{10}{3}$ (d) $\frac{3}{7}$
9. If 5 a – 4 b = 0 , then a : b = ...
   (a) 4 : 5   (b) 4 : 9   (c) 5 : 4   (d) 5 : 9

10 If $\frac{a}{3} = \frac{b}{5}$, then $5a - 3b + 4 = \dots\dots$ *(El-Monofia 19)*

(a) 3 (b) 4 (c) 5 (d) 6

11 If $\frac{a}{3} = \frac{b}{4}$, then $8a - 6b + 4 = \dots\dots$ *(El-Kalyoubia 20)*

(a) 3 (b) 4 (c) 5 (d) 6

12 If $\frac{3a}{5b} = \frac{1}{2}$, then $\frac{a}{b} = \dots\dots$ *(Red Sea 11 – Alex. 20)*

(a) $\frac{6}{5}$ (b) $\frac{5}{6}$ (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

13 If $2a = 3b$, then $\frac{3a}{2b} = \dots\dots$ *(El-Dakahlia 18)*

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{9}{4}$ (d) $\frac{4}{9}$

14 If $4x = 5y$, then $\frac{5y}{4x} = \dots\dots$ *(Qena 11)*

(a) 1 (b) 2 (c) 3 (d) 4

15 If $3a = 5b$, then $\frac{3a}{b} = \dots\dots$ *(El-Fayoum 17)*

(a) 3 (b) 5 (c) $\frac{3}{5}$ (d) $\frac{5}{8}$

16 If $2x = 7y$, then $\left(\frac{x}{y}\right)^{-1} = \dots\dots$ *(El-Fayoum 09)*

(a) $\frac{2}{7}$ (b) $\frac{7}{2}$ (c) $\frac{49}{4}$ (d) $\frac{4}{49}$

17 If a , b , 2 and 3 are proportional , then $\frac{b}{a} = \dots\dots$ *(El-Kalyoubia 17)*

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) 3 (d) 2

18 If a , x , b and $2x$ are proportional quantities , then $\frac{a}{b} = \dots\dots$ *(Aswan 17)*

(a) 2 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

19 If 5 a , 2 , 3 b and 7 are four proportional quantities , then $\frac{a}{b} = \dots\dots$ *(Souhag 13)*

(a) $\frac{3}{7}$ (b) $\frac{6}{35}$ (c) $\frac{3}{5}$ (d) $\frac{3}{2}$

20 If $4x^2 = 9y^2$, then $\frac{x}{y} = \dots\dots$ *(Beni Suef 16)*

(a) $\frac{9}{4}$ (b) $\frac{3}{2}$ (c) $\pm\frac{2}{3}$ (d) $\pm\frac{3}{2}$

21 If $\frac{5a - 7b}{2a + 3b} = 0$, then $\frac{b}{a} = \dots\dots$

(a) $\frac{5}{7}$ (b) $\frac{7}{5}$ (c) $\frac{3}{10}$ (d) $\frac{10}{3}$

22 If $\frac{a + 2b}{a - b} = \frac{2}{3}$, then $\frac{b}{a} = \dots\dots$ *(Alexandria 11 – El-Monofia 20)*

(a) $\frac{1}{8}$ (b) 8 (c) $-\frac{1}{8}$ (d) -8

23 If a , b , c and d are proportional quantities , then $\dots\dots$

(a) $\frac{b}{d} = \frac{a}{c}$ (b) $\frac{a}{c} = \frac{d}{b}$ (c) $\frac{b}{c} = \frac{a}{d}$ (d) $ab = cd$

24 $\dfrac{18\,x^2 y}{\cdots\cdots\cdots} = \dfrac{3\,x}{5\,y}$

(a) $20\,x\,y^2$ (b) $30\,x\,y$ (c) $30\,x\,y^2$ (d) $15\,x\,y$

25 $\dfrac{a+b}{a} = \dfrac{\cdots\cdots}{a^2}$

(a) $a^2 + b^2$ (b) $a^2 + ab$ (c) $a^2 + a^2\,b$ (d) $2\,a + 2\,b$

26 If $4\,x^2 + 9\,y^2 = 12\,x\,y$, then $\dfrac{x}{y} = \cdots\cdots$ *(El-Kalyoubia 09)*

(a) $\dfrac{3}{2}$ (b) $\dfrac{2}{3}$ (c) $-\dfrac{2}{3}$ (d) $-\dfrac{3}{2}$

27 The ratio between the area of a square shaped region of side length ℓ cm. to the area of another square shaped region of side length $2\,\ell$ cm. is ……… *(El Monofia 13)*

(a) 1 : 2 (b) ℓ : 4 (c) 1 : 4 (d) 4 : 1

2 Find each of the following :

1 The first proportional for the numbers : … , $\sqrt{8}$, 7 and $14\sqrt{2}$

2 The third proportional for the quantities : a , (a + b) , … and $(a^2 - b^2)$

3 The fourth proportional for the quantities : (a + b) , (a − b) , (a − b) and ..

3 Find the value of x in each of the following , if :

1 $(2\,x - 3) : (x - 5) = 1 : 4$ « 1 »

2 $(x - 5) : (5\,x + 3) = 2 : 3$ « −3 »

3 $(x^2 - 8) : (2\,x^2 + 1) = 1 : 3$ « ±5 »

4 $(x^2 + 10\,x) : (2\,x^2 - 3) = 24 : 5$ where x is an integer. « 2 »

4 If $\dfrac{x - 2\,y}{x + 3\,y} = \dfrac{1}{3}$, **find :** $\dfrac{y}{x}$ *(Aswan 15)* « $\dfrac{2}{9}$ »

5 If $\dfrac{2\,x + 3}{2\,x - 3} = \dfrac{2\,y + 5}{2\,y - 5}$, **prove that :** $\dfrac{x}{y} = \dfrac{3}{5}$

6 If $x^2 - 4\,y^2 = 3\,x\,y$, **find :** $x : y$ « −1 : 1 or 4 : 1 »

7 If $3\,x^2 - 10\,x\,y + 7\,y^2 = 0$, $x \neq y$, **find the ratio :** $x : y$ « 7 : 3 »

8 If $\frac{x}{y} = \frac{2}{3}$, **find the value of the ratio :** $\frac{3x+2y}{6y-x}$ *(Souhag 19 – El-Menia 20)* « $\frac{3}{4}$ »

9 If $\frac{a}{b} = \frac{3}{5}$, **find the value of :** $7a+9b : 4a+2b$ *(Qena 15 Cairo 20)* « 3 »

10 If $4a = 3b$, **then find the value of :**

1. $\frac{4a+b}{2a-b}$ « 8 »
2. $\frac{b^2-a^2}{a^2-b^2}$ « -1 »

11 If $\frac{a}{b} = \frac{1}{3}$, $\frac{c}{d} = \frac{7}{2}$, **find the ratio :** $\frac{2ac+bd}{bc-3ad}$ « $\frac{4}{3}$ »

12 If $7x-3y : x+y = 3:1$, **find the ratio :** $12x+9y : 11x-3y$ « 2 : 1 »

13 If $\frac{21x+a}{7x+b} = \frac{a}{b}$, where $x \neq 0$, **then find the value of :** $\frac{a+2b}{2a}$ *(Ismailia 13)* « $\frac{5}{6}$ »

14 Find the number that if it is added to each of the numbers 3 , 5 , 8 and 12 , they become proportional. *(South Sinai 17 – Assiut 18)* « 2 »

15 Find the number which is subtracted from each of the following numbers to be proportional 16 , 21 , 14 and 18 « 6 »

16 **Prove that : a , b , c and d are proportional quantities if :**

1. $\frac{a+b}{b} = \frac{c+d}{d}$ *(El-Fayoum 09)*
2. $\frac{a}{b-a} = \frac{c}{d-c}$ *(El-Sharkia 15 – El-Fayoum 15 Aswan 20)*
3. $\frac{a-b}{a+b} = \frac{c-d}{c+d}$
4. $\frac{a^2-2c^2}{b^2-2d^2} = \frac{a^2}{b^2}$ where a , b , c and d are positive quantities.

17 If $a:b:c = 5:7:3$ and $a+b = 27.6$, **find the value of each of :** a , b and c « 11.5 , 16.1 , 6.9 »

18 If $a:b:c = 3:4:5$, **find the numerical value of the expression :** $\frac{a^2+b^2+c^2}{a(b+c)}$ « $\frac{50}{27}$ »

19 If $2a = 3b = 4c$, **find :** $a:b:c$ « 6 : 4 : 3 »

20 **Answer the following :**

1. Find the number which if it is added to the two terms of the ratio 7 : 11 , it will be 2 : 3 *(Alex. 14 – Cairo 17 – El-Fayoum 18 – Giza 19)* « 1 »

2. Find the number that if we subtract thrice of it from each of the two terms of the ratio $\frac{49}{69}$, the ratio becomes $\frac{2}{3}$ *(Giza 12 – El-Beheira 20)* « 3 »

3. Find the number which if its square is added to each of the two terms of the ratio 7 : 11 , it becomes 4 : 5 *(Suez 17 – El-Monofia 20)* « 3 or 3 »

4. Find the positive number which if we add its square to each of the two terms of the ratio 5 : 11 , it becomes 3 : 5 *(Kafr El-Sheikh 17 Giza 19 - Beni Suef 20)* « 2 »

5. What is the number which is subtracted from the antecedent of the ratio 15 : 13 and added to its consequent to become 3 : 4 ? *(Luxor 20)* « 3 »

6. Two integers , the ratio between them is 3 : 7 and if we subtracted 5 from each term , the ratio between them becomes 1 : 3 , find the two numbers. *(Alex. 18 – Ismailia 20)* « 15 , 35 »

7. Two integers , the ratio between them is 2 : 3 , if you add to the first 7 and subtract from the second 12 , the ratio between them becomes 5 : 3
Find the two numbers. *(El Beheira 15 Beni Suef 17 – Matrouh 18)* « 18 , 27 »

8. Two positive real numbers , the ratio between them is 4 : 7 and the square of the small number exceeds 5 times the great number by 39 , find the two numbers. « 12 , 21 »

Geometric Applications

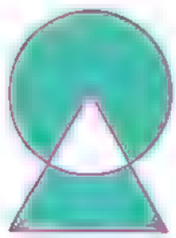
21 The ratio between the two dimensions of a rectangle is 4 : 7 and its perimeter is 88 cm. Find its area. « 448 cm^2 »

22 The ratio between the base length and the height of a triangle is 3 : 2 and its area is 48 cm^2. Find the length of the base and the height. « 12 cm. , 8 cm. »

23 **In the opposite figure :**

Alaa shaded $\frac{5}{6}$ the area of the circle , $\frac{2}{3}$ the area of the triangle.

Find the ratio between the area of the circle and the area of the triangle.



(Giza 08) « 2 : 1 »

Life Applications

24 A sum of money is divided between two persons with the ratio 2 : 3 , if the share of the first is L.E.30 , find the share of the second. « L.E. 45»

25 The shadow length of a tree is 3 m. and in the same moment the shadow length of Islam is 120 cm. If the height of Islam is 180 cm. , find the height of the tree. « $4\frac{1}{2}$ m. »

26 Through the interest of the Egyptian authorities in the villages , a budget of 1.85×10^6 pounds was set for one of the villages to build a school , a medical unit and a youth centre. If the cost of the school is $\frac{3}{2}$ of the cost of the medical unit and the cost of the medical unit is $\frac{5}{6}$ of the cost of the youth centre , what is the cost of each of them ?

« 7.5×10^5 , 5×10^5 , 6×10^5 »

27 If the rate of success in one of the governorates of the third preparatory is 83% and the rate of success for boys is 79% and the rate of success of girls is 89% , find the ratio between the number of boys and the number of girls in this governorate.

« 3 : 2 »

28 The length of a piece of wire is 152 cm. , it is divided into two parts of ratio 11 : 8 , a circular shape is made from the long part and a square shape is made from the short part. Find the ratio between the area of the square and the area of the circle. $\left(\pi = \frac{22}{7}\right)$ « 32 : 77 »

For excellent pupils

29 Four proportional numbers , the fourth proportional equals the square of the second proportional , the first proportional decreases the second proportional by 2 , the third proportional = 8 , find the four numbers.

« 2 , 4 , 8 , 16 or 4 , −2 , 8 , 4 »

---

30 If x , y , z and ℓ are four proportional numbers and $x + y = 8$, $y + z = 14$, $z + \ell = 24$, find the value of each of x , y , z and ℓ « 3 , 5 , 9 , 15 »

---

31 Find the positive number which if its multiplicative inverse is added to the consequent of the ratio $\frac{2}{3}$, it will become $\frac{3}{5}$ « 3 »

Wonders of numbers

- Choose an integer between 100 , 1000
- Multiply it by 7 , then multiply the product by 11 and multiply the product by 13
- Do it using different numbers and notice the product each time !

6 Follow properties of proportion

Interactive test

From the school book

● Remember ● Understand ○ Apply Problem Solving

1 Choose the correct answer from those given :

1 If $\frac{a}{b} = \frac{c}{d} = \frac{3}{5}$, then $\frac{a+c}{b+d} = $ ....

(a) $\frac{8}{5}$ (b) $\frac{5}{8}$ (c) $\frac{3}{5}$ (d) $\frac{5}{3}$

2 If $\frac{a}{b} = \frac{c}{d} = \frac{h}{m}$, then $\frac{a+c+h}{b+d+m} = $ .... *(El-Sharkia 20)*

(a) $\frac{a}{b} + \frac{c}{d} + \frac{h}{m}$ (b) $\frac{c}{h}$ (c) $\frac{c}{a}$ (d) $\frac{c}{d}$

3 If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{3}{5}$, then $\frac{a-2c+e}{b-2d+f} = $ ....

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{3}{5}$ (d) $\frac{2}{5}$

4 If $\frac{x}{2} = \frac{y}{3} = \frac{z}{5}$, then each ratio equals .... *(El-Fayoum 19)*

(a) $\frac{x+y+z}{3}$ (b) $\frac{x+2y-z}{3}$ (c) $\frac{x-y+z}{10}$ (d) $\frac{x-y}{5}$

5 If $\frac{4}{x} = \frac{7}{y} = \frac{a}{y-x}$, then a = ...

(a) – 3 (b) 3 (c) 11 (d) 28

6 If $\frac{l}{3} = \frac{m}{8} = \frac{l + \frac{1}{2} m}{b}$, then b = ...

(a) 24 (b) 11 (c) 8 (d) 7

7 If $\frac{x}{5} = \frac{y}{4} = \frac{x+2y}{k}$, then k = ... *(Kafr El-Sheikh 11 – El-Gharbia 17)*

(a) 9 (b) 13 (c) 14 (d) 8

8 If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, then $\frac{a+2c+3e}{b+2d+3f} = \frac{\dots\dots}{5f}$

(a) 5 a (b) 5 c (c) 5 e (d) 5 a + 5 c + 5 e

9 If $\frac{a}{2} = \frac{b}{3}$, then $\frac{b-a}{b+a} =$ ....

(a) $\frac{1}{5}$ (b) $\frac{1}{3}$ (c) $\frac{2}{5}$ (d) $\frac{3}{5}$

10 If $\frac{a}{b} = \frac{c}{d} = 5$, then $\frac{2a-3c}{2b-3d} =$ ....

(a) 10 (b) 15 (c) 5 (d) 1

11 If $\frac{6x}{4y} = \frac{3z}{9\ell} = 10$, then $\frac{3x+z}{2y+3\ell} =$ ....

(a) 50 (b) 30 (c) 20 (d) 10

12 If $\frac{a}{b} = \frac{c}{d} = m$, where $m \neq 0$, then $\frac{a \times c}{b \times d} =$ .... *(Cairo 17)*

(a) 2 m^2 (b) m^2 (c) m (d) 2 m

13 If $\frac{x}{5} = \frac{y}{7} = m$, then $\frac{2x+y}{17} =$ ....

(a) 3 m (b) 2 m (c) 17 m (d) m

14 If $\frac{a}{4} = \frac{b}{5} = k$, then $\frac{4a+4b}{9} =$ ....

(a) k (b) 2 k (c) 3 k (d) 4 k

15 If $\frac{a}{4} = \frac{b}{5}$, 2 a + 3 b = 46 , then a = ...

(a) 2 (b) 4 (c) 5 (d) 8

16 If $\frac{a}{b} = \frac{2}{3}$, $\frac{a}{c} = \frac{4}{5}$, then b : c = ... *(El-Gharbia 17)*

(a) 3 : 4 (b) 5 : 6 (c) 6 : 5 (d) 4 : 3

2 **If a , b , c and d are proportional quantities , prove that :**

1 $\frac{3a+c}{5a-2c} = \frac{3b+d}{5b-2d}$ *(Assiut 17)*

2 $\frac{3a-2c}{5a+3c} = \frac{3b-2d}{5b+3d}$ *(Suez 16 – Kafr El-Sheikh 18)*

3 $\frac{a^2+c^2}{ab+cd} = \frac{a}{b}$ *(El-Monofia 11)*

4 $\frac{a^2+c^2}{b^2+d^2}=\frac{a\,c}{b\,d}$ *(El-Monofia 16 – El-Kalyoubia 17 – El-Gharbia 18)*

5 $\frac{a\,c}{b\,d}=\left(\frac{a-c}{b-d}\right)^2$ *(Alex. 14 – Suez 18)*

6 $\left(\frac{a+b}{c+d}\right)^2=\frac{2a^2-3b^2}{2c^2-3d^2}$

7 $\sqrt{\frac{3a^2-5c^2}{3b^2-5d^2}}=\frac{a}{b}$ where a , b , c and d are positive quantities.

8 $\sqrt[3]{\frac{5a^3-3c^3}{5b^3-3d^3}}=\frac{a+c}{b+d}$ *(El-Kalyoubia 19)*

9 $\frac{a^2-2ac+c^2}{a\,c}=\frac{b^2-2bd+d^2}{b\,d}$ *(Ismailia 18)*

3 If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$, **prove that :**

1 $\frac{a+5c}{b+5d}=\frac{c-3e}{d-3f}$

2 $\frac{2a+7c-4e}{2b+7d-4f}=\frac{a-8e}{b-8f}$

3 $\frac{2a^4b^2+3a^2e^2-5e^4f}{2b^6+3b^2f^2-5f^5}=\frac{a^4}{b^4}$

4 $\sqrt{\frac{5a^2-7ce}{5b^2-7df}}=\frac{2a+c}{2b+d}$

4 If $\frac{x}{3}=\frac{y}{4}=\frac{z}{5}$, **prove that :**

1 $\frac{2y-z}{3x-2y+z}=\frac{1}{2}$ *(Giza 15 – North Sinai 18 – Port Said 19 – Beni Suef 20)*

2 $\sqrt{3x^2+3y^2+z^2}=2x+y$ *(El-Menia 12 – Souhag 16 – Damietta 19)*

5 If $x=\frac{y}{2}=\frac{z}{3}$, **then prove that :** $\frac{x+y-2z}{x-3z}=\frac{3}{8}$ *(Assiut 17)*

6 If $\frac{a}{2}=\frac{b}{3}=\frac{c}{4}$, **prove that :** $2a-5b+3c$ = one of the given ratios.

7 If $\frac{a}{2}=\frac{b}{3}=\frac{c}{4}=\frac{2a-b+5c}{3x}$, **then find the value of :** x

(El-Gharbia 16 – Qena 17 – Luxor 18 – Aswan 19 – El-Kalyoubia 20) « 7 »

8 If $\frac{a}{4x+y}=\frac{b}{x-4y}$, **prove that :** $\frac{a+b}{5x-3y}=\frac{a-b}{3x+5y}$ *(Damietta 12 – El-Dakahlia 19)*

9 If $\frac{x+y}{19}=\frac{y+z}{7}$, **prove that :** $\frac{x+2y+z}{13}=\frac{x-z}{6}$

10 If $\frac{y}{x-z} = \frac{x}{y} = \frac{x+y}{z}$, prove that each ratio is equal to 2 (unless $x + y = 0$) ,
then find $x : y : z$ *(El-Beheira 18)* « 4 : 2 : 3 »

11 If $\frac{x}{a-b+c} = \frac{y}{b-c+a} = \frac{z}{c-a+b}$, **prove that :** $\frac{x+y}{a} = \frac{y+z}{b}$ *(Port Said 09)*

12 If $\frac{x}{2a+b} = \frac{y}{2b-c} = \frac{z}{2c-a}$, **then prove that :** $\frac{2x+y}{4a+4b-c} = \frac{2x+2y+z}{3a+6b}$

(El-Beheira 17 – El-Kalyoubia 18 – Matrouh 19)

13 If $\frac{a}{2x-y} = \frac{b}{2y-x}$, **prove that :** $\frac{2a+b}{a+2b} = \frac{x}{y}$

14 If $\frac{a}{2x+y} = \frac{b}{3y-x} = \frac{c}{4x+5y}$, **prove that :** $\frac{a+2b}{4b+c} = \frac{7}{17}$

15 If $\frac{a}{2} = \frac{b}{7} = \frac{c}{3}$, **find the value of :** $\frac{a+2b}{b-c}$ *(North Sinai 09)* « 4 »

16 If $\frac{x+y}{7} = \frac{y+z}{5} = \frac{z+x}{8}$, **prove that :** $\frac{x+y+z}{x-z} = 5$ *(El-Monofia 16)*

17 If $\frac{a+b}{4} = \frac{b+c}{5} = \frac{c+a}{7}$, **prove that :** $\frac{a+b+c}{8} = \frac{a}{3}$ *(Kafr El Sheikh 15)*

18 If $\frac{x+y}{3} = \frac{y+z}{8} = \frac{z+x}{6}$, **prove that :** $\frac{x+y+z}{2x+3y+3z} = \frac{17}{50}$ *(Kafr El-Sheikh 20)*

19 If $\frac{x+y}{5} = \frac{y+z}{8} = \frac{z+x}{7}$, **prove that :** $\frac{x}{2} = \frac{y}{3} = \frac{z}{5}$ *(New Valley 17)*

20 If $\frac{x+y}{25} = \frac{x-y}{11} = \frac{x+y-z}{8}$, **prove that :** $x : y : z = 18 : 7 : 17$

21 If $\frac{a+3b}{x+6y} = \frac{3b+5c}{6y+10z} = \frac{5c+a}{10z+x}$, **prove that :** $\frac{a}{b} = \frac{x}{2y}$ and find $a : b : c$ « $x : 2y : 2z$ »

22 If $\frac{a}{3x+4y} = \frac{b}{5x-2y} = \frac{c}{y+2x}$, **prove that :** $13x(3c-2a) + 5y(a+2b) = 0$

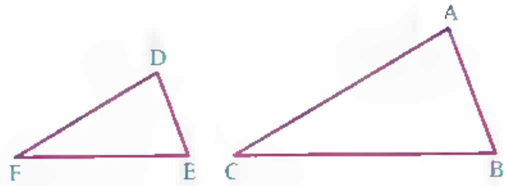
23 If $\frac{x}{7} = \frac{y}{3}$, **prove that :** $(2x - 3y)$, $(x + 2y)$, 10 and 26 are proportional. *(Luxor 19)*

24 If $\frac{a}{b} = \frac{3}{5}$ and $\frac{a}{c} = \frac{3}{7}$, **find the value of the expression :** $a + b + c$ in terms of a « 5 a »

25 If $\frac{a}{b} = \frac{2}{3}$, $\frac{a}{c} = \frac{3}{5}$ and $a + b + c = 75$, **find the value of each of :** a , b and c

(Red Sea 16) « 18 , 27 , 30 »

Geometric Application



26 **In the opposite figure :**

If $\Delta ABC \sim \Delta DEF$

where $DF : AC = 2 : 3$ and the perimeter of $\Delta DEF = 22$ cm.

, find the perimeter of : ΔABC « 33 cm. »

For excellent pupils

27 If $\frac{a}{x-y+z} = \frac{b}{x+y-z} = \frac{c}{y+z-x}$, **prove that :** each ratio $= \frac{ax+by+cz}{x^2+y^2+z^2}$

---

28 If $\frac{2x+y}{x} = \frac{4y+z}{y} = \frac{4z+3x}{z}$, find the ratio $x : y : z$

, **then prove that :** $\frac{2x+y+z}{3x-y+2z} = \frac{4}{3}$ « 1 : 3 : 3 »

---

29 If $\frac{a+2b}{5} = \frac{3b-c}{3} = \frac{c-a}{2}$, **prove that :**

1 $a + b - c = \text{zero}$

2 $\frac{3b-a}{2b+c} = \frac{5}{7}$

Wonders of numbers

2520 is the smallest number that can be divided by each of the numbers from 1 to 10 without any remainder.

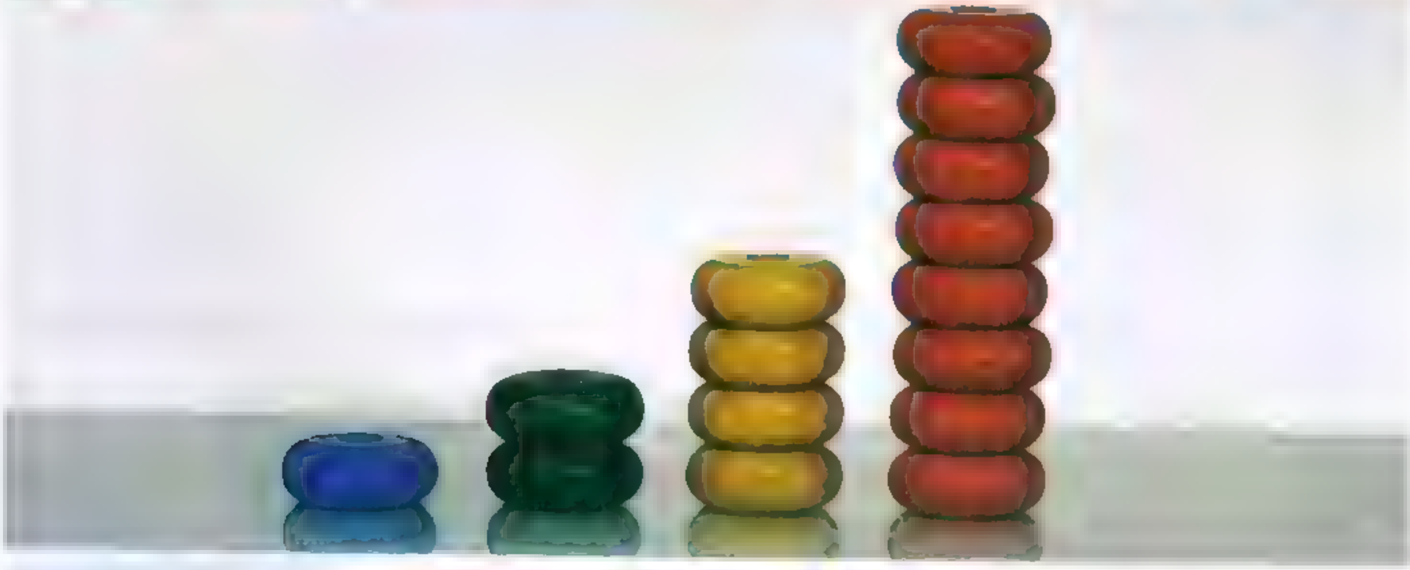
Try it yourself !

EXERCISE **7**

Continued proportion

Interactive test

From the school book



Remember · Understand · Apply · Problem Solving

1 **Find the middle proportional between :**

1. (From the school book) 3 , 27
2. 9 , 25
3. −2 , −8 *(Giza 09)*
4. $\frac{1}{5}$, 125
5. $2a$, $8ab^2$
6. $(\ell + m)^2$, $(\ell - m)^2$

2 **Find the third proportional of each of the following :**

1. 6 , 12
2. x^2 , $-5x$
3. x^2 , $-3x^2$

3 **If b is the middle proportional between a and c , prove that :**

1. $\frac{a}{c} = \frac{b^2}{c^2}$ *(Giza 14)*
2. $\frac{2a+3b}{2b+3c} = \frac{a}{b}$ *(Ismailia 17)*
3. $\frac{a-b}{b-c} = \frac{a+3b}{3c+b}$
4. (From the school book) $\frac{a^2+b^2}{b^2+c^2} = \frac{a}{c}$ *(El-Kalyoubia 18 – Cairo 20)*
5. $\left(\frac{b-c}{a-b}\right)^2 = \frac{c}{a}$
6. $\frac{a^3+b^3}{b^3+c^3} = \frac{a^2}{cb}$ *(El-Monofia 11)*
7. $\frac{a^3-4b^3}{b^3-4c^3} = \frac{b^3}{c^3}$
8. (From the school book) $\frac{2c^2-3b^2}{2b^2-3a^2} = \frac{c}{a} = \frac{c^2}{b^2}$
9. $\frac{a^2+ab+b^2}{b^2+bc+c^2} = \frac{a^2-b^2}{b^2-c^2}$
10. $\frac{a^2}{b^2} + \frac{b^2}{c^2} = \frac{2a}{c}$ *(Aswan 16 – Port Said 17 – El-Dakahlia 19)*
11. (From the school book) $\frac{a+b+c}{a^{-1}+b^{-1}+c^{-1}} = b^2$
12. $\frac{ac}{b(b+c)} = \frac{a}{a+b}$ *(El-Gharbia 17)*

4 **If a , b , c and d are in continued proportion , prove that :**

1. $\frac{a-2b}{b-2c}=\frac{3b+4c}{3c+4d}$
2. $\frac{3a+5c}{3b+5d}=\frac{a-4c}{b-4d}$
3. $\frac{3a-5c}{a-b+c}=\frac{3b-5d}{b-c+d}$
4. $\frac{a-d}{a+b+c}=\frac{a-2b+c}{a-b}$
5. $\frac{c^2-d^2}{a-c}=\frac{bd}{a}$ *(Matrouh 17 – El-Beheira 18 – South Sinai 20)*
6. $\frac{a^2-3c^2}{b^2-3d^2}=\frac{b}{d}$ *(El-Beheira 15 – Alex. 17 – Beni Suef 18)*
7. $\frac{ab-cd}{b^2-c^2}=\frac{a+c}{b}$ *(Qena 16 – El-Monofia 17 – El-Monofia 20)*
8. $\frac{a}{b+d}=\frac{c^3}{c^2d+d^3}$ *(Alex. 19 – El-Fayoum 20)*
9. $\frac{a^2+b^2+c^2}{b^2+c^2+d^2}=\frac{ac}{bd}$ *(El-Dakahlia 11)*
10. $\frac{2a+3d}{3a-4d}=\frac{2a^3+3b^3}{3a^3-4b^3}$
11. $\frac{a+5b}{b+5c}=\sqrt{\frac{b}{d}}$
12. $\sqrt[3]{\frac{5a^3-3c^3}{5b^3-3d^3}}=\frac{a+c}{b+d}$ *(Alexandria 11)*
13. $\left(\frac{a+b}{b+c}\right)^3=\frac{a}{d}$ *(El-Sharkia 15)*
14. $\frac{a^2+d^2}{c(a+c)}=\frac{b}{d}+\frac{d}{b}-1$

5 **Choose the correct answer from those given :**

1. The third proportional of the two numbers 9 and – 12 is .... *(El-Beheira 11)*
(a) – 16 (b) 8 (c) 16 (d) 108

2. The middle proportional between a and c is .... *(Beni Suef 20)*
(a) $\sqrt{a+c}$ (b) $\frac{a+c}{2}$ (c) $\pm\sqrt{ac}$ (d) a c

3. If the number 6 is the positive proportional mean of the two numbers 2 and m , then m = ... *(Aswan 13)*
(a) 8 (b) 12 (c) 18 (d) 36

4. If x , y , z are in continued proportion , then x = ... *(Luxor 20)*
(a) $\pm\sqrt{yz}$ (b) y z (c) $\frac{y^2}{z}$ (d) $\frac{y}{z}$

5. If ℓ , m and n are in continued proportion , then $m^2-\ell n$ = ...
(a) – 1 (b) 0 (c) 1 (d) 2

6. If 7 , x and $\frac{1}{y}$ are in continued proportion , then $x^2 y$ = ... *(El-Beheira 19)*
(a) 7 (b) $\frac{1}{7}$ (c) 14 (d) 49

7. If $\frac{a}{b}=\frac{b}{c}=\frac{c}{5}=2$, then a = ... *(El-Monofia 12)*
(a) 5×2^2 (b) 40 (c) 10 (d) 2×5^3

8 If $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = 2$, then $\frac{a}{d} =$ .... *(El-Sharkia 13)*

(a) 2 (b) 4 (c) 8 (d) 16

9 If $6 a^2 b^2$, 3 a b and c are proportional quantities , then c = ...

(a) – 3 (b) 3 a b (c) $\frac{3}{2}$ (d) $\frac{2}{3}$

10 If a , 2 , 4 and b are in continued proportion , then a + b = ... *(El Dakahlia 20)*

(a) 2 (b) 4 (c) 6 (d) 9

11 The proportional mean between $(x - 2)$ and $(x + 2)$ is .... *(Cairo 09)*

(a) $\sqrt{x+2}$ (b) $x^2 - 4$ (c) $\pm\sqrt{x^2-4}$ (d) $\sqrt{x^2-4}$

12 The number which is added to each of the numbers 1 , 3 and 6 to become in continued proportion is .... *(Damietta 13)*

(a) 1 (b) 2 (c) 3 (d) 6

6 If a , 3 , 9 and b are in continued proportion , find the value of each of a and b

(Luxor 16) « 1 , 27 »

7 If 3 , ℓ , 12 and m are in continued proportion , find the value of each of ℓ and m « ± 6 , ± 24 »

8 **Complete the following :**

1 The third proportional of the two quantities $9 (r + 1)^2$ and $6 (r^2 - 1)$ is ....

2 The middle proportional of the two quantities $9 x^2 - 25 y^2$ and $\frac{3x + 5y}{3x - 5y}$ is ....

3 If $\frac{y}{x} = \frac{x}{z} = \frac{2}{5}$, then y = ... z

4 If a , b , c , d and e are in continued proportion and each ratio = the constant m , then $\frac{a}{e} =$ ....

5 If a , b and c are proportional , then $\frac{a^2 + b^2}{b^2 + c^2} = \frac{\cdots\cdots}{c}$

6 The real number x which makes $x + 1$, $x + 5$ and $x + 13$ proportional is ....

7 If 2 , $4 + x$ and 18 are proportional quantities , $x \in \mathbb{R}$, then $x =$ ....

9 Find the number that if we subtract it from each of the numbers 3 , 7 , 19 , then they become in continued proportion. *(Luxor 17)* « 1 »

10 If b is the middle proportional between a and c and a = 4 c = 4

, then find the value of : $a^2 + b^2 + c^2$ *(El-Fayoum 17)* « 21 »

11 If b is the middle proportional between a and c , c is the middle proportional between b and d ,

prove that : $\frac{a^2}{b^2}+\frac{b^2}{c^2}+\frac{c^2}{d^2}=\frac{a}{c}+\frac{b}{d}+\frac{a\,c}{b\,d}$

12 If $y^2 = x z$, **prove that :** $\frac{x(x-y)}{y(y-z)}=\frac{y^2}{z^2}$

13 If $b^2 = a c$ and $c^2 = b d$, **prove that :** $\frac{2a+3d}{3a-4d}=\frac{2a^3+3b^3}{3a^3-4b^3}$

14 If $\frac{a^2+b^2}{b^2}=\frac{b^2+c^2}{c^2}$, prove that b is the middle proportional between a and c where a c is a positive quantity. *(Alexandria 15 – Beni Suef 15)*

15 If a , b , c and d are in continued proportion , **prove that :** (b + c) is the middle proportional between (a + b) and (c + d)

16 If 5 a , 6 b , 7 c and 8 d are positive quantities in continued proportion ,

prove that : $\sqrt[3]{\frac{5a}{8d}}=\sqrt{\frac{5a+6b}{7c+8d}}$

17 If b is the middle proportional between a and c , **prove that :** $\frac{a^4+b^4+c^4}{a^{-4}+b^{-4}+c^{-4}}=b^8$

Geometric Applications

18 x , y and z are three proportional side lengths in a triangle , $x + y = 15$ cm. and $y + z = 22.5$ cm. **Find :** $x : y$ « 2 : 3 »

19 ABC is a triangle in which m (∠ C) = 60° , if the measures of its angles ∠ A , ∠ B and ∠ C respectively are in continued proportion.

, **find :** m (∠ A) and m (∠ B) « 60° , 60° »

For excellent pupils

20 If $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}=2$, **find the solution set of the equation :** $a x^2 - 2 b x + c = 0$ « $\{\frac{1}{2}\}$ »

21 If 5 is the middle proportional between x and y , find the middle proportional between $\left(x+\frac{1}{y}\right)$ and $\left(y+\frac{1}{x}\right)$ « ± 5.2 »

Unit 8

Direct variation and inverse variation

Interactive test

From the school book

● Remember ● Understand ● Apply ● Problem Solving

1 Complete the following :

1. If $x \propto y$, then $x =$ ....
2. If $z = \frac{m}{x^2}$ where m is a constant $\neq 0$, then $z \propto$ ....
3. If $y \propto x$, then $\frac{x_1}{x_2} = \frac{\dots\dots}{\dots\dots}$ *(New Valley 11)*
4. If x varies inversely as y, then $\frac{y_1}{y_2} = \frac{\dots\dots}{\dots\dots}$ *(Qena 11)*
5. If $y = \frac{3}{5} x$, then $y \propto$ .... *(Aswan 08)*
6. If $y \propto \frac{5}{x}$, then y varies inversely as .... *(Ismailia 09)*
7. If $x - 2y = 0$, then $x \propto$ .... *(El-Dakahlia 09)*
8. If $2xy = 5$, then $x \propto$ .... *(El-Sharkia 09)*
9. If $y \propto x$ and $y = 2$ as $x = 8$, then $y =$ .... when $x = 12$ *(El-Sharkia 11)*
10. If $y \propto \frac{1}{x}$ and $y = 3$ as $x = 20$, then $y =$ .... when $x = 12$ *(El-Beheira 12)*
11. If $y \propto x$ and $y = 2$ as $x = 4$, then $y =$ .... x *(Cairo 11)*
12. If $y \propto x$ and $y = 6$ as $x = 4$, then $\frac{y}{x} =$ .... *(in the simplest form)*
13. If $t = \frac{mk}{h}$ where m is a constant $\neq 0$, then t varies .... as k when h is constant, t varies .... as h when k is constant.

2 If y varies directly as x and $y = 20$ as $x = 7$

Find : x when $y = 40$

« 14 »

3 If a varies inversely as b and a = 12 as b = 8 , **find :**

1. The value of a as b = 1.5
2. The value of b as a = 2

« 64 , 48 »

4 If $y \propto x$ and y = 14 when $x = 42$, **find :**

(El-Monofia 15 , Port Said 18 – South Sinai 19 – Port Said 20)

1. The relation between x and y
2. The value of y when $x = 60$

« $y = \frac{1}{3}x$, 20 »

5 If $y \propto \frac{1}{x}$ and y = 3 when $x = 2$, **find :** (Alex 17 , North Sinai 19 , Cairo 20)

1. The relation between x and y
2. The value of y when $x = 1.5$

« $xy = 6$, 4 »

6 If $y \propto \frac{1}{x}$ and $x = 3$ as y = 10 , find y when :

$x \in \{1 , 2 , 3 , 4 , 5\}$

« 30 , 15 , 10 , 7.5 , 6 »

7 If y ∝ the multiplicative inverse of the expression $\frac{1}{x^2}$, then find the relation between x and y , if y = 4 as $x = 3$, then find the value of y as $x = 9$

(El-Sharkia 08) « $y = \frac{4}{9}x^2$, 36 »

8 If $y \propto x^3$ and y = 64 as $x = 2$, find the relation between x and y and find the value of y as $x = \frac{1}{2}$

(Luxor 20) « $y = 8x^3$, 1 »

9 If y varies inversely as $\sqrt{x}$ and y = 2 as $x = 16$

, find the value of y as $x = 32$

« $\sqrt{2}$ »

10 If $y^2 \propto x^3$, find the relation between x and y where y = 3 as $x = 2$

(Qena 09) « $y^2 = \frac{9}{8}x^3$ »

11 If $y^2 \propto \frac{1}{\sqrt[3]{x}}$ and $x = 8$ as y = 3 , find x as y = 1.5

« 512 »

12 If $y \propto (x + 1)$ and $x = 3$ when y = 2 , then find the relation between x and y

(Matrouh 09) « $y = \frac{1}{2}(x + 1)$ »

13 If $\frac{5x - 3y}{3x + 5y} = 1$ for all the values of $x \in \mathbb{R}_+$, $y \in \mathbb{R}_+$, **prove that :** $y \propto x$

14 If $\frac{a+2b}{6}=\frac{b+3c}{3}$, then prove that : $a \propto c$ *(El-Fayoum 17)*

15 If $\frac{21x-y}{7x-z}=\frac{y}{z}$, prove that : $y \propto z$ *(Cairo 15 – El Kalyoubia 18 – Damietta 19)*

16 If $x^2 y^2 - 6xy + 9 = 0$, then prove that : y varies inversely as x

(Damietta 13 – South Sinai 14)

17 If $4a^2 + 9b^2 = 12ab$, prove that : a varies directly as b *(Matrouh 17)*

18 If $x^4 y^2 - 14x^2 y + 49 = 0$, prove that : $y \propto \frac{1}{x^2}$ *(Alex 19)*

19 If $(4x+7y) \propto (x+2y)$ where $x \in \mathbb{R}$ and $y \in \mathbb{R}$, then prove that : $y \propto x$

20 If $\left(\frac{a}{y}-\frac{a}{x}\right) \propto (x-y)$ where a is a constant , $x \neq y \neq 0$,

then prove that : x varies inversely as y

21 Which of the following tables represents the direct variation and which of them represents the inverse variation and which does not represent the direct variation nor the inverse variation with mentioning the reason in each case :

x	y
3	20
5	12
4	15
6	10

x	y
2	9
4	18
12	54
16	72

x	y
5	9
10	18
15	27
25	45

x	y
3	6
−2	−9
−18	1
9	−2

22 **From the data in the following table , answer the following questions :**

x	2	4	6
y	6	3	2

1. Show the type of variation between x and y
2. Find the constant of variation.
3. Find the value of y at $x = 3$
4. Find the value of x at $y = 2\frac{2}{5}$

(Damietta 16 – Ismailia 18) « 12 , 4 , 5 »

23 **From the opposite table :**

x	1	2	b	4	6
y	12	a	36	48	72

1. Show the type of variation between x and y
2. Find the value of each of a and b

« a = 24 , b = 3 »

24 If $y = z + 5$, z changes inversely with X and $y = 6$ when $X = 2$, then find the relation between y and X , then find the value of y when $X = 1$ *(El Monofia 17)* « $y = \frac{2}{X} + 5$, 7 »

25 If $y = a + b$ where a is a constant , b varies directly with X , $y = 3$ when $X = 0$ and $y = 5$ when $X = 3$, find the relation between X and y then find the value of y when $X = 7$

« $y = 3 + \frac{2}{3} X$, $7\frac{2}{3}$ »

26 If $y = a - 9$ and $y \propto \frac{1}{X^2}$ and $a = 18$ when $X = \frac{2}{3}$, find the relation between y and X , then deduce the value of y when $X = 1$ *(Kafr El Sheikh 18 – Suez 18 – Luxor 19)* « $y = \frac{4}{X^2}$, 4 »

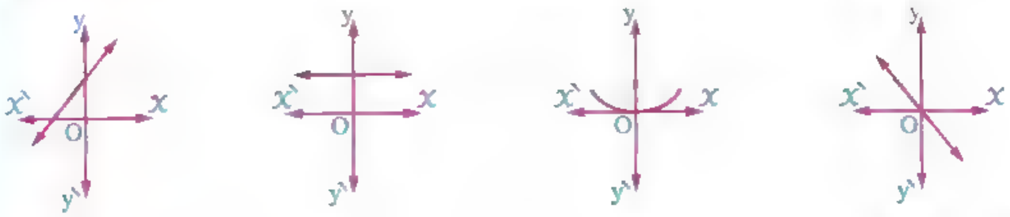
27 If $y = 2 + a$, a varies inversely as X and $a = 5$ when $X = 2$, **find :**

1. The relation between y and X
2. The value of y when $X = 5$ *(El-Sharkia 17)* « $y = 2 + \frac{10}{X}$, 4 »

28 If $X = \ell + 9$ and $\ell \propto y$, then find the relation between ℓ and y known that : $X = 24$ when $y = 5$, then find the value of y when $\ell = 12$ « $\ell = 3y$, 4 »

29 **Choose the correct answer from those given :**

1. The graphical form representing the direct variation between X and y is ………



(a) (b) (c) (d) *(El-Sharkia 16)*

2. The relation which represents a direct variation between the two variables X and y is ……… *(Souhag 20)*

(a) $X y = 5$ (b) $y = X + 3$ (c) $\frac{X}{3} = \frac{4}{y}$ (d) $\frac{X}{5} = \frac{y}{2}$

3. Which of the following relations represents an inverse variation between the two variables X and y ? *(El-Beheira 15)*

(a) $y = X + 5$ (b) $y = 4X$ (c) $\frac{X}{y} = \frac{5}{7}$ (d) $X y = 11$

4. If $y = m X$ where m is a constant $\neq 0$, which of the following is wrong ?

(a) $y \propto X$ (b) $X \propto y$ (c) $X = \frac{1}{m} y$ (d) $X \propto \frac{1}{y}$

5 If y varies inversely as x^2 , k is a constant , then ....

(a) $y = k x^2$ (b) $y = k - x^2$ (c) $y = \frac{k}{x^2}$ (d) $y = \frac{k}{x}$

6 If x and y are two variables , $\frac{x_1 y_1}{x_2 y_2} = 1$, then y $\propto$ ....

(a) x (b) $\frac{1}{x}$ (c) x^2 (d) $\frac{1}{x^2}$

7 If $y \propto x$ and $y = 5$ when $x = 3$, then the constant proportional equals ....

(a) 15 (b) 5 (c) 3 (d) $\frac{5}{3}$

8 If y varies inversely with x and $x = \sqrt{3}$ when $y = \frac{2}{\sqrt{3}}$, then the constant proportional equals .... *(Beni Suef 15 El-Beheira 16 – New Valley 20)*

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) 2 (d) 6

9 If $x y^5 =$ constant , then x varies inversely as .... *(Ismailia 08)*

(a) $\frac{1}{5}$ (b) y^5 (c) y (d) y^2

10 If $y \propto \frac{1}{\sqrt{x}}$, then x varies .... *(Matrouh 09)*

(a) directly as y^2 (b) inversely as y^2 (c) inversely as y (d) inversely as $\sqrt{y}$

11 If $y^2 + 4x^2 = 4xy$, then .... *(Alexandria 15 – South Sinai 19)*

(a) $y \propto x$ (b) $y \propto x^2$ (c) $y \propto \frac{1}{x}$ (d) $y \propto \frac{1}{x^2}$

12 If $x^2 y^2 + \frac{1}{4} = x y$, then .... *(El-Monofia 16)*

(a) $x \propto y$ (b) $y \propto x$ (c) $2x \propto 5y$ (d) $y \propto \frac{1}{x}$

13 If $y = 3x - 6$, then y $\propto$ .... *(El-Sharkia 14)*

(a) x (b) $3x$ (c) $x - 2$ (d) $3x - 6$

14 If $\frac{y+3}{y} = \frac{x+2}{x}$ where $x \neq y \neq$ zero , then y $\propto$ .... *(Ismailia 14)*

(a) x (b) $\frac{1}{x}$ (c) $x + 2$ (d) $x + 5$

15 If $y - x = \frac{2}{x} - \frac{2}{y}$ where $x \neq y \neq 0$, then ....

(a) $y \propto x + 1$ (b) $y \propto x$ (c) $y \propto \frac{1}{x}$ (d) $y \propto \frac{1}{x^2}$

16 If the total cost of a trip is (y) , some of it is constant (a) and the other is directly proportional with the number of participants (x) , then .... *(Ismailia 11)*

(a) $y = a x$ (b) $y = \frac{a}{x}$

(c) $y = a + \frac{m}{x}$ (m is a constant $\neq 0$) (d) $y = a + m x$ (m is a constant $\neq 0$)

Geometric Application

30 If (h) the height of a right circular cylinder (its volume is constant) varies inversely as the square of radius length (r) and h = 27 cm. when r = 10.5 cm.

, find h when r = 15.75 cm. *(Port Said 20)* « 12 cm. »

Life Applications

31 A car moves with a uniform velocity where the distance varies directly with the time (t). If the car covered a distance of 150 km. in 6 hours , find the distance covered by that car in 10 hours.

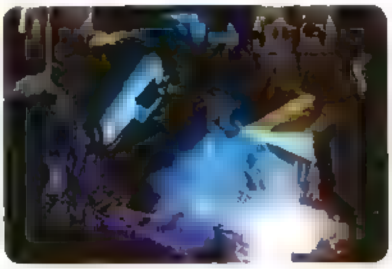
(El Kalyoubia 13) « 250 km. »

32 If the weight of a body on the moon (W) is directly proportional with its weight on the ground (R) If the body weighs 84 kg. on the ground and its weight on the moon is 14 kg. What will its weight be on the moon if its weight on the ground is 144 kg.?

« 24 kg. »

33 If the number of hours (n) needed for carrying out a work varies inversely as the number of workers (X) who carry out this work.

If the work is carried out by 6 workers within 4 hours , what is the needed time for carrying out the work by 8 workers ?



(El-Sharkia 11) « 3 hours »

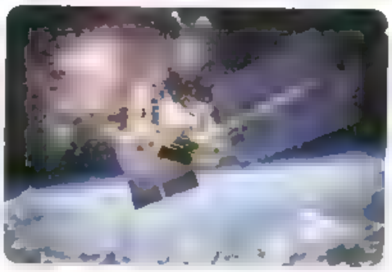
34 If the distance covered by a bicycle (d) varies directly with the square of the time (t) , $d = \frac{81}{16}$ km. when $t = \frac{1}{4}$ hour, find the value of t when d = 144 km.

(Assiut 12) « $1\frac{1}{3}$ hour »

35 If the value of speed v that water passes through a hose nuzzle inversely changes with the square of the hose nuzzle radius length r and v = 5 cm./s. when r = 3 cm., find v when r = 2.5 cm.

[illegible]

36 If the weight of a body varies inversely as the square of its distance from the centre of the earth. If a satellite of weight 500 w. kg. is projected up to the space , what will its weight be when it becomes at a distance of 640 km. far from the surface of the earth to the nearest one (kg.) (Consider the radius length of the earth 6390 km.)



« 413 w.kg. »

For excellent pupils

37 If $x \propto y$ and $z \propto l$, **then prove that :** $(x + y)(z + l) \propto (x - y)(z - l)$

38 If $(a + b) \propto \frac{a}{b}$, $(a^2 - ab + b^2) \propto \frac{b}{a}$, **then prove that :** $a^3 + b^3 = \text{constant}$

Summary of Unit 2

The ratio :

- The value of the ratio **does not change** if each of its terms is multiplied or divided by the same non-zero real number.
- The value of the ratio ($\neq 1$) **changes** if we add or subtract (to or from) each of its two terms the same non-zero real number.
- If the ratio between two numbers is a : b , then :

$\boxed{\text{The first number} = a\,m}$, $\boxed{\text{The second number} = b\,m}$, $m \neq 0$

The proportion :

- If $\frac{a}{b} = \frac{c}{d}$, then a , b , c and d are proportional quantities.
- If a , b , c and d are proportional quantities , then $\frac{a}{b} = \frac{c}{d}$
- If $\frac{a}{b} = \frac{c}{d}$, then $\boxed{a \times d = b \times c}$

i.e. The product of the extremes = the product of the means.

- If $a \times d = b \times c$, then $\boxed{\frac{a}{b} = \frac{c}{d}}$, $\boxed{\frac{a}{c} = \frac{b}{d}}$, $\boxed{\frac{d}{b} = \frac{c}{a}}$ and $\boxed{\frac{b}{a} = \frac{d}{c}}$
- If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$

i.e. $\frac{\text{The antecedent of the first ratio}}{\text{The antecedent of the second ratio}} = \frac{\text{The consequent of first ratio}}{\text{The consequent of second ratio}}$

- If $\frac{a}{b} = \frac{c}{d}$, then $a = c\,m$, $b = d\,m$ where m is a constant $\neq 0$
- If a , b , c and d are proportional quantities and we assume that : $\frac{a}{b} = \frac{c}{d} = m$, then $a = b\,m$, $c = d\,m$
- If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = ...$ and m_1 , m_2 , m_3 , ... are non-zero real numbers , then $\frac{m_1 a + m_2 c + m_3 e + ...}{m_1 b + m_2 d + m_3 f + ...}$ = one of the given ratios.

- The quantities a , b and c are said to be in continued proportion if $\frac{a}{b} = \frac{b}{c}$

  a is called the **first proportional** , **c** is called **the third proportional**

  and **b** is called the **middle proportional (proportional mean)**

  , $\boxed{b^2 = a\,c}$ *i.e.* $\boxed{b = \pm\sqrt{a\,c}}$

- If $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$, then $\boxed{c = d\,m}$, $\boxed{b = d\,m^2}$ and $\boxed{a = d\,m^3}$

- **The direct variation and inverse variation :**

Direct variation

- If y varies directly as x

  and is written as $y \propto x$, then :

1. $y = m\,x$ $\left(i.e.\ \frac{y}{x} = m\right)$

   where m is a constant $\neq 0$

2. $\frac{y_1}{y_2} = \frac{x_1}{x_2}$

3. The relation between x and y is represented graphically by a straight line passing through the origin point.

- To prove that $y \propto x$,

  we prove that : $y = m\,x$

  where m is a constant $\neq 0$

Inverse variation

- If y varies inversely as x

  and is written as $y \propto \frac{1}{x}$, then :

1. $y = \frac{m}{x}$ $\left(i.e.\ x\,y = m\right)$

   where m is a constant $\neq 0$

2. $\frac{y_1}{y_2} = \frac{x_2}{x_1}$

3. The relation between x and y is not a linear relation.

- To prove that $y \propto \frac{1}{x}$,

  we prove that : $x\,y = m$

  where m is a constant $\neq 0$

Exams on Unit Two

Model 1

Answer the following questions :

1 **Choose the correct answer from those given :**

1. If $4x^2 = 9y^2$, then $\frac{x}{y} = \cdots\cdots$

   (a) $\frac{9}{4}$ (b) $\frac{3}{2}$ (c) $\pm\frac{3}{2}$ (d) $\pm\frac{2}{3}$

2. If $x^2 y = 5$, then $\cdots\cdots$

   (a) $y \propto x$ (b) $y \propto x^2$ (c) $y \propto \frac{1}{x}$ (d) $y \propto \frac{1}{x^2}$

3. If $\frac{a}{2} = \frac{b}{3}$, then $\frac{b-a}{b+a} = \cdots\cdots$

   (a) $\frac{1}{5}$ (b) $\frac{1}{3}$ (c) $\frac{2}{5}$ (d) $\frac{3}{5}$

4. If $\frac{x}{2} = \frac{y}{3} = \frac{4x-2y}{z}$, then $z = \cdots\cdots$

   (a) -2 (b) $-\frac{1}{2}$ (c) $\frac{1}{2}$ (d) 2

5. The second proportional for the quantities $12\,a\,b^2$, · , $21\,a\,b$, $14\,b^2$ is $\cdots\cdots$

   (a) $8\,a\,b^2$ (b) $8\,b^3$ (c) $24\,a\,b$ (d) $24\,b^4$

6. If $y \propto x$ and $x = 1$ when $y = 4$, then the constant proportional equals $\cdots\cdots$

   (a) 1 (b) -4 (c) 4 (d) $-\frac{1}{4}$

2 [a] If $\frac{a}{4} = \frac{b}{3}$, find the value of : $\frac{a\,b + a^2}{a\,b - b^2}$

[b] If b is the middle proportional between a and c , **prove that :** $\sqrt{\frac{a^2+b^2}{b^2+c^2}} = \frac{a}{b}$

where a , b and c are positive quantities.

3 [a] If y varies inversely with x^2 and $x = 3$ when $y = 4$, **find :**

1. The relation between y and x
2. The value of x when $y = 9$

[b] If a , b , c and d are proportional quantities , **then prove that :** $\frac{a+2c}{b+2d} = \frac{c-a}{d-b}$

4 [a] If $\frac{a}{x+y} = \frac{b}{y-2x} = \frac{c}{x+4y}$, **find the value of :** $\frac{b+2c}{2a+b}$

[b] If $y = 3 + a$ and $a \propto \frac{1}{x}$ and $y = 5$ when $x = 1$, find the relation between x and y

, then find the value of y when $x = 2$

---

5 [a] If a , b , c and d are in continued proportion ,

prove that : $\frac{2a^3 + 3b^3}{2a + 3d} = a^2$

[b] Find the positive number if its square is added to the antecedent of the ratio 29 : 46 and its square is subtracted from its consequent it becomes 3 : 2

Model 2

Answer the following questions :

1 **Choose the correct answer from those given :**

1 If a , b , 2 and 3 are proportional quantities , then $\frac{b}{a} =$ ....

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) 3 (d) 2

2 If 2 , 6 and $x + 15$ are proportional , then $x =$ ....

(a) 1 (b) 2 (c) 3 (d) 4

3 If $xy = 12$, then y varies directly with ....

(a) $\frac{1}{x}$ (b) $x - 12$ (c) x (d) $x + 12$

4 If y varies inversely with x and $x = \sqrt{3}$ when $y = \frac{2}{\sqrt{3}}$, then the constant of proportion equals ....

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) 2 (d) 6

5 If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = m$ (where $m \in \mathbb{R}^*$) , then $\frac{ace}{bdf} =$ ....

(a) m (b) 3 m (c) m^3 (d) $3m^3$

6 If $\frac{a}{3} = \frac{b}{5}$, then $5a - 3b + 7 =$ ....

(a) 3 (b) 9 (c) 7 (d) 5 a b

2 [a] If $\frac{a}{b-a} = \frac{c}{d-c}$, **prove that :** a , b , c and d are proportional.

[b] If y = a − 1 , a varies inversely as x^2 and a = 4 when $x = 2$, **find :**

1. The relation between y and x
2. The value of x when y = 8

3 [a] If $\frac{x+y}{5} = \frac{y+z}{3} = \frac{z+x}{6}$

, **prove that :** $\frac{x-z}{2} = \frac{x+y+z}{7}$

[b] If a , b , c and d are proportional quantities

, **prove that :** $\frac{2a+3b}{2c+3d} = \frac{a}{c}$

4 [a] If 5 a = 3 b , **then find the value of :** $\frac{7a+9b}{4a+2b}$

[b] If b is the middle proportional between a and c

, **prove that :** $\frac{a-b}{a-c} = \frac{b}{b+c}$

5 [a] If $4x^2 + y^2 = 4xy$

1. **Prove that :** $y \propto x$
2. **Find the value of :** x when y = 8

[b] If a : b : c = 4 : 5 : 3 , **prove that :** $\frac{a-b+c}{a+b-c} = \frac{1}{3}$

Wonders of numbers

From wonders of the **number 7** is that if one of its multiples up to 63 is multiplied by **15873** , every time the product is a number consisting of the same digits.

- $7 \times 15873 = 111\,111$
- $14 \times 15873 = 222\,222$
- $21 \times 15873 = 333\,333$

Try other multiples

A Research Project

On Unit Two

Project aims:

- Using ratio and proportion to solve problems.
- Differentiation between direct variation and inverse variation.
- Using mathematics to solve life problems.
- Associating mathematics with social studies.
- Associating mathematics with science.

Do a research project on the following topics

"Drawing scale is an application of ratio and proportion. It is considered one of the most important elements of a map".

Discuss the following points using available resources :

- Define drawing scale and mention its importance.
- Stick a map of Egypt with a given drawing scale.
- Determine two governorates on this map and measure the distance between them. Then use the drawing scale on this map to calculate the real distance between these two governorates.
- Search for the law of uniform velocity and then calculate the time that a car , moving at uniform velocity of 120 km./hr. , takes between these two governorates.
- What is the type of variation between the time and the distance when the velocity is constant ?

UNIT 3 Statistics

Exercises of the unit :

9. Collecting data.

10. Dispersion.

- Unit Exam.

A research project on unit three

Scan the QR code to solve an interactive test on each lesson

9 Collecting data

From the school book

● Remember ● Understand ● Apply ● Problem Solving

1 Choose the correct answer from those given :

1 .... is a secondary resource of collecting data. *(El-Fayoum 12)*

(a) Personal interview (b) Questionnaires

(c) Data base of the employees (d) Observing and measuring

2 .... is a primary resource of collecting data.

(a) Central agency for statistics (b) Data of the school pupils in the previous year

(c) Questionnaires (d) Data of the employees in one of the companies

3 The method of mass population is suitable for ....

(a) searching the formation of the sand of the Western Desert.

(b) examining the sweetness of water for one of the wells.

(c) finding out the ratio of existing a metal in one of the mines.

(d) getting the number of the students who had the full mark in maths exam in a class.

4 The method of samples is suitable for all the following except .....

(a) examining a patient's blood.

(b) knowing population.

(c) checking the production of a factory.

(d) finding out the ratio of existing gas somewhere.

5. Selecting a sample of layers of a statistical society is called .... sample. *(El Beheira 17)*

(a) random (b) class (layer) (c) deliberate (d) bunch

6. A factory has 125 workers , 75 of them are technicians and 50 are engineers , it is wanted to take a sample of layers of size 50 individuals such that it represents each layer according to its size , then the number of engineers of the sample equals ...... *(El-Monofia 16)*

(a) 30 (b) 20 (c) 25 (d) 15

2 Which of the following statistical data is primary and which of them is secondary ?

1. A survey for the pupils in your class about the place to which will be the next trip.
2. If you count the number of seats existing in each class of your school.
3. If you make an investigation about the number of the successful pupils in each school subject in your school in the first session last year from the registered notebooks in the school.
4. If you go to a government authority in your governorate to collect data about the babies registered in each health office through March last year.
5. Searching the internet sites for the results of one team of sports teams in the league in Egypt in the year 2008 – 2009

3 Compare between the methods of mass population and samples , showing the advantages and disadvantages of each of them.

4 Mention the suitable method (mass population or samples) for collecting data in each of the following statistical societies :

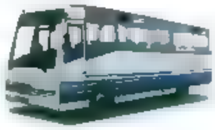
1. The educational level of a class formed from 25 students.
2. The range of validity of drinking water in a well for drinking.
3. The ratio of oil existing in an exploratory location.
4. The range of spread of a disease in one of the crops.
5. The counting of factories in one of the industrial cities.

5 200 employees were surveyed about their favourite food during break time. Every one was given a digit number from 1 to 200 , then a sample representing 10% was selected to be interviewed about their favourite food :

(a) Hot drinks. (b) Light meals. (c) Soft drinks.

Determine using your calculator the digits of target employees in this sample.

6 A school makes a study about how the pupils come to school , if the number of pupils in the school is 320 , each pupil is given a number from 1 to 320
A sample of 10% from this number is selected to ask them how they come to school.

- on foot 
- public transport 
- taxi 
- bike 
- private car 

Determine the digits of the sample using the calculator.

7 The administration of a hotel wanted to conduct a survey to 300 customers on the service level produced. Every customer got a digit from 201 to 500 , 10% of them were selected as a random sample to ask them about the service level.
Determine using the calculator the digits of the marked customers in this sample.

8 At a faculty , there are 4000 university students in the first grade , 3000 in the second grade , 2000 in the third grade and 1000 in the fourth grade. If we want to draw a layer sample of 500 students , where each layer is represented in this sample according to its size, calculate the number of students in each layer in the sample. « 200 , 150 , 100 , 50 »

9 One of the factories of cars produces 3 models of cars in the year , their numbers are :
300 cars from the first model.
500 cars from the second model.
200 cars from the third model.
The directorate of the factory wanted to select a sample of 5% of production to represent each model according to its size.

- Determine the number of the selected sample.
- Determine the number of each model in the sample. « 50 , 15 , 25 , 10 »

10 It is wanted to select a random layer sample to represent each layer due to its size from a society consisting of 5000 individuals and it is divided into two layers.
The number of the first layer is 1500 individuals.
If the number of the second layer in the sample is 140 individuals
, calculate the number of individuals in the sample. « 200 »

11 **There is a need to draw a random layer sample to represent all the layers according to their sizes from a society of a total 40000 values divided into three layers as follows :**

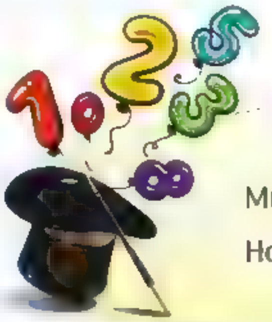
Number of the layer	1	2	3
Number of values in the layer	12000	20000	8000

If the number of values in the first layer is 240 , calculate the size of the whole sample. « 800 »

12 **A society has 2000 individuals divided into 4 layers. It is wanted to select a random sample such that each layer is represented due to its size.**
The investigator designed the following table :

Number of the layer	1	2	3	4	Total
Number of individuals of the layer	500	700	....	450	2000
Number of the layer in the sample	....	....	7	....	....

Complete the table. « 350 , 10 , 14 , 9 , 40 »



Wonders of numbers

Multiply your age in years × 39 × 259
How do you feel about the product ?

10 Dispersion

From the school book

interactive test

● Remember ● Understand ○ Apply ● Problem Solving

1 **Choose the correct answer from those given :**

1 .... is one of the measures of the dispersions. *(New Vally 20)*

(a) The median (b) The arithmetic mean

(c) The standard deviation (d) The mode

2 The simplest and easiest method of measuring dispersion is .... *(Ismailia 20)*

(a) the range. (b) the standard deviation.

(c) the arithmetic mean. (d) the mode.

3 The difference between the greatest value and the smallest value in a set of individuals is called .... *(El-Sharkia 18 – Souhag 18 – Port Said 19)*

(a) the range. (b) the arithmetic mean.

(c) the median. (d) the standard deviation.

4 The positive square root of the average of squares of deviations of the values from their mean is called .... *(Port Said 18 – Kafr El Sheikh 18 – El-Fayoum 19 – El-Kalyoubia 20)*

(a) the range. (b) the arithmetic mean.

(c) the standard deviation. (d) the mode.

5 The mean of the values : 7 , 3 , 6 , 9 and 5 equals .... *(Alex. 17 – North Sinai 17 – El-Fayoum 18)*

(a) 3 (b) 6 (c) 4 (d) 12

6 The range of the set of values : 23 , 22 , 15 , 18 and 17 is .... (Cairo 15)

(a) 8 (b) 18 (c) 19 (d) 23

7 If 67 is the greatest value of a set and if the range equals 27 , then the smallest value of this set equals .... (El-Menia 16)

(a) 67 (b) 40 (c) 27 (d) 94

8 The most repeated value in a set of values represents .... (Damietta 13 , Luxor 16)

(a) the median. (b) the range.

(c) the mode. (d) the mean.

9 If the mean of the numbers : $3k-3$, $3k-1$, $2k+1$, $2k+3$ and $2k+5$ is 13 , then $k =$ .... (Alexandria 11)

(a) -5 (b) 10 (c) 5 (d) $\frac{1}{5}$

10 If the range of the values : $6+k$, $6-k$, $6+5k$ and $6-2k$ is 14 where $k \in \mathbb{N}$, then $k =$ .... (El-Sharkia 20)

(a) 1 (b) 2 (c) 3 (d) 4

11 If the range of the values 2 , 7 , a , 6 is 8 where $a > 0$, then $a =$ .... (El-Sharkia 14)

(a) 4 (b) 9 (c) -1 (d) 10

12 Which of the following values of a makes the range of the numbers : 53 , a , 58 , 57 , 60 and 55 equal to 9 ? (El-Dakahlia 16)

(a) 63 (b) 61 (c) 51 (d) 50

13 $\frac{\text{Sum of values}}{\text{Number of these values}} =$ .... (Suez 19 – Alex. 20)

(a) range (b) standard deviation (c) mean (d) mode

14 If $2x + 2y = 10$, $x \in \mathbb{R}^+$, $y \in \mathbb{R}^+$, then the arithmetic mean of the values x and y is .... (Suez 16)

(a) $\frac{2}{5}$ (b) $\frac{5}{2}$ (c) 5 (d) 2

15 The set which has more dispersion of the following sets is .... (El Kalyoubia 15)

(a) 28 , 17 , 30 , 36 , 20 (b) 20 , 19 , 29 , 37 , 43

(c) 31 , 35 , 26 , 37 , 41 (d) 25 , 39 , 19 , 5 , 27

16 The commonest measure of dispersion and the most accurate is the .... (Damietta 14 , El-Menia 18)

(a) range. (b) mean. (c) standard deviation. (d) median.

17 If all individuals are equal in values , then .... (South Sinai 17 , Luxor 20)

(a) $x - \overline{x} > 0$ (b) $x - \overline{x} < 0$ (c) $\sigma = 0$ (d) $\overline{x} = 0$

18 The standard deviation of the values 5 , 5 , 5 , 5 equals ..... *(El-Dakahlia 20)*

(a) 0 (b) 5 (c) 6 (d) 2

19 If $\Sigma (X - \overline{X})^2 = 48$ of a set of values and the number of these values is 12 , then $\sigma =$ .... *(Cairo 17 – El-Monofia 19)*

(a) – 4 (b) – 2 (c) 2 (d) 4

2 **Calculate the standard deviation for the next data :**

1 16 , 32 , 5 , 20 , 27 *(El Monofia 17 , El Gharbia 18 , El Monofia 19 , Port Said 20)*

2 72 , 53 , 61 , 70 , 59 *(Luxor 19 , Damietta 20)* « 7.1 »

3 15 , – 12 , – 9 , 27 , – 6 « 15.3 »

4 22 , 20 , 20 , 20 , 18 « 1.3 »

3 **Which of the following sets has more dispersion , using the standard deviation ?**

Set (A) : 7 , 8 , 9 , 10 , 11 *Set (B) :* 21 , 20 , 11 , 19 *Set (C) :* 29 , 30 , 30 , 35

4 **Calculate the mean and standard deviation of each of the following data :**

1 73 , 54 , 62 , 71 , 60 *(Assiut 17 – Qena 20)* « 64 , 7.07 »

2 13 , 14 , 17 , 19 , 22 (to the nearest 3 decimals digits) *(El Sharkia 17,*

3 65 , 61 , 70 , 64 , 70 , 76 , 70 « 68 , 4.6 »

4 23 , 12 , 17 , 13 , 15 , 16 , 8 , 9 , 37 , 10 « 16 , 8.2 »

5 The following values represent marks of five pupils in a test : 8 , 9 , 6 , 12 , 10 **Calculate :**

1 The mean of the marks. *(El-Dakahlia 17)* « 9 »

2 The standard deviation of the marks. « 2 »

6 **The opposite table shows the temperature in some cities :**

1 Calculate the mean and standard deviation of the maximum temperature.

2 Calculate the mean and standard deviation of the minimum temperature.

City	Max.	Min.
Ismailia	25	11
Suez	26	12
El-Arish	24	10
Nakhl	24	6
Taba	22	7
El-Tur	26	16
Hurghada	27	15
Rafah	26	11

« 25 , 1.5 , 11 , 3.2 »

7 📖 **The following frequency distribution shows the number of children of some families in a new city :** *(El-Beheira 16 – Alex. 19 – El-Monofia 20)*

Number of children	zero	1	2	3	4
Number of families	8	16	50	20	6

Calculate the mean and the standard deviation of the number of children. « 2 , 1 »

8 📖 **The following are the frequency distribution for a number of defective units found in 100 boxes of manufactured units :** *(El-Beheira 14 – El-Beheira 17 – Souhag 18)*

Number of defective units	zero	1	2	3	4	5
Number of boxes	3	16	17	25	20	19

Find the standard deviation of the defective units. « 1.4 »

9 **The following frequency distribution shows the number of goals which have been scored by 30 players from 5 penalty kicks for each player during a training :**

Number of scored goals	0	1	2	3	4	5
Number of players	2	4	5	8	7	4

Find the mean and standard deviation of the number of scored goals. « 2.9 , 1.4 »

10 📖 **The following frequency distribution shows the ages of 10 children :**

Age in years	5	8	9	10	12	Total
Number of children	1	2	3	3	1	10

Calculate the standard deviation of the ages in years.

(Alex. 17 – Giza 18 – Qena 19 – Cairo 20) « 1.7 years »

11 **The following table shows the frequency distribution of the number of students who won in an art competition from a school having 20 classes :**

Number of students	0	1	2	3	4	5	Total
Number of classes	1	3	5	6	3	2	20

Find the mean and the standard deviation of the number of students. « 2.65 , 1.3 »

12 **The following table represents the frequency distribution of sets of temperature degrees in some of world cities :**

Sets of temperature degrees	5 –	15 –	25 –	35 –	45
Frequency	7	9	11	15	8

Find the mean and the standard deviation of the temperature degrees. « [illegible] »

---

13 **Calculate the mean and the standard deviation for the following frequency distribution :** *(Qena 16 – El-Gharbia 17)*

Sets	0 –	4 –	8 –	12 –	16 20	Total
Frequency	3	4	7	2	9	25

« 11.6 , 5.7 »

---

14 **The following table represents the daily wages of a set of workers in a factory :** *(Kafr El-Sheikh 20)*

Sets of wages	20 –	30 –	40 –	50 –	60 –	70 –
Number of workers	10	12	8	6	3	1

Find the mean and standard deviation of the wages. « 40.75 , 13.4 »

---

15 **The following distribution table shows the amount of gasoline that a set of cars consumes :**

Number of kilometres per litre	5 –	7 –	9 –	11 –	13 –	15 – 17	Total
Number of cars	3	6	10	12	5	4	40

Find the standard deviation of the number of kilometres per litre. « 2.7 »

---

16 **The following frequency distribution shows the value of electricity bill for 200 participants :**

The value of the bill in pounds	5 –	15 –	25 –	35 –	45 –	55	Total
Number of participants (frequency)	19	50	85	25	15	6	200

Find the mean and the standard deviation of the values of these bills. « [illegible] , 11.5 »

For excellent pupils

17 **The two frequency tables represent the marks of the students of two classes A and B of third preparatory in an exam :**

Class A

Sets of marks	0 –	10 –	20 –	30 –	40 – 50	Total
Number of students	2	5	11	15	7	40

Class B

Sets of marks	0 –	10 –	20 –	30 –	40 – 50	Total
Number of students	2	3	18	7	10	40

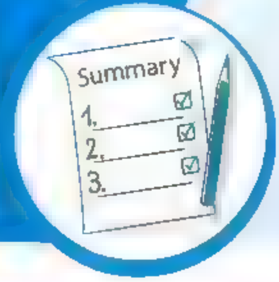
1. Represent both of distributions using the frequency polygon in one figure.
2. Find the mean and standard deviation for both frequency distributions.
3. Which class is more homogeneous in getting marks ? « 30 ، 10.7 ، 30 ، 11 »

Wonders of numbers

111111111 × 111111111

= 12345678987654321

Summary of Unit 3



Resources of collecting data :

- **Primary resources (field resources)**

  These are the resources from which the researcher gets data directly.

- **Secondary resources (historical resources) :**

  These are the resources from which the researcher gets data that previously collected and registered by some authorities , formal organisations or persons.

Methods of collecting data :

- **Method of mass population :**

  It is based on collecting the data related to the phenomenon under study from all individuals of the statistical society and it is used to inventory all individuals of society.

- **Method of samples :**

  It is based on collecting data related to the phenomenon under study from a representative sample of the society , and applying the research on it , then generalizing the results on the whole society.

The samples :

- The sample is a small part from a large society that looks like the society and represents it well.

- **Not a random sample (deliberate sample) :**

  It is a sample in which we select its individuals of the statistical society by a certain way to satisfy the objectives of the research.

- **Simple random sample :**

  It is a sample used for the homogeneous societies which are not naturally divided into groups or classes.

- **Layer random sample :**

  It is a sample used in the statistical societies which are heterogeneous or made up of qualitative sets that are different in characteristics.

- The number of individuals of the layer in the sample

$$= \frac{\text{the total number of individuals in the layer}}{\text{the total number of individuals in the society}} \times \text{the number of individuals of the sample}$$

«approximated the result to the nearest unit»

The dispersion :

- The dispersion is a measure that expresses how much the sets are homogeneous.
- The range of a set of values is the difference between the greatest value and the smallest value in the set.
- Standard deviation is the most important , common and accurate measure of dispersion. We can calculate it by calculating the positive square root of the average of squares of deviations of the values from their mean.
- **The standard deviation (σ) of a set of values** $= \sqrt{\frac{\sum (x - \overline{x})^2}{n}}$

  Where :

  x denotes a value of the values ,

  $\overline{x}$ denotes the mean of the values ,

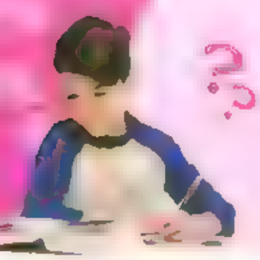
  n denotes the number of values
- **The standard deviation (σ) of a frequency distribution** $= \sqrt{\frac{\sum (x - \overline{x})^2 k}{\sum k}}$

  Where :

  x represents the value or the centre of the set , k represents the frequence of the value or the set ,

  $\sum k$ is the sum of frequences and $\overline{x}$ (the mean) $= \frac{\sum (x \times k)}{\sum k}$

Exam on Unit Three



Answer the following questions :

1 **Choose the correct answer from those given :**

1. The difference between the greatest value and the smallest value of a set of individuals is called ....
   (a) the range. (b) the arithmetic mean.
   (c) the median. (d) the standard deviation.

2. If $\sum (x - \overline{x})^2 = 36$ of a set of values and the number of these values $= 9$, then $\sigma =$ ....
   (a) 2 (b) 4 (c) 18 (d) 27

3. Selecting a sample of layers of a statistical society is called .... sample.
   (a) random (b) class (layer) (c) deliberate (d) bunch

4. The range of the set of values : 5 , 14 , 4 , 37 , 15 , 16 and 7 is ....
   (a) 30 (b) 33 (c) 32 (d) 22

5. The positive square root of the average of squares of deviations of the values from its arithmetic mean is called ....
   (a) the range. (b) the arithmetic mean.
   (c) the median. (d) the standard deviation.

6. The standard deviation of the values : 5 , 5 , 5 , 5 equals ....
   (a) zero (b) 5 (c) 6 (d) 2

2 **Calculate the mean and standard deviation of the following data :** 16 , 32 , 5 , 20 , 27

3 **If the number of persons in 50 families is as follows :**

Number of persons	2	3	4	5	6	7	8
Number of families	5	7	8	12	9	5	4

Calculate the mean and the standard deviation of the number of persons.

4 **The following frequency distribution shows the marks of 20 students in an exam :**

Sets	zero –	2 –	4 –	6 –	8 –
Frequency	1	3	6	5	5

Calculate the standard deviation.

A Research Project

On Unit Three

Project aims :

- Collecting and organizing data in frequency tables with sets.
- Calculating the range of a set of individuals.
- Calculating the mean and the standard deviation of a frequency distribution with sets.
- Appreciating the role of statistics in daily life.

Do a research project on the following topic :

"Standard deviation is considered the most accurate, important and common measure of dispersion measures".

Discuss the following points using available resources :

- Choose two of dispersion measures and point out their advantages and disadvantages.
- Record your classmates' marks at one of the maths exams and at a social studies' exam , then do the following :
  1. Find the range of your class marks in each of the two subjects.
  2. Form the frequency table with sets for the marks of math and according to this table's data , calculate the mean and the standard deviation of your class marks in maths.
  3. Form the frequency table with sets for the marks of social studies and according to this table's data , calculate the mean and the standard deviation of your class marks in social studies.
  4. Indicate the subject in which the level of your class is more homogeneous.

Accumulative basic skills

TIMSS Problems

Choose the correct answer from the given ones :

1 $\{3\} \subset$ .... *(Alex. 16)*

(a) $(3 , 7)$ (b) $]3 , 7]$ (c) $]3 , 7[$ (d) $\{3 , 7\}$

2 $[2 , 7] - \{2 , 7\} =$ .... *(Matrouh 17)*

(a) $[1 , 6]$ (b) $\varnothing$ (c) $]2 , 7[$ (d) $\{0\}$

3 The next number in the pattern : $\sqrt{3} , \sqrt{12} , \sqrt{27} , \sqrt{48}$ is ..... *(New Valley 20)*

(a) $\sqrt{50}$ (b) $\sqrt{75}$ (c) $\sqrt{60}$ (d) $\sqrt{90}$

4 $2^{2017} = 2^{2016} +$ .... *(Luxor 17)*

(a) 1 (b) 2 (c) 2016 (d) 2^{2016}

5 If $[-1 , x] \cap [y , 5] = [2 , 3]$, then $x^y =$ ....

(a) 8 (b) $\frac{1}{5}$ (c) 9 (d) -1

6 When the side length of a square increases by the ratio 10% , then its area increases by the ratio .... %

(a) 10 (b) 15 (c) 20 (d) 21

7 The ratio between the area of a square shaped region of side length x cm. to the area of another square shaped region of side length $2x$ cm. is .... *(Beni Suef 17)*

(a) 1 : 2 (b) x : 4 (c) 1 : 4 (d) 4 : 1

8 If F is an odd number , then the next odd number directly is .... *(Giza 17 - South Sinai 19)*

(a) F^2 (b) $F^2 + F$ (c) $F + 1$ (d) $F + 2$

9 If M represents a negative number , which of the following represents a positive number ? *(Kafr El-Sheikh 17)*

(a) M^3 (b) M^2 (c) 2 M (d) $\frac{M}{2}$

10 Half of the number 2^{20} is .... *(Damietta 17)*

(a) 2^{10} (b) 1^{20} (c) 2^{19} (d) 1^{10}

11 If $(X-3)^{zero} = 1$, then $X \in$ .... *(El-Monofia 18)*

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{3\}$ (c) $\mathbb{R} - \{4\}$ (d) $\mathbb{R} - \{1\}$

12 $\left(\frac{\sqrt{5}+1}{2}\right)^{1000} \left(\frac{\sqrt{5}-1}{2}\right)^{1000} =$ .... *(El-Monofia 18)*

(a) zero (b) 1 (c) $\frac{5^{1000}-1}{4}$ (d) 4^{1000}

13 $3^X + 3^X + 3^X =$ .... *(Suez 16)*

(a) 9^X (b) 3^{3X} (c) 3^{X+1} (d) 3^{X+3}

14 $2^5 + 2^5 + 2^5 + 2^5 =$ .... *(Luxor 16)*

(a) 2^7 (b) 2^6 (c) 2^4 (d) 2^{20}

15 If $X - y = 5$, $X + y = \frac{1}{5}$, then $X^2 - y^2 =$ .... *(Kafr El-Sheikh 17 – Aswan 20)*

(a) $\frac{1}{25}$ (b) 1 (c) 25 (d) 5

16 If $X + y = y X = 5$, then $X^2 y + y^2 X =$ .... *(Aswan 16 – Ismailia 20)*

(a) 10 (b) 15 (c) 20 (d) 25

17 If $(X-y)^2 = 20$, $X^2 + y^2 = 10$, then $X y =$ .... *(Alex. 16)*

(a) 10 (b) 5 (c) – 5 (d) 20

18 If $1 < X < 3$, $X \in \mathbb{R}$, then $(3X - 1) \in$ .... *(Suez 16 – Giza 20)*

(a) $[2 , 8[$ (b) $[2 , 8]$ (c) $]2 , 8[$ (d) $\{2 , 8\}$

19 The S.S. of the inequality : $5 - 3X > 11$ in $\mathbb{R}$ is .... *(Kafr El-Sheikh 17)*

(a) $]-\infty , -2[$ (b) $]-2 , \infty[$ (c) $]-\infty , -2]$ (d) $[-2 , 2]$

20 The sum of the two square roots of $2\frac{1}{4}$ is .... *(El-Monofia 17 North Sinai 19)*

(a) zero (b) $\frac{3}{2}$ (c) 3 (d) $\frac{9}{4}$

21 Four times the number $2^8 =$ .... *(Alex. 17 Souhag 19)*

(a) 2^{32} (b) 8^8 (c) 2^{10} (d) 4^8

22 If $X = \sqrt{3} + \sqrt{2}$, $y = \frac{1}{\sqrt{3}+\sqrt{2}}$, then $(X+y)^2 =$ .... *(El-Gharbia 17)*

(a) 8 (b) zero (c) 9 (d) 12

23 If $2^{x} = \frac{1}{8}$, then $x =$ .... *(El-Monofia 17)*

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) 3 (d) -3

24 A book consists of 56 pages. How many pages the number 5 appears in the pages serial of this book ?

(a) 6 (b) 7 (c) 12 (d) 13

25 If we put on one side of a road of length 12 km. some light poles from the begining to the end of the road, where the distance between each two consecutive poles is $\frac{1}{2}$ km. , then the number of poles is ....

(a) 12 (b) 24 (c) 25 (d) 23

26 The decimal that lies between 0.07 and 0.08 is .... *(Alex. 17)*

(a) 0.00075 (b) 0.0075 (c) 0.075 (d) -0.75

27 The square of double the number $\frac{1}{2}$ is ....

(a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) 1 (d) 2

28 If three times a number $= 45$, then $\frac{1}{5}$ of this number $=$ .... *(El-Menia 19)*

(a) 15 (b) 5 (c) 3 (d) 9

29 If $\frac{5}{4} + \frac{5}{x} = \frac{5}{2}$, then $x =$ .... *(El-Monofia 20)*

(a) 2 (b) 4 (c) 5 (d) $\frac{5}{2}$

30 $]-1 , 3] \cap \{-3 , -1\} =$ .... *(Assiut 18)*

(a) $\varnothing$ (b) $\{-3\}$ (c) $\{-1\}$ (d) $\{3\}$

31 $[2 , 7] -]2 , 7[=$ .... *(Beni Suef 18)*

(a) $\varnothing$ (b) $\{2\}$ (c) $\{7\}$ (d) $\{2 , 7\}$

32 $\mathbb{Z}^{-} \cup \mathbb{N} =$ .... *(Luxor 17 – Alex. 18)*

(a) $\varnothing$ (b) $\mathbb{N}$ (c) $\mathbb{Z}$ (d) $\mathbb{R}$

33 The expression : $(x-2)^2 - x^2$ is of the .... degree. *(Kafr El-Sheikh 20)*

(a) first. (b) second. (c) third. (d) fourth.

34 The solution set of the equation : $x - 1 = |-1|$ in $\mathbb{N}$ is .... *(Suez 18)*

(a) $(1 , 2)$ (b) 2 (c) $\{2\}$ (d) $\{-2\}$

35 If $17x + 8 = 11$, then $17x + 11 =$ .... *(Ismailia 19)*

(a) 8 (b) 11 (c) 14 (d) 17

Second

Trigonometry and Geometry

UNIT

4 Trigonometry

Exercises of the unit :

1. The main trigonometrical ratios of the acute angle.
2. The main trigonometrical ratios of some angles.

- Unit Exams.

A research project on unit four

Scan the QR code to solve an interactive test on each lesson

1 The main trigonometrical ratios of the acute angle

interactive test

From the school book

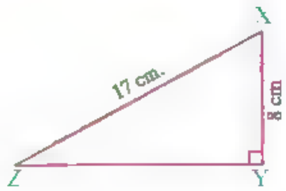
● Remember ● Understand ○ Apply Problem Solving

1 **Complete the following :**

In the opposite figure :

XYZ is a right-angled triangle at Y

in which XY = 8 cm. , XZ = 17 cm.



1 sin X = ... , sin Z = ...

2 cos X = ... , cos Z = ...

3 tan X = ... , tan Z = ...

2 **Choose the correct answer from those given :**

1 For any acute angle A , tan A = ... *(Ismailia 12)*

(a) $\frac{\cos A}{\sin A}$ (b) sin A cos A (c) $\frac{\sin A}{\cos A}$ (d) sin A + cos A

2 If X , y are the measures of two complementary angles and sin $X = \frac{3}{5}$

, then cos y = ... *(Giza 17 – El-Beheira 18 – Giza 20)*

(a) $\frac{4}{5}$ (b) $\frac{3}{5}$ (c) $\frac{3}{4}$ (d) $\frac{5}{3}$

3 For any two acute angles A and B , if sin A = cos B , then m (∠ A) + m (∠ B) = ...

(a) 30° (b) 60° (c) 90° (d) 180°

4 If sin 70° = cos X where X is the measure of an acute angle , then X = ...

(El-Kalyoubia 18)

(a) 60° (b) 45° (c) 10° (d) 20°

5 In ΔABC, if $m(\angle A) = 85°$ and $\sin B = \cos B$, then $m(\angle C) = \dots\dots$

(El-Beheira 17 – El-Dakahlia 19)

(a) 30° (b) 45° (c) 50° (d) 60°

6 In ΔABC, if $m(\angle B) = 90°$, then $\sin A + \cos C = \dots\dots\dots$ *(El-Monofia 17)*

(a) 2 sin A (b) 2 sin C (c) 2 sin B (d) 2 cos A

ΔABC is a right-angled triangle at A, then cosine angle B : sine angle C equals $\dots\dots$

(El Sharkia 18)

(a) $\frac{3}{5}$ (b) $\frac{4}{3}$ (c) $\frac{3}{4}$ (d) 1

DEF is a right-angled triangle at E, which of the following relations is false ?

(El-Dakahlia 16)

(a) $\tan D \times \tan F = 1$ (b) $\sin D = \cos F$ (c) $\cos D = \sin F$ (d) $\cos D = \sin E$

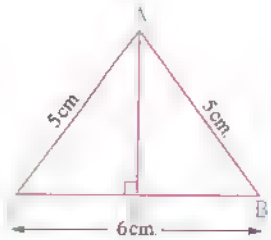
ABC is a right-angled triangle at B, where $3\,AC = 5\,BC$, then $\tan A = \dots\dots$

(El Sharkia 20)

(a) $\frac{3}{5}$ (b) $\frac{5}{3}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

10 **In the opposite figure :**

$\cos B = \dots\dots\dots$ *(El-Gharbia 12)*



(a) $\frac{4}{5}$ (b) $\frac{3}{5}$

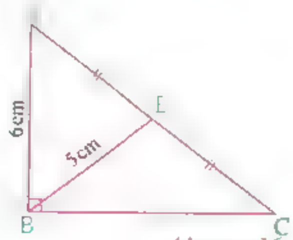
(c) $\frac{5}{6}$ (d) $\frac{5}{4}$

11 **In the opposite figure :**

ΔABC is a right-angled triangle at B

, $\overline{BE}$ is a median, $BE = 5$ cm.

, $AB = 6$ cm., then $\sin C = \dots\dots\dots$



(Aswan 16)

(a) $\frac{5}{6}$ (b) $\frac{3}{5}$

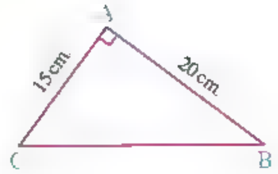
(c) $\frac{6}{5}$ (d) $\frac{5}{3}$

3 If the ratio between the measures of two supplementary angles is 3 : 5, find the degree measure of each one. *(El Beheira 14 – Aswan 15 – El Gharbia 19)* « 67° 30̀ , 112° 30̀ »

4 If the ratio between the measures of two complementary angles is 3 : 4, find the degree measure of the greater angle in measure. « 51° 25̀ 43̀̀ »

5 If the ratio between the measures of the interior angles of a triangle is 3 : 4 : 7, find the degree measure of each angle.

6 **In the opposite figure :**



ABC is a triangle in which : m ($\angle$ A) = 90°

, AC = 15 cm. and AB = 20 cm.

Prove that : cos C cos B − sin C sin B = zero

(El-Beheira 17 – El-Kalyoubia 18 – El-Menia 19 – Giza 20)

7 XYZ is a right-angled triangle at Z where : XZ = 7 cm. and XY = 25 cm.

Find the value of each of the following :

1. tan X × tan Y
2. $\sin^2 X + \sin^2 Y$ *(Port Said 18)* « 1 , 1 »

8 ABC is a right-angled triangle at B in which : BC = 4 cm. and AC = 5 cm.

Deduce that : $\sin^2 A - \cos^2 A = 2 \sin^2 A - 1$

9 ABC is a right-angled triangle at B , if AB : AC = 3 : 5

, find the main trigonometrical ratios of $\angle$ A *(Aswan 13)* « $\frac{4}{5}$, $\frac{3}{5}$, $\frac{4}{3}$ »

10 XYZ is a right-angled triangle at Y , if YZ = 2 XY

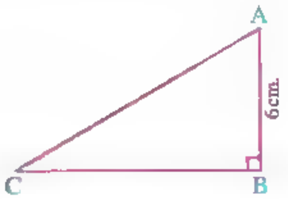
, **find :** the value of each of tan Z , tan X , cos Z , cos X « $\frac{1}{2}$, 2 , $\frac{2}{\sqrt{5}}$, $\frac{1}{\sqrt{5}}$ »

11 ABC is a right-angled triangle at B , if 2 AB = $\sqrt{3}$ AC

, find the main trigonometrical ratios of the angle C

(Alexandria 15 – El-Dakahlia 18 – Aswan 19) « $\frac{\sqrt{3}}{2}$, $\frac{1}{2}$, $\sqrt{3}$ »

12 **In the opposite figure :**

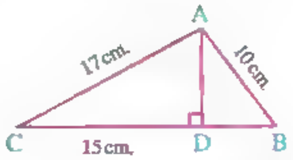


ABC is a right-angled triangle at B

, AB = 6 cm. , tan C = $\frac{3}{4}$, **find :**

1. The length of each of $\overline{BC}$ and $\overline{AC}$
2. sin A + cos A *(Ismailia 12 – El-Monofia 16)* « 8 cm. , 10 cm. , $\frac{7}{5}$ »

13 **In the opposite figure :**



$\overline{AD} \perp \overline{BC}$, AC = 17 cm. ,

DC = 15 cm. , AB = 10 cm.

Find : The value of 3 tan C + sin B *(Ismailia 14)* « $\frac{12}{5}$ »

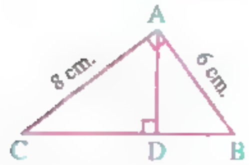
14 **In the opposite figure :**

m (∠ BAC) = 90° , $\overline{AD} \perp \overline{BC}$

, AB = 6 cm. , AC = 8 cm.

Find : 1 tan (∠ BAD)

2 cos (∠ DAC) + cos (∠ DAB)



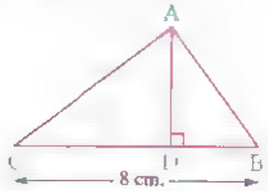
(El Gharbia 16) « $\frac{3}{4}$, $\frac{7}{5}$ »

15 **In the opposite figure :**

Δ ABC is an acute-angled triangle

, BC = 8 cm. , $\overline{AD} \perp \overline{BC}$

Find the value of : AB cos B + AC cos C



(El-Sharkia 17) « 8 cm. »

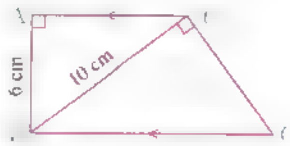
16 **In the opposite figure :**

ABCD is a trapezium in which ∠ A is right

, $\overline{AD}$ // $\overline{BC}$, m (∠ BDC) = 90°

, AB = 6 cm. , BD = 10 cm.

Find : tan (∠ ADB) and the length of $\overline{DC}$



(El-Dakahlia 17) « $\frac{3}{4}$, 7.5 cm. »

17 ABCD is an isosceles trapezoid in which : $\overline{AD}$ // $\overline{BC}$, AD = 4 cm. , AB = 5 cm. and BC = 12 cm.

Prove that : $\frac{5 \tan B \cos C}{\sin^2 C + \cos^2 B} = 3$ *(New Valley 17)*

18 ABCD is a trapezoid in which : $\overline{AD}$ // $\overline{BC}$, m (∠ B) = 90° , AB = 3 cm.

, AD = 6 cm. and BC = 10 cm.

Prove that : cos (∠ DCB) – tan (∠ ACB) = $\frac{1}{2}$

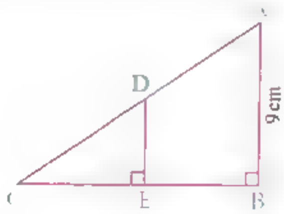
19 **In the opposite figure :**

ABC is a right-angled triangle at B in which :

AB = 9 cm. , D ∈ $\overline{AC}$, E ∈ $\overline{BC}$

where $\overline{DE} \perp \overline{BC}$ and 4 DE = 3 EC

Calculate : The area of Δ ABC



« 54 cm^2 »

20 ABC is an isosceles triangle in which : AB = AC and sin $\frac{A}{2} = \frac{4}{5}$

Find cos B without using the calculator. *(Red Sea 13)* « $\frac{4}{5}$ »

21 If Δ ABC is a right-angled triangle at C , **prove that :** sin B + cos B > 1

22 ABC is a right-angled triangle at B and sin A = 0.6

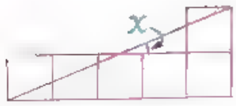
Find : The value of sin A cos C + cos A sin C *(Kafr El Sheikh 13)* « 1 »

23 ABC is a right-angled triangle at B and 7 tan A – 24 = 0

Find : The value of 1 – tan A sin C «$\frac{1}{25}$»

---

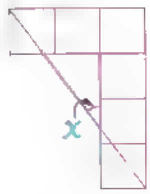
24 **If the following figures are formed from congruent squares , then find the required under each figure :**

1


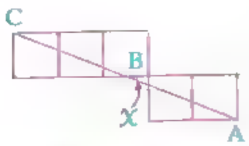
Find : tan X

2

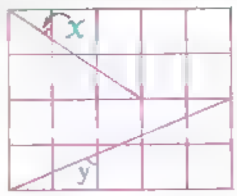

Find : tan X

3


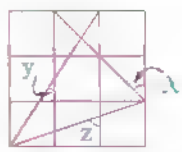
Find : cos X

4


If A , B and C are collinear.
Find : tan X

5


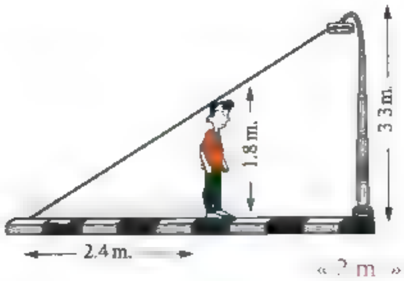
Find : tan $X + \dfrac{1}{\tan y}$

6


Find : tan X + tan y – tan z

Life Applications

25 A man of height 1.8 m. stands in front of a lamppost of height 3.3 m.
If the length of the man's shade (when the lamppost is turned on) is 2.4 m. , find the distance between the man and the base of the lamppost.

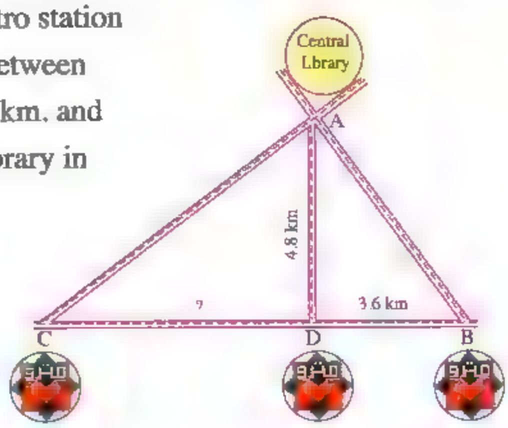


«2 m»

---

26 In one of the governorates , we want to build a metro station between two other stations such that the distance between that station and one of the other two stations is 3.6 km. and the shortest distance between it and the Central Library in the governorate is 4.8 km.

If you know that the two ways between the Central Library and the two metro stations B and C are orthogonal.

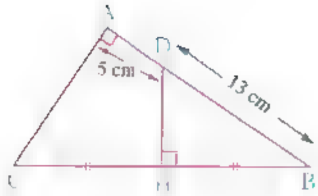


Find in two different methods :

The distance between the metro station we want to build and the metro station C «6.4 km»

For excellent pupils

27 **In the opposite figure :**



m (∠ A) = 90° , $\overline{DH} \perp \overline{BC}$

where H is the midpoint of $\overline{BC}$

, AD = 5 cm. and BD = 13 cm.

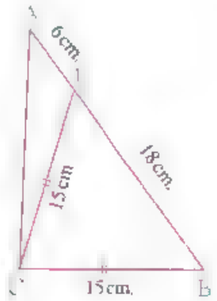
Find with proof tan B

(Damietta 17) « $\frac{2}{3}$ »

---

28 **From the opposite figure :**

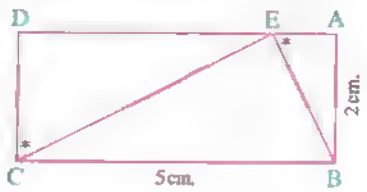
Find : tan (∠ BAC)



« $\frac{4}{5}$ »

---

29 **In the opposite figure :**



ABCD is a rectangle in which :

AE < ED , AB = 2 cm. , BC = 5 cm.

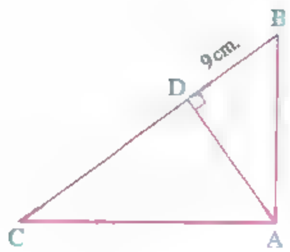
, m (∠ AEB) = m (∠ ECD)

Find : tan (∠ CED)

« $\frac{1}{2}$ »

---

30 **In the opposite figure :**



ABC is a triangle , D ∈ $\overline{BC}$ where :

$\overline{AD} \perp \overline{BC}$, BD = 9 cm.

If sin (∠ BAD) = cos (∠ CAD) = $\frac{3}{5}$

, **find :** the area of Δ ABC

« 150 cm^2 »

---

31 In any right-angled triangle ABC at B

, **prove that :** $\sin^2 A + \sin^2 C = 1$

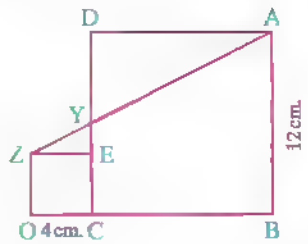
32 ○ **In the opposite figure :**

ABCD is a square ,

ECOZ is a square ,

AB = 12 cm. , CO = 4 cm.

Find : tan (∠ AZE)



« $\frac{1}{2}$ »

---

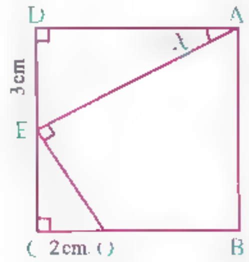
33 ○ **In the opposite figure :**

ABCD is a square , E ∈ $\overline{DC}$,

O ∈ $\overline{BC}$, $\overline{AE} \perp \overline{EO}$

, DE = 3 cm. , CO = 2 cm.

Find : tan X



« $\frac{1}{3}$ »

---

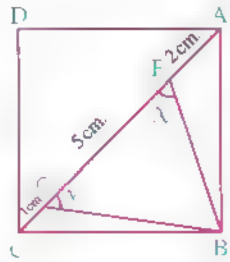
34 ○ **In the opposite figure :**

ABCD is a square ,

E ∈ $\overline{AC}$, O ∈ $\overline{AC}$

where AE = 2 cm. , EO = 5 cm. , OC = 1 cm.

Find : The value of tan X + tan y



« $3\frac{1}{3}$ »

2 The main trigonometrical ratios of some angles

interactive test

From the school book

● Remember   Problem Solving

1 Without using the calculator , find each of the following :

1) $\sin 45° - \cos 45°$

2) $\cos 60° + \sin 30°$

3) $\sin 30° + \cos 60° - \tan 45°$

4) $\sin 60° + \cos 30° + \tan 60°$

5) $\sin^2 45° + \cos^2 45°$

6) $4 \cos 30° \tan 60°$

7) $\tan^2 60° - 2 \sin 45° \cos 45°$ *(North Sinai 17)*

8) $\sin^2 60° - \tan 60° \cos 30° + \cos 60° \sin 30°$ *(Assiut 17)*

9) $2 \sin 30° \cos 60° + \sqrt{2} \sin 45°$

10) $(\cos 30° - \cos 60°)(\sin 30° + \sin 60°)$

11) $\dfrac{\sin 30°}{\cos 60°} - \cos 30° \sin 60°$ *(Ismailia 17)*

12) $\dfrac{\cos^2 60° + \cos^2 30° + \tan^2 45°}{\sin 60° \tan 60° - \sin 30°}$ *(El-Gharbia 17)*

2 Without using the calculator , prove each of the following :

1) 📖 $\sin 60° = 2 \sin 30° \cos 30°$ *(Souhag 18 – Giza 19 – North Sinai 20)*

2) 📖 $\cos 60° = 2 \cos^2 30° - 1$ *(Port Said 18 – North Sinai 19 – South Sinai 20)*

3) $2 \cos^2 30° - 1 = 1 - 2 \sin^2 30°$ *(El Sharkia 15)*

4 $\cos 60° = \cos^2 30° - \sin^2 30°$ *(Alex. 17 – El-Fayoum 18 – South Sinai 19)*

5 $\tan 60° = \dfrac{2 \tan 30°}{1 - \tan^2 30°}$ *(Matrouh 17 – Damietta 19 – Alex. 20)*

6 $\cos^2 60° = 5 \sin^2 30° - \tan^2 45°$ *(Kafr El-Sheikh 11 – El-Menia 14 – Suez 17)*

7 $\sin^3 30° = 9 \cos^3 60° - \tan 45°$ *(Luxor 17)*

8 $\dfrac{\sin 30° \cos 45° + \sin 45° \cos 30°}{\sin 45° \cos 60° + \cos 45° \sin 60°} = \tan^2 45°$

9 $\sin 30° = \sqrt{\dfrac{1 - \cos 60°}{2}}$

---

3 Choose the correct answer from those given :

1 If $\cos X = \frac{1}{2}$ where X is an acute angle , then m $(\angle X) =$ .... *(Cairo 13)*

(a) 90° (b) 60° (c) 45° (d) 30°

2 If $\sin X = \frac{1}{2}$ where X is an acute angle , then m $(\angle X) =$ ..... *([illegible] 20)*

(a) 90° (b) 60° (c) 45° (d) 30°

3 If $\tan X = \frac{1}{\sqrt{3}}$ where X is the measure of an acute angle , then $\tan 2X =$ ....

(a) $\frac{2}{\sqrt{3}}$ (b) $2\sqrt{3}$ (c) $\sqrt{3}$ (d) 3

4 If $\cos X = \frac{\sqrt{3}}{2}$ where X is the measure of an acute angle , then $\sin 2X =$ ....

(El-Gharbia 18 - Red Sea 19)

(a) 1 (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{2}}$

5 If $2 \sin X = \tan 60°$ where X is an acute angle , then m $(\angle X) =$ .... *([illegible] 11)*

(a) 30° (b) 45° (c) 60° (d) 40°

6 If X is the measure of an acute angle , $2 \sin X - 1 = 0$, then $X =$ .... *([illegible])*

(a) 60° (b) 90° (c) 45° (d) 30°

7 If $\tan 3X = \sqrt{3}$ where $3X$ is an acute angle , then m $(\angle X) =$ .... *([illegible] 18 – North Sinai 20)*

(a) 20° (b) 30° (c) 45° (d) 60°

8 If $\sin 2X = \frac{\sqrt{3}}{2}$, then $X =$ .... (where $2X$ is the measure of an acute angle) *([illegible])*

(a) 20° (b) 30° (c) 45° (d) 60°

9 If $\cos \frac{X}{2} = \frac{1}{2}$ where $\frac{X}{2}$ is an acute angle , then m ($\angle X$) = ...

(a) 30° (b) 45° (c) 60° (d) 120°

10 If $\cos (X + 10°) = \frac{1}{2}$ where $(X + 10°)$ is an acute angle , then X = ...

(a) 30° (b) 40° (c) 50° (d) 70°

11 If $\tan (2X - 5°) = 1$ where X is an acute angle , then X = ...

(a) 45° (b) 35° (c) 25° (d) 15°

12 If $\sin (X + 5°) = \frac{1}{2}$ where $(X + 5°)$ is the measure of an acute angle , then $\tan (X + 20°)$ = ... *(El Dakahlia 11)*

(a) $\frac{\sqrt{2}}{2}$ (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) 1

13 If X and y are complementary angles where $X : y = 1 : 2$, then $\sin X + \cos y$ = ...

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{\sqrt{3}}{2}$ (d) 1

(El Beheira 15)

14 In Δ ABC , if m ($\angle$ A) : m ($\angle$ B) : m ($\angle$ C) = 3 : 4 : 5 , then cos B = ...

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) $\frac{\sqrt{3}}{2}$

(El-Gharbia 16)

15 The tangent of an acute angle of the right isosceles triangle is equal to ....

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) 1 (d) $\frac{\sqrt{2}}{2}$

(El-Dakahlia 16)

16 Δ ABC is right-angled at A , if tan B = 1 , then tan C sin C cos C = ...

(a) zero (b) 1 (c) 2 (d) $\frac{1}{2}$

(Red Sea 16)

17 If the straight line : $y = X \sin 30° + c$ passes through the point (4 , 6) , then c = ...

(a) 4 (b) 6 (c) 8 (d) 2

(El Monofia 16)

4 **Find the value of X in each of the following :**

1. $X \sin^2 45° = \tan^2 60°$ *(Souhag 17)* « 6 »

2. $X \sin 30° \cos^2 45° = \sin^2 60°$ *(South Sinai 16 – Alex. 19 – Assiut 20)* « 3 »

3. $X \sin 45° \cos 45° \tan 60° = \tan^2 45° - \cos^2 60°$ « $\frac{\sqrt{3}}{2}$ »

4. $4 X = \cos^2 30° \tan^2 30° \tan^2 45°$ *(Alex. 17 – El-Fayoum 19 – Suez 20)* « $\frac{1}{16}$ »

5 **Find the value of X in each of the following :**

1. $\tan X = 4 \sin 30° \cos 60°$ where X is an acute angle. *(Damietta 18 – El-Gharbia 19 – Giza 20)* « 45° »

2. $\sin X = \sin 60° \cos 30° - \cos 60° \sin 30°$ where $0° < X < 90°$ *(Cairo 17)* « 30° »

3. $2 \sin X = \sin 30° \cos 60° + \cos 30° \sin 60°$ where X is an acute angle. *(Giza 18)* « 30° »

4. $6 \sin X \cos 45° \sin 45° = 1 - \cos^2 60°$ where $0° < X < 90°$ *(Aswan 13)* « 14° 28̀ 39̀ »

5. $\cos X = \frac{\sin 60° \sin 30°}{\tan 45° \sin^2 45°}$ where X is an acute angle. *(El-Dakahlia 18)* « 30° »

6. $\cos (3 X + 6°) = \sin 30°$ where $(3 X + 6°)$ is an acute angle. « 18° »

7. $\sqrt{3} \sin X \tan 30° = \tan 45° \cos 2 X$ where X is an acute angle. *(El-Monofia 20)* « 30° »

6 **Find E in each of the following where E is the measure of an acute angle :**

1. $\sin^2 45° = \cos E \tan 30°$ *(Damietta 16 – El-Monofia 17 – Beni Suef 19)* « 30° »

2. $\sin E \sin^2 60° = 3 \sin^2 45° \cos^2 45° \cos 60°$ *(Beni Suef 18)* « 30° »

3. $3 \tan E - 4 \sin^2 30° = 8 \cos^2 60°$ « 45° »

7 If $\tan X = \frac{1}{\sqrt{3}}$, X is an acute angle , **find :** $\sin X \tan \left(\frac{3 X}{2}\right) + \cos 2 X$ *(Damietta 13)* « 1 »

7 If $\sin X = \tan 30° \sin 60°$ where X is the measure of an acute angle , then find without using the calculator the value of : $4 \cos X \sin X$ *(El-Kalyoubia 20)* « $\sqrt{3}$ »

9 **Complete the following tables where the using angles are acute angles :**

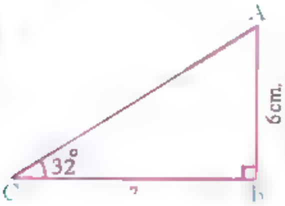
Ratio \ Measure of angle	30°	....	....
sin	....	....	....
cos	....	$\frac{1}{2}$	....
tan	....	....	1

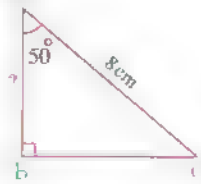
Without using the calculator

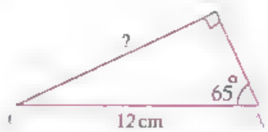
Ratio \ Measure of angle	34° 12′	....	....	....
sin	....	....	....	0.6
cos	....	0.6217	....	....
tan	....	....	2.2203	....

By using the calculator

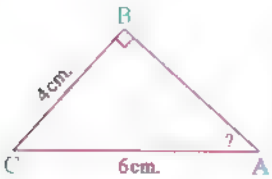
10 **Find the length of the side marked by the sign (?) in each of the following figures to the nearest two decimal digits :**

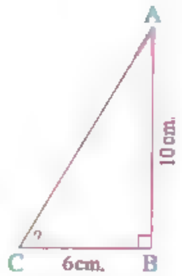
1 

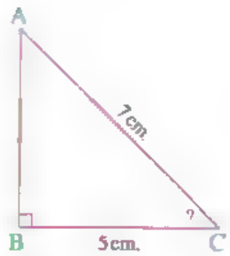
2 

3 

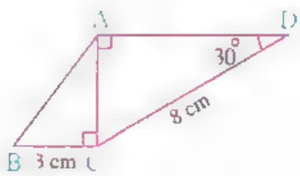
11 **Find in each of the following figures the measure of the angle marked by the sign (?) in degrees , minutes and seconds :**

1 

2 

3 

12 **In the opposite figure :**



m (∠ D) = 30°

, m (∠ CAD) = m (∠ ACB) = 90°

, BC = 3 cm. , CD = 8 cm.

Find : 1 tan B 2 m (∠ BAD) *(El-Sharkia 18)* « $\frac{4}{3}$, 126° 52′ 12″ »

13 ABC is an isosceles triangle in which AB = AC = 7 cm. and BC = 10 cm.

Find : 1 m (∠ B) 2 The area of Δ ABC « 44° 24′ 55″ , $10\sqrt{6}$ cm² »

14 ABC is an isosceles triangle in which AB = AC = 12.6 cm. and m (∠ C) = 84° 24′

Find the length of $\overline{BC}$ to the nearest one decimal number. « 2.5 cm »

---

15 **In the opposite figure :**

ABC is a right-angled triangle at B ,

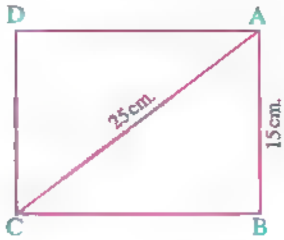
m (∠ A) = 2 m (∠ C)

Find : The value of $\cos^2 A + \tan^2 C$

(El-Sharkia 13) « $\frac{7}{12}$ »

---

16 **In the opposite figure :**



ABCD is a rectangle in which :

AB = 15 cm. and AC = 25 cm.

Find :

1. m (∠ ACB)
2. The area of the rectangle ABCD [illegible] (El-Fayoum 20) « 36° 52′ 12″ , 300 cm² »

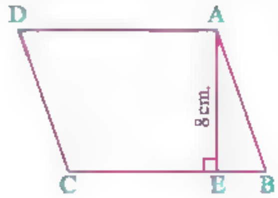
---

17 ABCD is a rectangle whose diagonal length AC = 24 cm. , m (∠ ACB) = 25°

Find : The length of $\overline{BC}$ « 21.8 cm. »

---

18 **In the opposite figure :**



ABCD is a parallelogram of surface area 96 cm²

BE : EC = 1 : 3 , $\overline{AE} \perp \overline{BC}$ and AE = 8 cm.

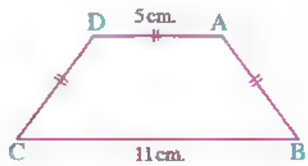
Find : 1. The length of $\overline{AD}$ 2. m (∠ B)

3. The length of $\overline{AB}$ to the nearest one decimal (Use more than one way)

« 12 cm. , 69° 26′ 38″ , 8.5 cm. »

---

19 **In the opposite figure :**



ABCD is an isosceles trapezium in which :

AB = AD = DC = 5 cm., BC = 11 cm. **Find :**

1. m (∠ B) , m (∠ A)
2. The area of the trapezium ABCD *(Matrouh 13)* « 53° 7′ 48″ , 126° 52′ 12″ , 32 cm² »

20 ABCD is a trapezium in which : $\overline{AD} \mathbin{/\mkern-6mu/} \overline{BC}$ and m $(\angle ABC) = 90^\circ$

If AB = 12 cm. , AD = 16 cm. and BC = 25 cm. , **find :**

1. The length of $\overline{DC}$
2. m $(\angle C)$
3. sin $(\angle DCB)$ – tan $(\angle ACB)$

« 15 cm. , $53^\circ\ 7'\ 48''$, $\frac{8}{25}$ »

Life Applications

21 A ladder $\overline{AB}$ is of length 6 metres, its upper edge A lies on a vertical wall and its other edge B on a horizontal floor. If C is the projection of the point A on the surface of the floor and its angle of slope on the surface of the floor was of measure 60° , then find the length of $\overline{AC}$ *(Kafr El-Sheikh 17)* « $3\sqrt{3}$ m. »

22 A person walks up an inclined plane which makes with the horizontal plane an angle of measure 22°. If this person walks 500 m. up the plane , calculate the height of this plane above the ground surface to the nearest metre. « 187 m. »

23 The wind broke the upper point of a tree to make an angle of measure 60° with the ground level , if the top of the tree meets the ground 4 metres away from the bottom of the tree , find the height of the tree to the nearest metre. *(El-Fayoum 14)* « 15 m. »

For excellent pupils

24 **Find the value of A where A is the measure of an acute angle if :**

$\cos A \times \tan A = \frac{1}{2}$ « 30° »

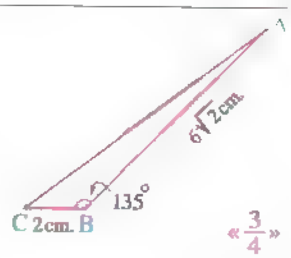
25 **In the opposite figure :**

If m $(\angle B) = 135^\circ$

, AB = $6\sqrt{2}$ cm.

, BC = 2 cm.

, **find :** tan C



« $\frac{3}{4}$ »

Summary of unit 4

The main trigonometrical ratios of the acute angle :

- If Δ ABC is a right-angled triangle at B , then :

- $\sin A = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{BC}{AC}$
- $\cos A = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{AB}{AC}$
- $\tan A = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{BC}{AB}$
- $\sin C = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{AB}{AC}$
- $\cos C = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{BC}{AC}$
- $\tan C = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{AB}{BC}$

- $\tan A = \frac{\sin A}{\cos A}$

- If $\angle$ A , $\angle$ B are complementary angles , then : $\sin A = \cos B$ and $\cos A = \sin B$

  and vice versa

  i.e. If $\angle$ A and $\angle$ B are acute angles and $\sin A = \cos B$ or $\cos A = \sin B$

  , then $m(\angle A) + m(\angle B) = 90^\circ$

The trigonometrical ratios of some angles :

- $\sin 30^\circ = \frac{1}{2}$
- $\cos 30^\circ = \frac{\sqrt{3}}{2}$
- $\tan 30^\circ = \frac{1}{\sqrt{3}}$

- $\sin 60^\circ = \frac{\sqrt{3}}{2}$
- $\cos 60^\circ = \frac{1}{2}$
- $\tan 60^\circ = \sqrt{3}$

- $\sin 45^\circ = \frac{1}{\sqrt{2}}$
- $\cos 45^\circ = \frac{1}{\sqrt{2}}$
- $\tan 45^\circ = 1$

Exams on Unit Four

Model 1

Answer the following questions :

1 **Choose the correct answer from those given :**

1 For any acute angle A , tan A = ...

(a) $\frac{\cos A}{\sin A}$ (b) $\sin A \cos A$ (c) $\frac{\sin A}{\cos A}$ (d) $\sin A + \cos A$

2 If $\cos X = \frac{\sqrt{2}}{2}$ where X is the measure of an acute angle , then $\sin 2X$ = ...

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{-\sqrt{2}}{2}$ (c) 1 (d) $\frac{2}{\sqrt{2}}$

3 ABC is a right-angled triangle at B where 3AC = 5 BC

, then tan A = ...

(a) $\frac{3}{5}$ (b) $\frac{5}{3}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

4 If $\cos(X + 15^\circ) = \frac{1}{2}$, then $\sin(75^\circ - X)$ = ...

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 1

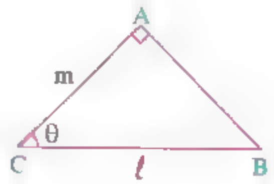
5 If XYZ is an isosceles triangle and right at Z , then tan X = ...

(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$ (c) 1 (d) $\frac{1}{3}$

6 **In the opposite figure :**

If the length of $\overline{BC}$ is ℓ and the length of $\overline{AC}$ is m

, then which of the following equations can be used to find ℓ ?



(a) $\ell = \frac{m}{\cos \theta}$ (b) $\ell = \frac{m}{\sin \theta}$

(c) $\ell = m \cos \theta$ (d) $\ell = m \sin \theta$

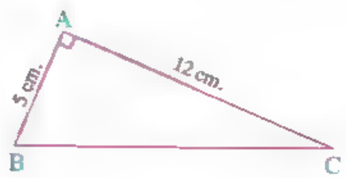
---

2 **[a] In the opposite figure :**

ABC is a triangle in which : m (∠ A) = 90°

, AC = 12 cm. and AB = 5 cm.

Prove that : cos C cos B – sin C sin B = zero



[b] Find the value of X in each of the following :

1 $X \sin 30° = \sin 30° \cos 60° + \cos 30° \sin 60°$

2 $2 \sin X = \tan^2 60° - 2 \tan 45°$ where X is the measure of an acute angle.

---

3 [a] ABCD is an isosceles trapezoid in which :

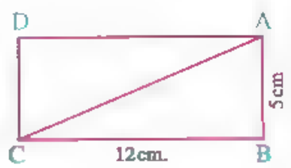
$\overline{AD} \,//\, \overline{BC}$, AD = 4 cm. , AB = 5 cm. and BC = 12 cm.

Find the value of : $\frac{\tan B \cos C}{\cos^2 C + \sin^2 C}$

[b] Find the value of : $\frac{\sin 30° \cos 45° + \cos 30° \sin 45°}{\sin 45° \cos 60° + \cos 45° \sin 60°}$

---

4 [a] **In the opposite figure :**



ABCD is a rectangle in which :

AB = 5 cm. and BC = 12 cm.

Find :

1 The length of $\overline{AC}$

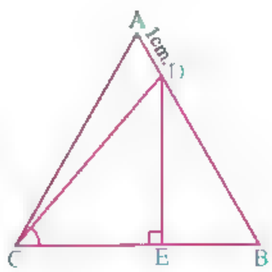
2 The value of : 5 tan (∠ ACD) – 13 sin (∠ DAC)

[b] The wind broke the upper point of a tree to make an angle of measure 30° with the ground level , if the top of the tree meets the ground 3 metres away from the bottom of the tree , find the height of the tree to the nearest metre.

---

5 [a] If ABC is a right-angled triangle at B , sin A + cos C = 1 , **find :** m (∠ A)

[b] In the opposite figure :



ABC is an equilateral triangle of side length 5 cm.

, D ∈ $\overline{AB}$ where AD = 1 cm.

, $\overline{DE} \perp \overline{BC}$

Find : tan (∠ DCE)

Model 2

Answer the following questions :

1 **Choose the correct answer from those given :**

1. If $\tan(X + 15°) = 1$ where X is the measure of an acute angle , then $X = \dots\dots$
   (a) 60° (b) 45° (c) 30° (d) 15°

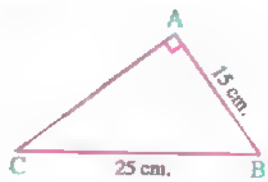
2. If $\sin 30° = \cos E$ where E is an acute angle , then $m(\angle E) = \dots\dots\dots$
   (a) 15° (b) 30° (c) 60° (d) 90°

3. If X is an acute angle , $2 \sin X - 1 = 0$, then $m(\angle X) = \dots\dots\dots$
   (a) 60° (b) 90° (c) 45° (d) 30°

4. If A and B are two complementary angles where $A : B = 1 : 2$, then $\sin A + \cos B = \dots\dots$
   (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{\sqrt{3}}{2}$ (d) 1

5. If $m(\angle A) = 70°$, $\sin B = \cos B$ in ΔABC , then $m(\angle C) = \dots\dots$
   (a) 50° (b) 45° (c) 70° (d) 65°

6. **In the opposite figure :**
   ABC is a triangle in which :
   $m(\angle A) = 90°$, AB = 15 cm.
   , BC = 25 cm. , then $\tan B = \dots\dots\dots$



   (a) $\frac{3}{5}$ (b) $\frac{4}{5}$ (c) $\frac{4}{3}$ (d) $\frac{3}{4}$

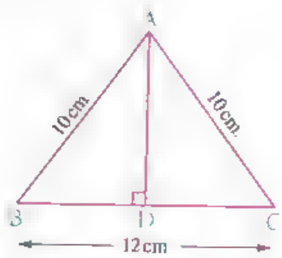
2 **[a] Find the value of X in degrees if :** $\tan 2X = 4 \sin 30° \cos 30°$ where $0° < X° < 90°$

[b] If the ratio between the measures of two complementary angles is 3 : 5 , find the measure of each one by degree measure.

3 **[a] In the opposite figure :**

ABC is a triangle where AB = AC = 10 cm.
, BC = 12 cm. , $\overline{AD} \perp \overline{BC}$

Find : 1 $\cos B$ 2 $m(\angle B)$
3 $\sin(90° - B)$



[b] Without using calculator , prove that : $\tan^2 60° - \tan^2 45° = \sin^2 60° + \cos^2 60° + 2 \sin 30°$

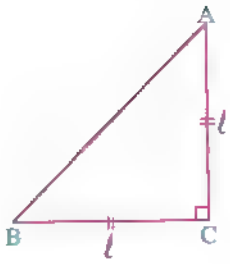
4 **[a] In the opposite figure :**

ABC is an isosceles triangle and right at C

, the length of each leg is ℓ length unit.

Find : 1 The ratio among the lengths of sides of the triangle AC : BC : AB

2 tan B , sin A



[b] In the opposite figure :

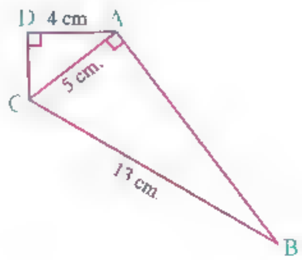
m (∠ ADC) = m (∠ BAC) = 90°

, AD = 4 cm. , AC = 5 cm. and BC = 13 cm.

Calculate the value of each of :

1 tan (∠ ACB) + tan (∠ ACD)

2 sin (∠ B) cos (∠ CAD) + cos (∠ B) sin (∠ CAD)



5 **[a]** ABC is a right-angled triangle at B

1 **Prove that :** $\sin^2 A + \cos^2 A = 1$

2 If AB = 5 cm. , AC = 13 cm. , **find :** m (∠ C)

[b] Find without using the calculator : $\sin 45° \cos 45° + 3 \sin 30° \cos 60° - \cos^2 30°$

A Research Project

On Unit Four

Project aims :

- Finding the measure of an angle knowing one of its trigonometrical ratios.
- Applying Pythagoras' theorem.
- Associating mathematics with sports.
- Associating mathematics with history.
- Associating mathematics with science.

Do a research project on the following to

"Football game is considered one of the most popular team sports around the world. It is the most practiced in most countries".

Discuss the following points using available resources :

- Write down about the start of football and how it has developed over the years.
- Mention the dimensions of the football pitch, the dimensions of the goal and the dimensions of the penalty area.
- How far is the penalty point to the goal line ?
- If a player kicks a ball at the penalty point towards the goal and the ball then hits the horizontal crossbar exactly at the middle, calculate the following providing the ball moves in a straight line :

1. The distance that the ball covered to hit the horizontal crossbar.
2. The measure of the angle that the path of the ball made with the ground.
3. To score a goal, in this case, write the interval which the measure of the angle that the ball path made with the ground belongs to.
4. The average velocity that the ball moved in if it hits the horizontal crossbar in 0.4 seconds after it was kicked.

UNIT 5 Analytical geometry

Exercises of the unit :

3. Distance between two points.

4. The two coordinates of the midpoint of a line segment.

5. The slope of the straight line.

6. The equation of the straight line given its slope and the intercepted part of y-axis.

- Unit Exams.

A research project on unit five

Scan the QR code to solve an interactive test on each lesson

EXERCISE **3**

Distance between two points

Interactive test

From the school book

Remember   Understand   Apply   Problem Solving

1 **Find the length of $\overline{AB}$ in each of the following cases :**

1. A (1 , 2) , B (4 , 6)
2. A (2 , – 1) , B (5 , – 5)
3. A (– 2 , 7) , B (3 , – 5)
4. A (– 2 , 5) , B (3 , 0)
5. A (15 , 0) , B (6 , 0)
6. A (6 , 0) , B (0 , – 8)

2 **Choose the correct answer from those given :**

1. The distance between the two points (3 , a) and (– 1 , a) is ...... length unit.
   (a) 16   (b) 9   (c) 5   (d) 4
2. The distance between the point ($\sqrt{3}$, 1) and the origin point is ....
   (a) 4   (b) 3   (c) 2   (d) 1
3. If the distance between the two points (a , 0) , (0 , 1) is one length unit , then a = ...
   (El-Gharbia 20)
   (a) 1   (b) – 1   (c) 0   (d) 2
4. The radius length of the circle whose centre is (7 , 4) and passes through (3 , 1) equals .... length units.
   (a) 5   (b) – 5   (c) 2.5   (d) 25
5. If ABCD is a square and A (3 , 5) and B (4 , 2) , then the area of the square ABCD equals .... area unit.
   (a) $\sqrt{10}$ (b) 10   (c) $4\sqrt{10}$ (d) 40

6 If ABCD is a rhombus and A (– 1 , 7) , B (– 3 , 1) , then the perimeter of the rhombus ABCD = ... length unit.

(a) 40 (b) $4\sqrt{52}$ (c) $8\sqrt{10}$ (d) $2\sqrt{10}$

7 In the Cartesian coordinates plane , the point that is at the distance 2 length unit from the origin may be .... *(Cairo 09)*

(a) (1 , 2) (b) (2 , 1) (c) (0 , 2) (d) (– 3 , 5)

8 The distance between the point (- 5 , - 2) and y-axis is ...... length unit. *(El Gharbia 16)*

(a) – 5 (b) – 2 (c) 2 (d) 5

9 The distance between the point (5 , $\tan^2 60°$) and the X-axis is ...... length unit. *(Suez 17)*

(a) 5 (b) $\sqrt{5}$ (c) 3 (d) $\sqrt{3}$

10 The distance between the point (ℓ , – 4) and y-axis is ...... length unit, where $\ell \in \mathbb{R}$ *(Damietta 18)*

(a) 4 (b) ℓ (c) – 4 (d) $|\ell|$

11 The perpendicular distance between the two straight lines : y – 3 = 0 , y + 2 = 0 equals .... length units. *(Alex. 17 – El-Fayoum 17)*

(a) 5 (b) 1 (c) 2 (d) 3

12 A circle its centre is the origin and its radius length is 2 length unit , which of the following points belongs to the circle ? *(El Gharbia 14 – Beni Suef 16 – El Beheira 17)*

(a) (1 , 2) (b) (– 2 , 1) (c) $(\sqrt{3} , 1)$ (d) $(\sqrt{2} , 1)$

3 If A (3 , 1) , B (1 , 2) and C (5 , 4) , **prove that :** BC = 2 AB *(El-Monofia 13 – Luxor 16)*

4 **Prove that :** The points A (4 , 3) , B (1 , 1) and C (– 5 , – 3) are collinear. *(Assiut 14 – Kafr El Sheikh 15 – El-Fayoum 17)*

5 If A (- 2 , 2) and B (1 , – 1) , then prove that the point C (3 , 4) lies on the axis of symmetry of $\overline{AB}$

6 **Show which of the following sets of points are collinear :** *(Cairo 08)*

1 A (1 , 4) , B (3 , – 2) and C (– 3 , 16)

2 A (7 , 0) , B (– 3 , 6) and C (22 , 9)

3 A (– 1 , 4) , B (3 , – 14) and C (– 5 , – 6)

7 Show the type of Δ ABC such that A (– 2 , 4) , B (3 , – 1) and C (4 , 5) according to its side lengths.

(New Valley 16 – Giza 17 – Damietta 19 – El-Beheira 20)

8 **Show the type of each of the following triangles according to its angles if its vertices are :**

1. A (2 , 1) , B (4 , – 2) and C (7 , 5)
2. A (3 , 5) , B (– 1 , 1) and C (5 , – 5)
3. A (4 , 4) , B (3 , – 1) and C (– 2 , 4)
4. A (0 , 0) , B (6 , 0) and C (0 , 8)
5. A (1 , – 1) , B (2 , 1) and C (– 3 , – 2)

9 Prove that the triangle whose vertices are : A (5 , – 5) , B (– 1 , 7) and C (15 , 15) is right-angled at B , then find its area.

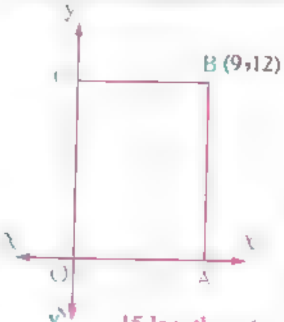
(Beni Suef 13 - El Monofia 14 – Qena 16) « 120 square units »

10 If the points A (5 , 0) , B $\left(7 , 2\sqrt{3}\right)$ and C $\left(3 , 2\sqrt{3}\right)$ are three points in a Cartesian coordinates plane , **prove that :** Δ ABC is equilateral and find its area. [illegible]

11 **In the opposite figure :**

If ABCO is a rectangle

, then find the length of : $\overline{AC}$



« 15 length units »

12 **In each of the following , prove that the points A , B , C and D are vertices of a parallelogram where :**

1. A (– 1 , 1) , B (0 , 5) , C (5 , 6) and D (4 , 2) *(Suez 11)*
2. A (– 2 , 4) , B (5 , – 3) , C (7 , 1) and D (0 , 8) *(Souhag 08)*

13 **Prove that :** The points A (0 , 1) , B (4 , 5) , C (1 , 8) and D (– 3 , 4) are vertices of a rectangle and find its diagonal length.

(Souhag 09) « $5\sqrt{2}$ length units»

14 **Prove that :** The points A (3 , 3) , B (0 , 3) , C (0 , 0) and D (3 , 0) in the Cartesian coordinates plane are vertices of a square and calculate the length of its diagonal and its area.

(Luxor 09) « $3\sqrt{2}$ length units , 9 square units»

15 ABCD is a quadrilateral where A (5 , 3) , B (6 , - 2) , C (1 , – 1) and D (0 , 4)

Prove that : ABCD is a rhombus , then find its area. [illegible]

16 **Prove that :** The points A $(-2 , 5)$, B $(3 , 3)$ and C $(-4 , 2)$ are non-collinear and if D $(-9 , 4)$

, prove that : the figure ABCD is a parallelogram. *(Port Said [illegible])*

---

17 ABCD is a quadrilateral where A $(2 , 4)$, B $(-3 , 0)$, C $(-7 , 5)$ and D $(-2 , 9)$

Prove that : The figure ABCD is a square. *(El-Beheira [illegible] – Cairo [illegible] – El-Menofia 20)*

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18 **Prove that :** The points A $(3 , -1)$, B $(-4 , 6)$ and C $(2 , -2)$ are located on a circle whose centre is M $(-1 , 2)$, then find the circumference of the circle where $\pi = 3.14$

(Cairo 15 – North Sinai 16 – El-Kalyoubia 18 – Alex. 19 – Aswan 20) « 31.4 length units »

---

19 If the distance between the point $(x , 5)$ and the point $(6 , 1)$ equals $2\sqrt{5}$ length units.

, find : the value of x *(El-Menofia 18 – Matrouh [illegible] – El-Ka[illegible] – Da[illegible] 20)* « 4 or 8 »

---

20 **Find the value of a in each of the following cases :**

1. If the distance between the two points $(a , 7)$, $(-2 , 3)$ equals 5 length unit.

   (Matrouh 17 – Alex. 18 – El Menia 19 – El-Fayoum 20) « 1 or – 5 »

2. If the distance between the two points $(a , 7)$, $(3a - 1 , -5)$ equals 13 length unit.

   « – 2 or 3 »

---

21 If A $(x , 3)$, B $(3 , 2)$ and C $(5 , 1)$ and AB = BC , then find the value of x

(Port Said 14 – El-Beheira 15 – El-Beheira 17 – El-Beheira 19) « 5 or 1 »

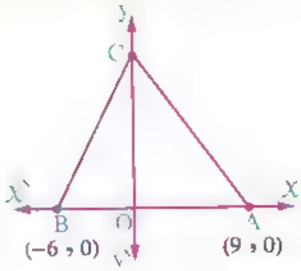
---

22 **In the opposite figure :**

If AB = AC

, find : the length of $\overline{CO}$

« 12 length units »



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23 If the axis of symmetry of $\overline{CD}$ is passing through the point A $(6 , m)$ where C $(3 , 1)$, D $(-3 , 7)$, then find the value of m *(El-Dakahlia 16)* « 10 »

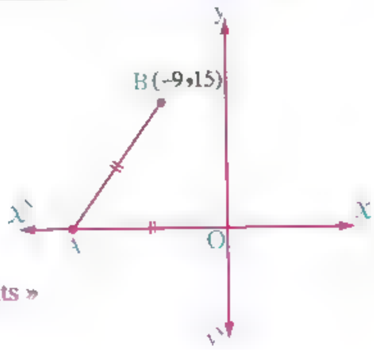
---

24 **In the opposite figure :**

If A $\in$ the x-axis

and AO = AB

, find : the length of $\overline{AB}$

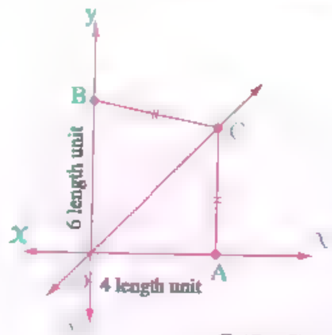


(El Dakahlia 18) « 17 length units »

25 **In the opposite figure :**

$A \in \overleftrightarrow{XX'}$, $B \in \overleftrightarrow{yy'}$ where OA = 4 length unit , OB = 6 length unit , the straight line $\overleftrightarrow{OC}$ represents the function $f : f(X) = X$ and AC = BC

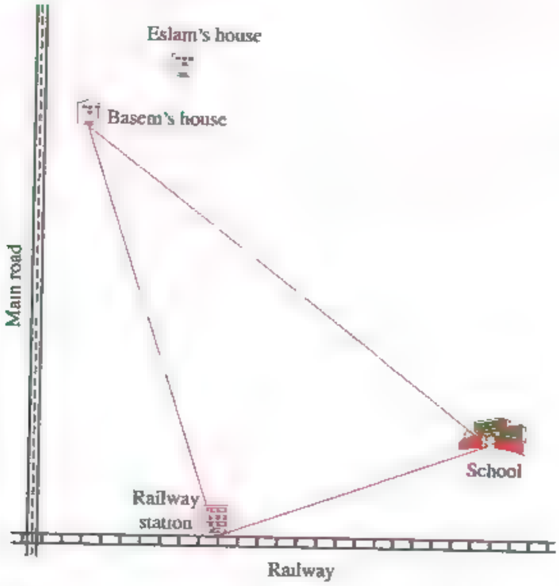
Find the coordinates of the point C



« C (5 , 5) »

Life application

26



If the distance between Basem's house and the main road is 1 km. and the distance between Basem's house and the railway lines is 9 km.

Eslam's house is 3 km. away from the main road and 10 km. away from the railway lines.

The school is 10 km. away from the main road and 2 km. away from the railway lines.

The railway station is 4 km. away from the main road.

1. Which is nearer to school , Basem's house or Eslam's house ?
2. Is the way (school – railway station) perpendicular to the way (Basem's house – railway station) ? Mention the reason.

For excellent pupils

27 If the points A (4 , – 2) , B (X , 2) and C (3 , 5) are three points in the Cartesian coordinates plane , find the value of X which makes Δ ABC a right-angled triangle at B and find its area.

« 0 or 7 , 12 square units or 12.5 square units »

4 The two coordinates of the midpoint of a line segment

From the school book

● Remember ● Understand ○ Apply ⁂ Problem Solving

1 **Find the coordinates of the midpoint of $\overline{AB}$ in each of the following cases :**

1. A (3 , 5) , B (7 , 1)
2. A (5 , – 3) , B (– 1 , 3)
3. A (– 5 , 4) , B (5 , – 4)
4. A (0 , 4) , B (8 , 0)
5. A (2 , 4) , B (6 , 0)
6. A (7 , – 6) , B (– 1 , 0)

---

2 If the point (x , 0) is the midpoint of $\overline{AB}$ where A (1 , – 5) and B (2 , 5) ,
find the value of : x « $\frac{3}{2}$ »

---

3 If the point (5 , 3) is the midpoint of $\overline{AB}$ where A (15 , y) and
B (– 5 , – 2) , **find the value of :** y « 8 »

---

4 If C (6 , – 4) is the midpoint of $\overline{AB}$ where A (5 , – 3) , find the coordinates of the point B
(Aswan 17 El-Dakahlia 18 – Beni Suef 19 Cairo 19) « (7 , 5) »

---

5 **If C is the midpoint of $\overline{AB}$, then find x , y in each of the following cases :**

1. A (1 , 5) , B (3 , 7) , C (x , y) « 2 , 6 »
2. A (– 3 , y) , B (9 , 11) , C (x , – 3) *(Aswan 18)* « 3 , – 17 »
3. A (x , – 6) , B (9 , – 11) , C (– 3 , y) « 15 , 8.5 »
4. A (x , 3) , B (6 , y) , C (4 , 6) *(Cairo 15 Matrouh 17)* « 2 , 9 »

6 **Choose the correct answer from those given :**

1 If (4 , – 3) is the midpoint of $\overline{XY}$ where X (5 , – 2) , then Y is .... (El-Menia 16)

(a) (4.5 , 4.5) (b) (4 , 3) (c) (3 , 4) (d) (3 , – 4)

2 If the point of the origin is the midpoint of $\overline{AB}$ where A (5 , – 2) , then the point B is .... (Port Said 19)

(a) (2 , 5) (b) (5 , – 2) (c) (– 2 , – 5) (d) (– 5 , 2)

3 If C (– 3 , y) is the midpoint of $\overline{AB}$ where A (x , – 6) and B (1 , – 8) , then x + y = ... (Qena 18)

(a) – 11 (b) 11 (c) – 18 (d) – 14

4 If $\overline{AB}$ is a diameter in a circle where A (3 , – 5) and B (5 , 1) , then the centre of the circle is .... (El-Fayoum 18 – Matrouh 19)

(a) (4 , – 2) (b) (4 , 2) (c) (2 , – 2) (d) (8 , – 2)

5 If ABCD is a square where A (3 , 4) and C (5 , 6) , then the midpoint of its diagonal is .... (El Menia 18)

(a) (8 , 10) (b) (10 , 8) (c) (4 , 5) (d) (15 , 24)

6 If M (1 , 2) is the intersection point of the two diagonals of the parallelogram ABCD where A (2 , 5) , then C is ....

(a) (0 , 2) (b) (0 , – 1) (c) (– 4 , 1) (d) (– 1 , 0)

7 If $\left(\frac{1}{2} , \frac{5}{2}\right)$ is the midpoint of $\overline{AB}$ where A (1 , – 1) and B (x , 6) , then x = ...

(a) 0 (b) 1 (c) 2 (d) $\frac{1}{2}$

8 If the x-axis bisects $\overline{AB}$ such that A (3 , 2) and B (– 2 , y) , then y = ... (El Dakahlia 17)

(a) 3 (b) 2 (c) – 2 (d) 4

7 **Complete the following :**

1 If A (3 , y) , B (5 , – 2) and C is the midpoint of $\overline{AB}$ where C (x , 2) , then x = ... and y = ...

2 If B ∈ $\overline{AC}$ where AB = BC , A (0 , 5) and C (– 4 , – 1) , then B is ....

3 If A , B , C and D are four collinear points and AB = BC = CD , A (1 , 3) and C (5 , 1) , then the point B is (.......... , ...) and the point D is (.......... , ...)

4 $\overline{AD}$ is a median in Δ ABC , M is the midpoint of $\overline{AD}$ where A (0 , 8) , B (3 , 2) and C (– 3 , 6) , then the point D is (.......... , ...) and the point M is (.......... , ...)

8 If A (1 , – 6) and B (9 , 2) , find the coordinates of the points which divide $\overline{AB}$ into four equal parts in length. (Souhag 18) « (5 , – 2) , (3 , – 4) , (7 , 0) »

9 If the origin point O is the midpoint of $\overline{AB}$ where A $(X - 2 , y)$ and B $(-2 , 2)$,

find : (X , y) « (4 , – 2) »

10 Find the value of each of a and b that satisfies that $(2a - 3 , a - b)$ is the midpoint of the line segment whose terminals are $(7 , -1)$ and $(3 , 7)$ *(El-Fayoum 12)* « 4 , 1 »

11 $\overline{AB}$ is a diameter in a circle M , if B (8 , 11) and M (5 , 7)

Find : 1 The coordinates of A 2 The circumference of the circle where $(\pi = 3.14)$

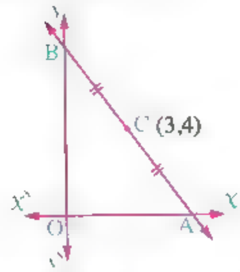
(El-Kalyoubia 16 North Sinai 17 Kafr El-Sheikh 18) « A (2 , 3) , 31.4 length unit »

12 ABC is a triangle where A (1 , 3) , B (5 , 1) and C (3 , 7) , if D is the midpoint of $\overline{AB}$ and E is the midpoint of $\overline{AC}$, by using the coordinates , **prove that :** $DE = \frac{1}{2} BC$

13 **In the opposite figure :**

C (3 , 4) is the midpoint of $\overline{AB}$

Find : The perimeter of the triangle OAB



([illegible] 17 El-Kalyoubia 20) « 24 length unit »

14 **In the opposite figure :**

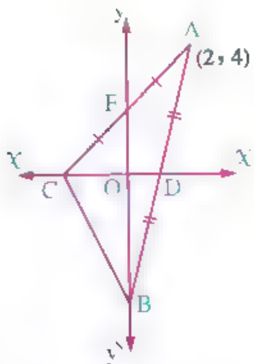
D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

If A (2 , 4)

, **then find :**

the length of $\overline{BC}$ and from it deduce the length of $\overline{DE}$



« $2\sqrt{5}$ length unit , $\sqrt{5}$ length unit »

15 $\overline{AD}$ is a median in Δ ABC , M is the midpoint of $\overline{AD}$ where M (0 , 6) , B (3 , 2) , C (– 3 , 6) , find the coordinates of the point A *(Kafr El-Sheikh 17)* « A (0 , 8) »

16 If A (– 1 , – 1) , B (2 , 3) , C (6 , 0) and D (3 , – 4) are four points in the Cartesian coordinates plane , **prove that :** $\overline{AC}$ and $\overline{BD}$ bisect each other. *(Suez 19)*

17 **Prove that :** The points A (3 , – 2) , B (– 5 , 0) , C (0 , – 7) and D (8 , – 9) are the vertices of a parallelogram. *(El-Fayoum 19)*

18 If the points A (3 , 2) , B (4 , – 3) , C (– 1 , – 2) and D (– 2 , 3) are the vertices of a rhombus , **find :**

1. The coordinates of the point of intersection of the two diagonals.
2. The area of the rhombus ABCD

---

19 ABCD is a parallelogram where A (3 , 2) , B (4 , – 5) and C (0 , – 3)
Find the coordinates of the intersection point of its diagonals , then find the coordinates of the point D

---

20 **Prove that :** The points A (6 , 0) , B (2 , – 4) and C (– 4 , 2) are the vertices of a right-angled triangle at B , then find the coordinates of D that make the figure ABCD a rectangle. *(Assiut 11 – Kafr El-Sheikh 14 – El-Beheira 19)* « D (0 , 6) »

---

21 **Prove that :** The points A (5 , 3) , B (3 , – 2) and C (– 2 , – 4) are the vertices of an obtuse-angled triangle at B , then find the coordinates of the point D that makes the figure ABCD a rhombus and find its area. « (0 , 1) , 21 square units »

---

22 **Prove that :** The points A (– 3 , 0) , B (3 , 4) and C (1 , – 6) are the vertices of an isosceles triangle of vertex A , then find the length of the drawn line segment from A perpendicular on $\overline{BC}$

---

23 ABC is a triangle where A (1 , 1) , B (3 , 1) and C (1 , 3)
Prove that : Δ ABC is an isosceles triangle then find its area.

---

24 ABCD is a parallelogram where A (3 , 4) , B (2 , – 1) and C (– 4 , – 3) , find the coordinates of D , take E ∈ $\overrightarrow{AD}$ where AE = 2 AD
What are the coordinates of the point E ? « (– 3 , 2) , (– 9 , 0) »

---

25 ABCD is a quadrilateral , X (2 , 3) , Y (m , 3) , Z (1 , – 1) and L (– 4 , n) are the midpoints of $\overline{AB}$, $\overline{AD}$, $\overline{BC}$ and $\overline{DC}$ respectively.
Find : The value of : m + n « – 4 »

For excellent pupils

26 ABCD is a trapezium in which BC = 2 AD and A (6 , 4) , B (4 , – 2) , C (– 2 , – 4)
Find the coordinates of D where $\overline{BC}$ // $\overline{AD}$
(Hint : Complete the parallelogram ABCE and use it to find D)

5

The slope of the straight line

Interactive test

From the school book

● Remember ● Understand ● Apply ● Problem Solving

1 Choose the correct answer from those given :

1 The slope of the straight line parallel to the X-axis is .... (El-Kalyoubia 18)

(a) – 1 (b) zero (c) 1 (d) undefined.

2 The slope of the straight line parallel to the y-axis is ....

(a) undefined. (b) zero (c) 1 (d) – 1

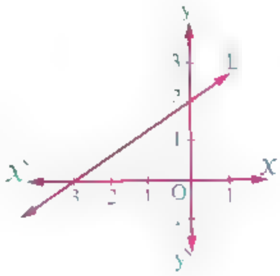
3 If $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = \frac{2}{3}$, then the slope of $\overleftrightarrow{CD}$ = ... (Alex 19)

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{-3}{2}$ (d) $\frac{-2}{3}$

4 If $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = \frac{1}{2}$, then the slope of $\overleftrightarrow{CD}$ = ... (Cairo 19)

(a) 2 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) 2

5 In the opposite figure :

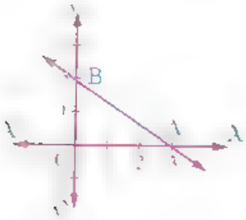


The slope of the straight line L equals .... (Red Sea 17)

(a) $\frac{2}{3}$ (b) $\frac{-2}{3}$

(c) $\frac{3}{2}$ (d) $\frac{-3}{2}$

6 **In the opposite figure :**



The slope of $\overleftrightarrow{AB} = \ldots\ldots\ldots\ldots$ *(Luxor 19)*

(a) $\frac{2}{3}$ (b) $-\frac{2}{3}$

(c) $\frac{3}{2}$ (d) $-\frac{3}{2}$

7 The slope of the straight line that makes with the positive direction of the X-axis a positive angle of measure θ equals .... *(Giza 17)*

(a) $\sin\theta$ (b) $\cos\theta$ (c) $\frac{\sin\theta}{\cos\theta}$ (d) $\sin\theta + \cos\theta$

8 If the slope of a straight line is more than zero , then the type of the positive angle which it makes with the positive direction of X-axis is .... *(Damietta 11)*

(a) zero. (b) acute. (c) right. (d) obtuse.

9 If m_1 and m_2 are the slopes of two perpendicular straight lines , then .... *([illegible])*

(a) $m_1 = m_2$ (b) $m_1 = -m_2$ (c) $m_1 m_2 = -1$ (d) $m_1 m_2 = 1$

10 If m_1 and m_2 are the slopes of two parallel straight lines , then .... *(Port [illegible])*

(a) $m_1 - m_2 = 0$ (b) $m_1 + m_2 = 0$ (c) $m_1 m_2 = 0$ (d) $m_1 - m_2 \neq 0$

11 The slope of the straight line which is parallel to the straight line passing through the two points (2 , 3) , (– 2 , 3) is .... *(Port Said 18)*

(a) undefined. (b) zero. (c) – 4 (d) – 1

12 If the straight line L is perpendicular to the straight line which passes through the two points (– 1 , 2) and (0 , 5) , then the slope of the straight line L = ...

(a) 3 (b) – 3 (c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

13 If m_1 and m_2 are the slopes of two perpendicular straight lines and $m_1 = 0.75$, then $m_2 = \ldots\ldots\ldots\ldots$ *(El-Sharkia 13)*

(a) $-\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $-\frac{4}{3}$ (d) $\frac{3}{4}$

14 If the two straight lines whose slopes are $-\frac{2}{3}$ and $\frac{k}{2}$ are parallel , then k = ... *(Alex. 17 - Matrouh 19)*

(a) $\frac{-3}{4}$ (b) $\frac{1}{3}$ (c) 3 (d) $\frac{-4}{3}$

15 If $-\frac{2}{3}$, $\frac{6}{k}$ are the slopes of two perpendicular straight lines , then k = ... *(Kafr El-Sheikh 19)*

(a) 4 (b) – 9 (c) – 4 (d) 9

16 If the straight line which passes through the two points (X , 5) and (2 , 3) is parallel to the straight line which passes through the two points (3 , 4) and (5 , 2) , then X = ...

(a) 2 (b) – 2 (c) zero (d) 1

17 The straight line which passes through the two points $(-1 , -1)$ and $(4 , 4)$ makes with the positive direction of X-axis a positive angle of measure .... (El Monofia 15 , North Sinai 17)

(a) $30°$ (b) $45°$ (c) $60°$ (d) $135°$

18 If the straight line which passes through the two points $(k , 0)$ and $(0 , 4)$ is perpendicular to the straight line which makes a positive angle of measure $45°$ with the positive direction of X-axis , then $k =$ .... (Aswan 13)

(a) 4 (b) -4 (c) 1 (d) -1

19 If the slope of the straight line L_1 is $\frac{a}{5}$ and the slope of the straight line L_2 is $\frac{-b}{3}$ where $a \neq 0 , b \neq 0$ and $L_1 \perp L_2$, then $a\,b =$ .... (El-Sharkia 19)

(a) $\frac{3}{5}$ (b) $\frac{-3}{5}$ (c) 15 (d) -15

20 ABC is a right-angled triangle at B where $A = (1 , 5)$ and $B = (0 , 1)$, then the slope of $\overleftrightarrow{BC}$ equals ....

(a) -4 (b) $-\frac{1}{4}$ (c) $\frac{1}{4}$ (d) 4

21 ABCD is a parallelogram where $A(-1 , 4)$ and $B(0 , 1)$, then the slope of $\overleftrightarrow{DC} =$ ....

(a) -3 (b) $-\frac{1}{3}$ (c) $\frac{1}{3}$ (d) 3

22 If ABCD is a square whose diagonals $\overline{AC}$ and $\overline{BD}$ where $A(3 , 5)$ and $C(5 , -1)$, then the slope of $\overleftrightarrow{BD} =$ ....

(a) -6 (b) -3 (c) $-\frac{1}{3}$ (d) $\frac{1}{3}$

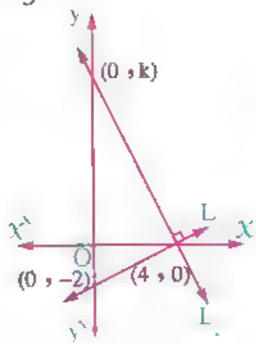
23 **In the opposite figure :**

If $L_1 \perp L_2$

, then $k =$ ....

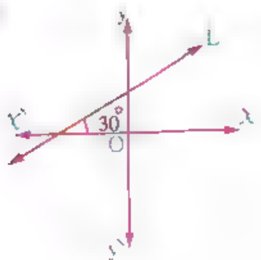
(a) 2 (b) 4

(c) 6 (d) 8



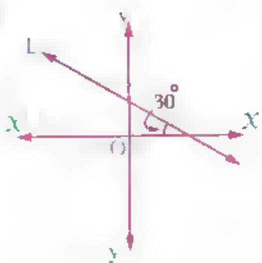
2 Write under each figure the slope of the straight line L :

1



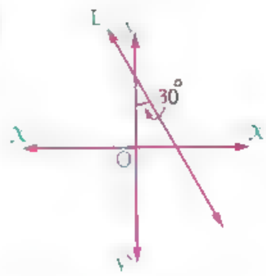
The slope of L is ....

2



The slope of L is ....

3



The slope of L is ....

3 **Find the slope of the straight line which makes with the positive direction of X-axis a positive angle of measure :**

1 zero°	2 30°	3 45°	4 57°
5 60°	6 90°	7 86° 42́	8 135°

4 **Using the calculator , find the measure of the positive angle which the straight line (whose slope is m) makes with the positive direction of X-axis in each of the following cases :**

1 $m = 0.3$ 2 $m = 0.3673$ 3 $m = 1.0246$ 4 $m = \frac{4}{5}$

5 **Prove that :** The straight line which passes through the two points (4 , 2) and (5 , 6) is parallel to the straight line which passes through the two points (0 , 5) and (– 1 , 1)

6 **Prove that :** The straight line passing through the two points A (– 3 , 4) and C (- 3 , 2) is perpendicular to the straight line passing through the two points B (1 , 2) and D (– 3 , 2)

7 **Prove that :** The straight line passing through the two points (2 , – 1) and (6 , 3) is parallel to the straight line that makes a positive angle of measure 45° with the positive direction of the X-axis. *(Alex. 17 – El-Menia 18 – Suez 20)*

8 **Prove that :** The straight line which passes through the two points $\left(4 , 3\sqrt{3}\right)$ and $\left(5 , 2\sqrt{3}\right)$ is perpendicular to the straight line which makes a positive angle of measure 30° with the positive direction of X-axis. [illegible]

9 In the Cartesian coordinates plane if A (1 , 5) , B (X – 1 , 3) , C (4 , 7) and D (2 , 1) are four points satisfying $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$, **find the value of :** X [illegible]

10 If the triangle whose vertices are Y = (4 , 2) , X = (3 , 5) , Z = (– 5 , a) is right-angled at Y , **find the value of :** a *(El-Monofia 17 – Damietta 17 – Assiut 20)* « – 1 »

11 If the straight line $\overleftrightarrow{AB}$ // the y-axis , where A (X , 7) and B (3 , 5) , **then find the value of :** X *(Luxor 19)* « 3 »

12 If the straight line $\overleftrightarrow{CD}$ // the X-axis , where C (4 , 2) and D (– 5 , y) , **find the value of :** y « 2 »

13 If the straight line L_1 passes through the two points (3 , 1) and (2 , k) and the straight line L_2 makes with the positive direction of the X-axis a positive angle whose measure is 45° , then find k if the two straight lines L_1 and L_2 are :

1 parallel 2 perpendicular *(Assiut 17 – Alex. 18 – Aswan 20)* « 0 , 2 »

---

14 Find the measure of the positive angle which the straight line L makes with the positive direction of X-axis if the straight line L passes through the two points (4 , 3) and (2 , – 5) « 75° 57̀ 50̋ »

---

15 Find the measure of the positive angle which the straight line L makes with the positive direction of X-axis if the straight line L passes through the two points (0 , 0) and (2 , – 2) « 135° »

---

16 Find the measure of the positive angle which the straight line L makes with the positive direction of X-axis if the straight line L is perpendicular to the straight lines which passes through the two points (– 2 , 5) and (4 , – 1) « 45° »

---

17 **Prove that :** The points A (1 , 1) , B (2 , 3) and C (0 , – 1) are collinear. *(Cairo 13)*

---

18 If the points (0 , 1) , (a , 3) and (2 , 5) are located on one straight line **, then find the value of :** a *([illegible] – Damietta 19 – Cairo 20)* « 1 »

---

19 If A (1 , 7) , B (– 1 , 5) and C (4 , 2) **, prove that :** $C \notin \overleftrightarrow{AB}$

---

20 If A (– 1 , – 1) , B (2 , 3) and C (6 , 0) **, prove that :** the triangle ABC is a right-angled triangle at B *(Suez 14 – Kafr El Sheikh 17 – Aswan 19)*

---

21 **Prove that :** The points A (– 1 , 1) , B (0 , 5) , C (4 , 2) and D (5 , 6) are the vertices of the parallelogram ABDC *(Luxor 12 – Beni Suef 18)*

---

22 Prove by using the slope that the points A (– 1 , 3) , B (5 , 1) , C (6 , 4) and D (0 , 6) are the vertices of the rectangle ABCD *(B[illegible] 17 – Port Said 18)*

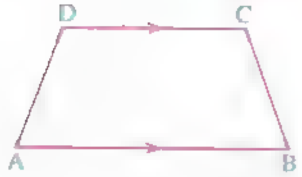
---

23 **Prove that :** The points A (1 , 3) , B (6 , 4) , C (7 , 9) and D (2 , 8) are the vertices of the rhombus ABCD

---

24 **Prove that :** The points A (– 1 , – 1) , B (2 , 3) , C (6 , 0) and D (3 , – 4) are the vertices of a square.

25 **In the drawn figure :**



ABCD is a trapezoid where $\overline{AB} // \overline{CD}$, A (9 , – 2) , B (3 , 2) , C (X , – X) and D (4 , – 3)

Find the coordinates of the point C

(Alex. 14 – Suez 19) « (1 , – 1) »

26 **Prove that :** The points A (4 , 3) , B (7 , 0) and C (1 , – 2) are the vertices of a triangle and if the point D (1 , 2) , then prove that the figure ABCD is a trapezoid and find the ratio between AD and BC « 1 : 2 »

For excellent pupils

27 Find the slope of the straight line which makes with the positive direction of X-axis a positive acute angle whose sine = $\frac{3}{5}$ « $\frac{3}{4}$ »

28 If the points A (1 , 1) , B (3 , 3) , C (0 , – 3 X) and D (X , y) are the vertices of the rectangle ABCD , **find the value of each of :** X and y « – 2 , 4 »

29 ABCD is a rhombus in which : A (3 , 2) , B (4 , k) and C (– 1 , 2)

Find : 1 The value of k « – 3 »

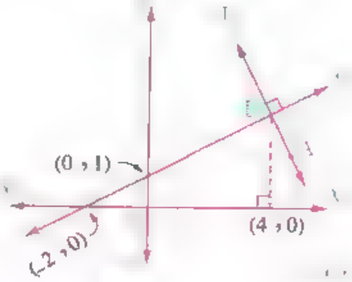
2 The length of $\overline{BD}$ « $6\sqrt{2}$ length unit »

30 **In the opposite figure :**

If $\overleftrightarrow{L_1} \perp \overleftrightarrow{L_2}$

, A $\in L_2$ where A (5 m , m)

, **find :** the value of m



Wonders of numbers

The two digits 8 , 5

- $8 \times 5 = 40$
- $88 \times 5 = 440$
- $888 \times 5 = 4440$
- $8888 \times 5 = 44440$

Try it yourself !

6 The equation of the straight line given its slope and the intercepted part of y-axis

Interactive test

From the school book

● Remember ● Understand ○ Apply ● Problem Solving

1 **Find the slope and the intercepted part of y-axis by each of the following straight lines :**

1. $y = 5x - 3$
2. $2y = 4 - x$
3. $2x - 3y - 6 = 0$
4. $y + x - 1 = 0$
5. $\frac{x}{2} + 3y = 6$
6. $\frac{x}{2} + \frac{y}{3} = 1$ *(Matrouh 19 , El-Kalyoubia 20)*

---

2 **Find the equation of the straight line if :**

1. Its slope = 2 and intercepts from the positive part of y-axis 7 units. *(Damietta 19 , Suez 20)*
2. Its slope = – 1 and intercepts from the positive part of y-axis 3 units.
3. Its slope = $2\frac{1}{2}$ and intercepts from the negative part of y-axis one unit.
4. Its slope = $-\frac{3}{4}$ and intercepts from the negative part of y-axis $2\frac{1}{2}$ units.
5. Its slope = zero and intercepts from the negative part of y-axis 2 units.

---

3 **Find the equation of the straight line :**

1. Passing through the point (3 , 2) and makes with the positive direction of x-axis a positive angle of measure 45° *(El-Sharkia 17)*
2. Which cuts a part of length 3 units from the negative part of y-axis and is parallel to the line whose equation is : $2x - 3y = 6$ *(El-Beheira 11)*
3. Which is perpendicular to the straight line : $3x - 4y + 7 = 0$ and intercepts from the positive part of y-axis a part of length 6 units.

4 Which intercepts a positive part from y-axis of length 5 units and perpendicular to the straight line which passes through the two points $(-2, 1)$ and $(2, 7)$

5 Which intercepts from the positive parts of the coordinate axes «X-axis and y-axis» two parts of lengths 4 and 9 length unit respectively.

(Luxor 17 – Kafr El-Sheikh 18 – El-Kalyoubia 19)

6 Which passes through the point $(2, -1)$ and its slope equals 2

7 Passing through the point $(-2, 3)$ and perpendicular to the straight line whose equation is : $y = \frac{1}{2}X - 5$ *(El-Dakahlia 13)*

8 Passing through the point $(3, -5)$ and it is parallel to the straight line : $X + 2y - 7 = 0$

(Aswan 20)

9 Which passes through the point $(3, 2)$ and is parallel to the straight line passing through the two points $(5, 6)$ and $(-1, 2)$ *(Helwan 09)*

10 Passing through the point $(1, 2)$ and perpendicular to the straight line passing through the two points $A(2, -3)$ and $B(5, -4)$

11 Passing through the point $(2, -2)$ and perpendicular to the straight line which makes a positive angle of measure $45°$ with the positive direction of X-axis.

12 Which passes through the two points $(2, -1)$ and $(1, 1)$

(El-Gharbia 13 – El-Kalyoubia 16)

13 Which passes through the two points $(4, 2)$ and $(-2, -1)$, then prove that it passes through the origin point. *(El-Beheira 17 – Cairo 19)*

14 Whose slope equals the slope of the straight line : $\frac{y-1}{X} = \frac{1}{3}$ and intercepts a negative part of y-axis of 3 length units. *(Damietta 18 – Suez 19)*

15 Which is perpendicular to $\overline{AB}$ from the point A where $A(-3, 6)$ and $B(2, 1)$

16 Which is perpendicular to $\overline{AB}$ from its midpoint where $A(1, 3)$ and $B(3, 5)$

17 Passing through the midpoint of the line segment $\overline{AB}$ where $A(4, 8)$ and $B(-2, 4)$ and parallel to the straight line whose equation is $2y = 4X - 5$

18 Passing through the midpoint of the line segment $\overline{AB}$ where $A(3, 6)$ and $B(-1, 4)$ and perpendicular to the straight line whose equation is $2y - 4X + 11 = 0$ *(Cairo 09)*

19 Passing through the point $(2, 3)$ and intercepts from the positive part of X-axis a part of length 4 units. *(El-Sharkia 18)*

4 Choose the correct answer from those given :

1 The straight line whose equation is : $3y = 2X - 6$, its slope = ... *(El-Sharkia 19)*

(a) 2 (b) $\frac{3}{2}$ (c) 6 (d) $\frac{2}{3}$

2 The slope of the straight line : $x - 5 = 0$ is .... (Damietta 19)

(a) 5 (b) $\frac{1}{5}$ (c) undefined. (d) zero

3 The straight line whose equation is : $3x - 3y + 5 = 0$ makes a positive angle with the positive direction of x-axis , its measure = ... (El-Monofia 11)

(a) 30° (b) 45° (c) 60° (d) 90°

4 The straight line whose equation is : $2x - 3y - 6 = 0$ intercepts from the negative part of y-axis a part of length .... units. (El-Fayoum 13 – Cairo 14 – Qena 17 – El-Kalyoubia 18)

(a) -6 (b) -2 (c) $\frac{2}{3}$ (d) 2

5 The straight line whose equation is : $2x + 5y - 10 = 0$ cuts from the positive part of x-axis a part of length .... units. (El-Dakahlia 11)

(a) $\frac{2}{5}$ (b) 2 (c) $\frac{5}{2}$ (d) 5

6 The equation of the straight line passing through the origin point and its slope = 1 is .... (El Kalyoubia 19)

(a) $y = x$ (b) $y = -x$ (c) $y = 2x$ (d) $y = 0$

7 The equation of the straight line which passes through the origin point and makes with the positive direction of x-axis an angle of measure 60° is .... (El Sharkia 19)

(a) $x = \sqrt{3}\,y$ (b) $y = \sqrt{3}\,x + 2$ (c) $y = 3x$ (d) $y = \sqrt{3}\,x$

8 The equation of the straight line which its slope = $\frac{1}{2}$ and cuts the y-axis at the point (0 , 3) is .... (El-Monofia 19)

(a) $2y = \frac{1}{2}x + 6$ (b) $y = \frac{1}{2}x$ (c) $y = \frac{1}{2}x + 3$ (d) $2y = \frac{1}{2}x + 3$

9 The equation of the straight line which passes through the point (2 , – 3) and is parallel to x-axis is .... (Kafr El-Sheikh 19)

(a) $x = 2$ (b) $y = 3$ (c) $x = -2$ (d) $y = -3$

10 The equation of the straight line which passes through the point (– 5 , 3) and is parallel to y-axis is .... (Beni Suef 19)

(a) $x = -5$ (b) $y = -5$ (c) $y = 3$ (d) $x = 3$

11 The equation of the straight line which intercepts a part of length 4 units from the positive part of y-axis and is parallel to the straight line : $y = 3x + 5$ is ....

(a) $y = 3x + 4$ (b) $y = 4x + 3$ (c) $y = 3x - 4$ (d) $y = -3x + 4$

12 The two straight lines : $y = 3x - 5$ and $2y = 6x + 5$ are .... (Giza 09)

(a) parallel (b) coincident

(c) intersecting and not perpendicular (d) perpendicular

13 If the two straight lines : $3x - 4y - 3 = 0$ and $ky + 4x - 8 = 0$ are perpendicular , then k = ... *(El Beheira 15 – Giza 16 – Red Sea 19)*

(a) – 4 (b) – 3 (c) 3 (d) 4

14 If the two straight lines : $x + y = 5$ and $kx + 2y = 0$ are parallel , then k = ... *(El-Dakahlia 15 – Souhag 16 – Qena 17 – El-Menia 19)*

(a) – 2 (b) – 1 (c) 1 (d) 2

15 If the straight line whose equation is : $y = kx + 5$ is parallel to x-axis , then k = ... *(El-Gharbia 18)*

(a) 0 (b) 1 (c) 2 (d) 3

16 The two straight lines : $y = ax + b$ and $y = cx + d$ are perpendicular , then .... $= -1$ *(El-Gharbia 08 – Souhag 16)*

(a) $a \times d$ (b) $b \times c$ (c) $a \times c$ (d) $b \times d$

17 The straight line passing through the two points (5 , 4) and (1 , 5) is perpendicular to the straight line ....

(a) $4x = 3 - 4y$ (b) $5y + x = 4$ (c) $y = 4x$ (d) $x + 2y = 4$

18 The slope of the straight line whose equation is : $3y = ax - 5$ and passes through the point (20 , 5) is ....

(a) – 1 (b) 1 (c) – 2 (d) $\frac{1}{3}$

19 If the straight line whose equation is : $ax + (2 - a)y = 5$ is parallel to the straight line which passes through (1 , 4) , (3 , 5) , then a = ... *(El-Dakahlia 19 – Kafr El Sheikh 20)*

(a) 3 (b) – 2 (c) 6 (d) 4

20 The area of the triangle in square units which is bounded by the straight lines $3x - 4y = 12$, $x = 0$, $y = 0$ equals .... *(El-Kalyoubia 15 – El-Fayoum 20)*

(a) 6 (b) 7 (c) 12 (d) – 6

21 **In the opposite figure :**

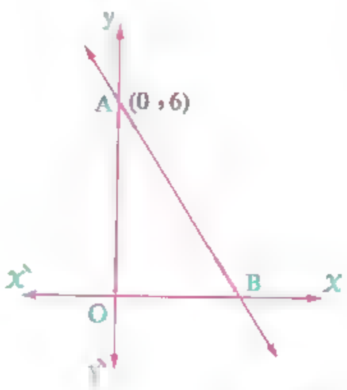
If the area of Δ AOB = 9 square unit , then the equation of $\overleftrightarrow{AB}$ is .... *(El-Monofia 17)*

(a) $y = 2x + 6$

(b) $y = 6 - 2x$

(c) $y = 2x - 6$

(d) $y = \frac{1}{2}x - 6$



22 **In the opposite figure :**

If ABCD is a square

, the equation of the straight line

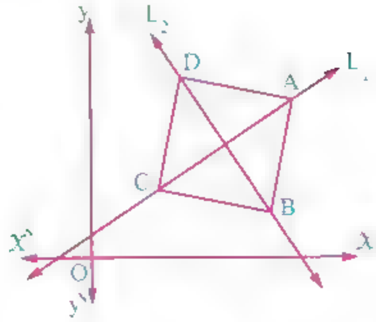
L_1 is : $y = \frac{2}{3} X + 1$

and the equation of the straight line

L_2 is : $y = k X + 14$, then $k =$ ....

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$

(c) $\frac{-2}{3}$ (d) $\frac{-3}{2}$



5 Complete the following :

1 The slope of the straight line whose equation is : $3 X - 4 y - 15 = 0$ is .... and the slope of its perpendicular straight line is ....

2 The straight line whose equation is : $2 X + 3 y - 6 = 0$ cuts the y-axis at the point ....

3 The equation of the X-axis is .... while the equation of the y-axis is ....

4 The equation of the straight line passing through the point (2 , –5) and its slope = zero is ....

5 The equation of the straight line which is parallel to the straight line : $y = 2 X - 3$ and passes through the origin point is ....

6 **In the opposite figure :**

$L_1 // L_2$, AB = 7 length units.

If the equation of L_1 is : $y = 2 X + 4$

, then the equation of L_2 is ....

(El-Sharkia 12)

7 **In the opposite figure :**

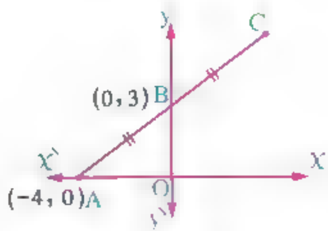
$B \in \overline{AC}$, A (– 4 , 0) , B (0 , 3) and AB = BC

, then :

(1) The point C is (.......... , ...)

(2) In Δ OAB , tan A = ...

(3) The equation of $\overleftrightarrow{AC}$ is y = ... X + ...



(El-Sharkia 11)

6 **Prove that :** The straight line which passes through the two points A (3 , 1) and B (1 , 2) is parallel to the straight line : $2 X + 4 y - 3 = 0$ (El-Sharkia 17)

7 **Prove that :** The straight line whose equation is : $2 X + y + 8 = 0$ is perpendicular to the straight line passing through A (2 , 3) and B (– 2 , 1) (Aswan 12)

8 Find the equations of the two straight lines which pass through the point $(-3 , 2)$ and parallel to the two axes.

---

9 Find the measure of the positive angle which is made by the straight line : $3X - 2y + 6 = 0$ with the positive direction of the X-axis , then find the coordinates of its intersection point with the y-axis.

---

10 **If the straight line whose equation is :** $2X - 3y - 6 = 0$ cuts the X-axis at the point A and the y-axis at the point B , **find :** *(El-Sharkia 13)*

1. The coordinates of the two points A and B
2. The equation of the straight line passing through the midpoint of $\overline{AB}$ and parallel to the y-axis.

---

11 If the straight line which passes through the two points $(2 , -1)$ and $(5 , 1)$ is parallel to the straight line whose equation is : $aX + 3y + 5 = 0$, **find the value of :** a [illegible]

---

12 If the straight line which passes through the two points $(5 , 2)$ and $(6 , -3)$ is perpendicular to the straight line whose equation is : $5y - aX + 3 = 0$, **find the value of :** a

---

13 If A $(2 , -3)$ and B $(5 , y)$, find the value of y if the straight line $\overleftrightarrow{AB}$ is parallel to the straight line L : $3y - 4X + 1 = 0$

---

14 If the straight line : $y - (2k - 1)X = 7$ and the straight line which makes with the positive direction of the X-axis a positive angle of measure 45° are parallel

, then find the value of : k *(El-Sharkia 16)* « 1

---

15 Find the equation of the axis of symmetry of $\overline{XY}$, where X $(3 , -2)$ and Y $(-5 , 6)$

(El-Dakahlia 12 , Port Said 14)

---

16 A $(5 , -6)$, B $(3 , 7)$ and C $(1 , -3)$, find the equation of the straight line which passes through the point A and the midpoint of $\overline{BC}$ [illegible]

---

17 ABC is a triangle whose vertices are A = $(0 , 6)$, B = $(5 , -1)$ and C = $(-2 , 1)$

Find the equation of the straight line passing through the vertex A and perpendicular to $\overline{BC}$

---

18 ABC is a triangle in which A $(1 , 2)$, B $(5 , -2)$ and C $(3 , 4)$, D is the midpoint of $\overline{AB}$ and $\overleftrightarrow{DE}$ // $\overleftrightarrow{BC}$ and intersects $\overline{AC}$ at E

Find : The length of $\overline{DE}$ The equation of $\overleftrightarrow{DE}$ [illegible]

---

19 ABCD is a square in which : A $(5 , 4)$ and C $(-1 , 6)$

Find the equation of $\overleftrightarrow{BD}$ *(El Monofia 15)*

20 ABCD is a rhombus , M is the point of intersection of its two diagonals where A (1 , 3) and C (6 , 0) , find the equation of the straight line which passes through the two points B and D *(Aswan 09)*

21 Find the equation of the straight line passing through two points A (2 , 3) and B (– 1 , – 3) Show that for any point C (2 k + 1 , 4 k + 1) , then $C \in \overleftrightarrow{AB}$ *(El Dakahlia 14)*

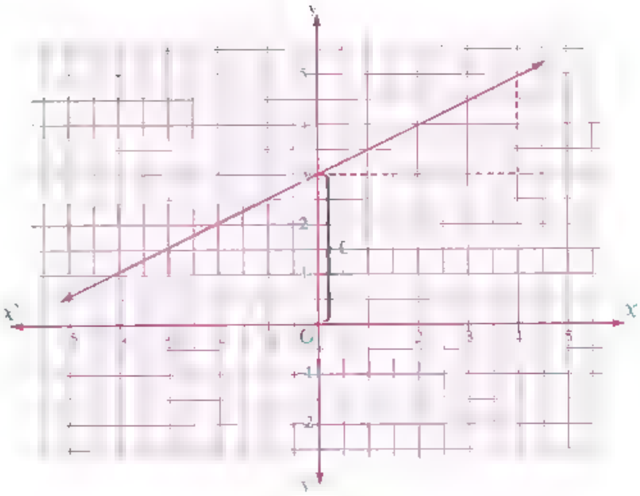
22 **Draw the straight line in each of the following cases :**

1. The slope $= -\frac{1}{2}$ and intercepts from the positive part of y-axis a part of one unit.
2. The slope = 2 and intercepts from the negative part of y-axis a part of 3 length units.
3. Intercepts from the positive parts of the two axes (X-axis , y-axis) two parts of lengths 2 and 3 length units respectively.

23 Find the slope of the straight line : $y - 2X - 3 = 0$, then find the length of the intercepted part from y-axis , also draw this line. *(Helwan 11)*

24 **From the opposite graph , find :**

1. The slope of the straight line (m)
2. The intercepted part of y-axis (c)
3. The equation of the straight line given (m) and (c)
4. The length of the intercepted part of X-axis.
5. The area of the triangle bounded by the straight line and the two axes.



25 **The opposite table represents a linear relation :**

X	1	2	3
$y = f(X)$	1	3	a

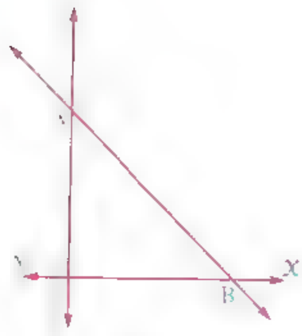
1. Find the equation of the straight line.
2. Find the length of the intercepted part from y-axis.
3. Find the value of a

(El Kalyoubia 13 – Alexandria 15)

26 The opposite figure represents $\overleftrightarrow{AB}$ whose equation is $y = kx + c$ and cuts from the two axes two equal parts and passes through the point (2 , 3)

Find : 1 The values of k , c

2 The area of the triangle ABO



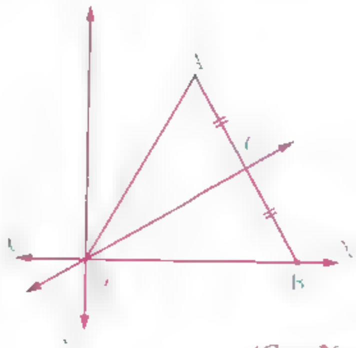
(El-Dakahlia 19)

27 **In the opposite figure :**

ABO is an equilateral triangle ,

C is the midpoint of $\overline{AB}$

Find the equation of the straight line $\overleftrightarrow{OC}$



(Giza 20)

28 **In the opposite figure :**

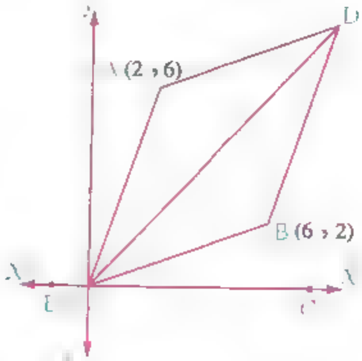
The points A (2 , 6) , O (0 , 0) , B (6 , 2) and D are the vertices of a rhombus.

Find :

1 The coordinates of the point D

2 The equation of $\overleftrightarrow{OD}$

3 m (∠ DOE)



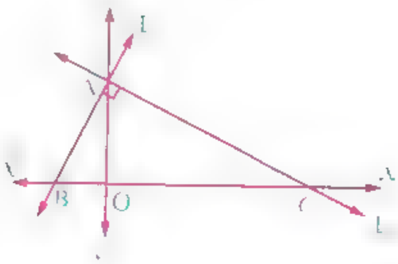
(El-Sharkia 14)

29 **In the opposite figure :**

If $L_1 \perp L_2$

and the equation of L_1 is : $2x - y + 2 = 0$

, find the equation of the straight line L_2



30 **In the opposite figure :**

$\overleftrightarrow{AB}$ cuts y-axis at the point A (0 , 8) and cuts X-axis at the point B

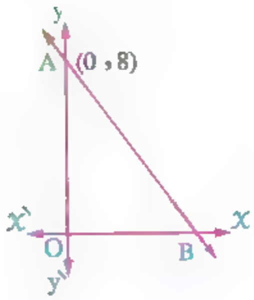
If tan (∠ ABO) = $\frac{4}{3}$, **find :**

1 **First :** m (∠ BAO)

Second : The coordinates of B

2 **First :** The slope of $\overleftrightarrow{AB}$

Second : The equation of the straight line passing through the point O and perpendicular to $\overleftrightarrow{AB}$



(El-Sharkia 13)

31 **In the opposite figure :**

The point C is the midpoint of $\overline{AB}$ where C (4 , 3) :

1 **Find the coordinates of each of :**

O , A and B

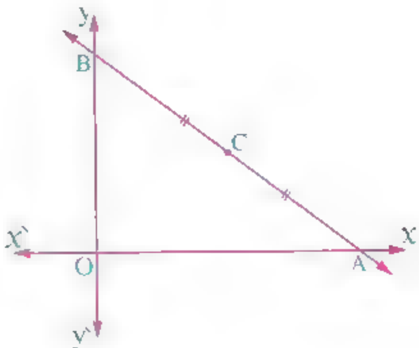
2 **Find the length of each of :**

$\overline{OA}$, $\overline{OB}$, $\overline{CA}$, $\overline{CB}$ and $\overline{CO}$

3 **Find the slope of each of :**

$\overleftrightarrow{AB}$, $\overleftrightarrow{OC}$, $\overleftrightarrow{OA}$ and $\overleftrightarrow{OB}$

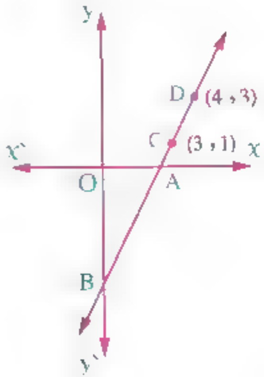
4 **Find the equation of each of :** $\overleftrightarrow{AB}$ and $\overleftrightarrow{CO}$



32 **In the opposite figure :**

The straight line $\overleftrightarrow{AB}$ passes through the two points C (3 , 1) , D (4 , 3) and intersects the two axes at A , B respectively.

Find the lengths of $\overline{AO}$, $\overline{OB}$, where O is the origin point.



33 **In the opposite figure :**

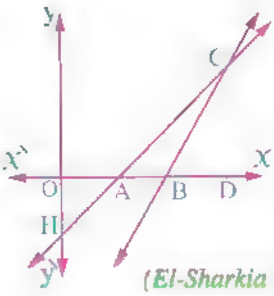
O is the origin point , A , B , D ∈ X-axis ,

the slope of $\overleftrightarrow{BC}$ = $\sqrt{3}$, the equation of $\overleftrightarrow{AC}$ is : $X - y = 3$

Find : 1 The slope of $\overleftrightarrow{AC}$ and the length of $\overline{OH}$

2 m (∠ CBD) and m (∠ CAD)

3 m (∠ ACB)



(El-Sharkia 16)

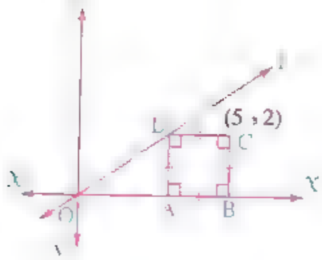
34 **In the opposite figure :**

ABCD is a square

, D $\in$ the straight line L

and C (5 , 2)

Find the equation of the straight line L

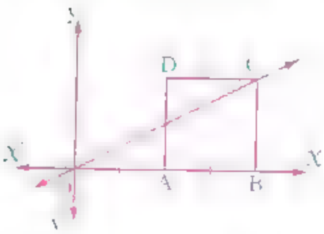


35 **In the opposite figure :**

ABCD is a square

, OA = AB

Find the equation of $\overleftrightarrow{OC}$



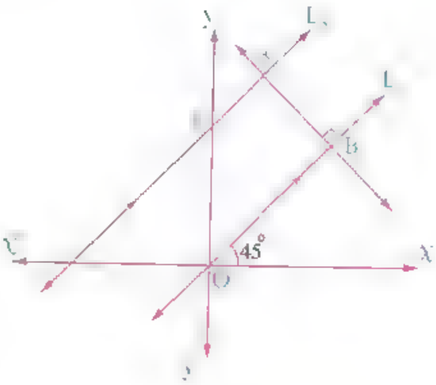
36 **In the opposite figure :**

L_1 and L_2 are parallel lines , L_1 makes with the positive direction of X-axis an angle of measure 45° and passes through the origin point O , A $\in L_2$ where A (1 , 5) , $\overleftrightarrow{AB} \perp L_1$, L_2 intersects the y-axis at the point C

Find : 1 The equation of the straight line L_1

2 The equation of the straight line L_2

3 The length of $\overline{AB}$



(El-Sharkia 15)

Life Applications

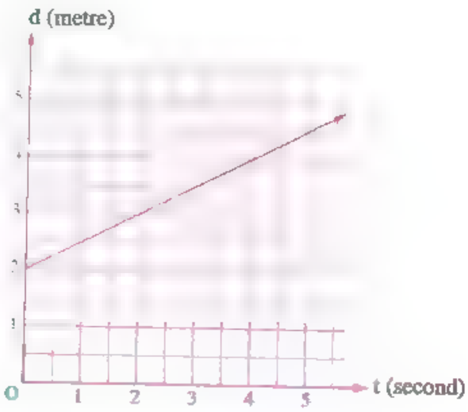
37 The opposite graph represents the motion of a particle moving with uniform velocity (v) where the distance (d) is measured in metre and the time (t) in seconds.

Find :

1 The distance at the beginning of the motion.

2 The velocity of the particle.

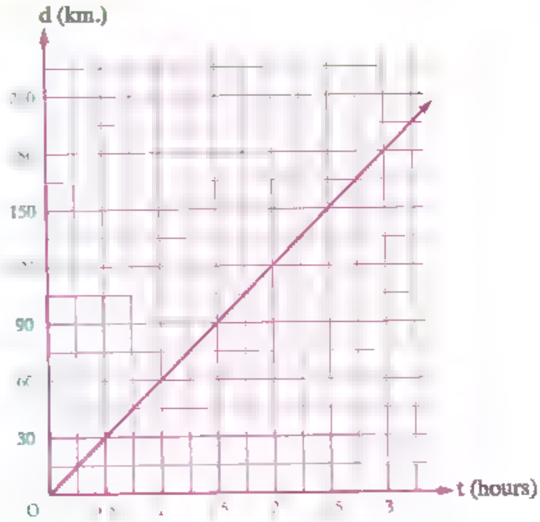
3 The equation of the straight line representing the motion of the particle.



4 The covered distance after 4 seconds from the beginning of the motion.

5 The time in which the particle covers a distance of 3.5 metres from the beginning of the motion.

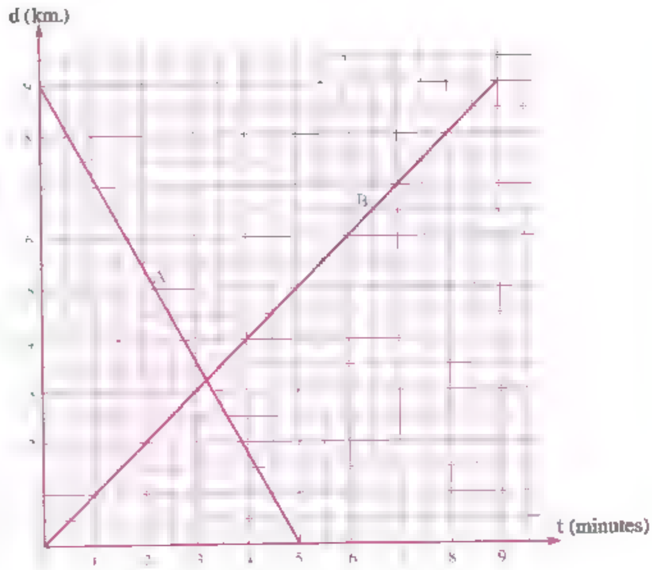
38 The opposite graph represents the relation between the distance the car covers (d in km.) and the time the car covers in (t in hour).



Find :

1. The covered distance after 90 minutes.
2. The time which the car took to cover a distance of 150 km.
3. The velocity of the car.
4. The equation of the straight line which represents the relation between the distance (d) and the time (t).

39 The opposite graph represents the relation between the covered distance (d) in km. and the time (t) in minutes for the two objects A and B in the two ends of a straight road moving in two opposite directions :



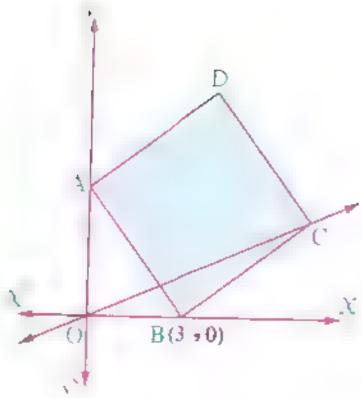
1. Did A and B move at the same time ?
2. After how many minutes did A and B meet together ?
3. What is the velocity of A ?
4. Write the equation of the straight line which represents the relation between the distance (d) and the time (t) of the motion of the object B

For excellent pupils

40 **In the opposite figure :**

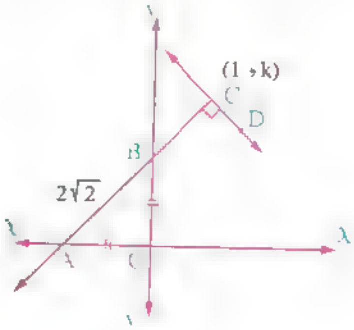
If the area of the square ABCD = 25 square units

, **find :** the equation of $\overleftrightarrow{CO}$



41 **From the opposite figure :**

Find : the equation of $\overleftrightarrow{CD}$



Wonders of numbers

From wonders of the number 37 is that if it multiplied by 3 or one of its multiples up to 27, every time you get number consisting of the same digits.

- $37 \times 3 = 111$
- $37 \times 6 = 222$
- $37 \times 9 = 333$

Try it yourself !

Summary of Unit 5

- If A (x_1 , y_1) and B (x_2 , y_2) , then :
  - The length of $\overline{AB}$

    $= \sqrt{(\text{difference between } x\text{-coordinates})^2 + (\text{difference between y-coordinates})^2}$

    $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
  - The midpoint of $\overline{AB} = \left(\frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2}\right)$
  - The slope of $\overleftrightarrow{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \tan \theta$

    (where θ is the measure of the positive angle which $\overleftrightarrow{AB}$ makes with the positive direction of the x-axis)
- If L_1 and L_2 are two straight lines of slopes m_1 and m_2 respectively, then :
  - $L_1 \parallel L_2$ if $m_1 = m_2$ and vice versa
  - $L_1 \perp L_2$ if $m_1 \times m_2 = -1$ and vice versa
- If the equation of a straight line is in the form : $y = m x + c$, then :
  - The slope of the straight line $= m$
  - The length of the intercepted part from y-axis $= |c|$

    and the straight line passes through the point $(0 , c)$
- If the equation of a straight line is in the form : $a x + b y + c = 0$, then :
  - The slope of the straight line $= \frac{-\text{coefficient of } x}{\text{coefficient of } y} = \frac{-a}{b}$
  - The length of the intercepted part from y-axis $= \left|\frac{-c}{b}\right|$

    and the straight line cuts y-axis at the point $\left(0 , \frac{-c}{b}\right)$
- The equation of the straight line which passes through the origin point O $(0 , 0)$ is $y = m x$, where m is the slope of the straight line.
- The equation of x-axis is $y = 0$
- The equation of y-axis is $x = 0$

Exams on Unit Five

Model 1

Answer the following questions :

1 **Choose the correct answer from those given :**

1. The distance between the point $(-7, -3)$ and y-axis is .... length unit.

   (a) -7 (b) -3 (c) 7 (d) 3

2. The point $(0, 4)$ bisects the distance between the two points $(-1, 1)$ and (x, y), then the point (x, y) is ....

   (a) $(1, 9)$ (b) $(-1, 9)$ (c) $\left(-\frac{1}{2}, \frac{3}{2}\right)$ (d) $(-1, 3)$

3. If the two straight lines whose slopes are $\frac{-1}{4}$ and $4k$ are perpendicular, then $k =$ ....

   (a) 1 (b) 4 (c) -4 (d) $\frac{1}{4}$

4. The slope of the straight line whose equation is : $x - 5 = 0$ is ....

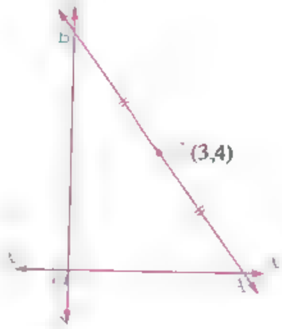
   (a) 5 (b) $\frac{1}{5}$ (c) undefined (d) 0

5. **In the opposite figure :**

   $C(3, 4)$ is the midpoint of $\overline{AB}$, then $OA =$ .... length unit

   (a) 3 (b) 4

   (c) 6 (d) 8



6. If the straight line which passes through the two points $(x, -1)$ and $(4, 2)$ is parallel to the straight line which passes through the two points $(3, 4)$ and $(-3, -2)$, then $x =$ ....

   (a) -3 (b) 2 (c) 7 (d) 1

---

2 **[a]** ABCD is a quadrilateral where $A(-1, 3)$, $B(5, 1)$, $C(7, 4)$ and $D(1, 6)$

Prove that : The figure ABCD is a parallelogram.

[b] Find the equation of the straight line which passes through the point $(3, 4)$ and is perpendicular to the straight line whose equation is : $5x - 2y + 7 = 0$

3 [a] If the point A (5 , 2) lies on the circle whose centre is M (1 , – 1) , **find :**

1 The area of the circle in terms of π

2 The equation of the straight line passing through the two points A and M

[b] If the points : A (3 , 2) , B (4 , – 3) , C (– 1 , – 2) and D (– 2 , 3) are the vertices of a rhombus , **find :**

1 The coordinates of the point of intersection of its diagonals.

2 The area of the rhombus ABCD

---

4 [a] If the distance between the point (x , 5) and the point (6 , 1) equals $2\sqrt{5}$ length unit , then find the value of x

[b] **Prove that :** The points A (– 3 , 0) , B (3 , 4) and C (1 , – 6) are the vertices of an isosceles triangle at vertex A , then find the length of the drawn line segment from A perpendicular to $\overline{BC}$

---

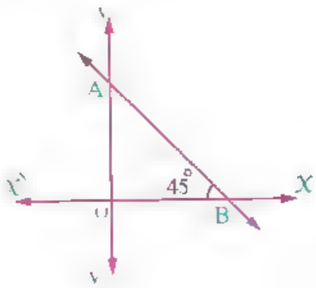
5 [a] Find the equation of the straight line which passes through the two points (3 , 7) , (6 , 13)

[b] **In the opposite figure :**

$\overleftrightarrow{AB}$ intercepts from the positive part of the x-axis 3 units

, m (∠ ABO) = 45°

Find the equation of $\overleftrightarrow{AB}$



Model 2

Answer the following questions :

1 **Choose the correct answer from those given :**

1 If ABCD is a rectangle where : A (– 4 , – 1) and C (4 , 5) , then the length of $\overline{BD}$ = ... length unit.

(a) 10 (b) 6 (c) 5 (d) 4

2 If (4 , – 3) is the midpoint of $\overline{AB}$ where A (3 , – 4) , then B is ....

(a) (5 , – 2) (b) (2 , 5) (c) (5 , 2) (d) (3.5 , – 3.5)

If the two straight lines : $3x - 4y - 3 = 0$ and $kx + 3y - 8 = 0$ are perpendicular , then $k =$ ....

(a) -4 (b) -3 (c) 3 (d) 4

The straight line whose equation is : $2x - 3y - 6 = 0$ intercepts from y-axis a part of length .... units.

(a) -6 (b) -2 (c) $\frac{2}{3}$ (d) 2

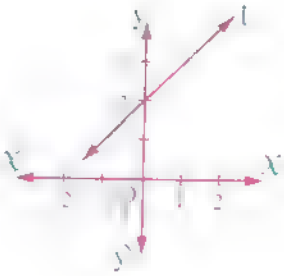
The equation of the straight line which passes through the point $(2 , -3)$ and is parallel to x-axis is ....

(a) $x = 2$ (b) $y = -3$

(c) $x = -2$ (d) $y = 3$

6 **In the opposite figure :**

Which of the following represents the equation of the straight line L ?



(a) $y = x$ (b) $y = 2$

(c) $y + x = 2$ (d) $y - x = 2$

---

2 **[a]** If ABCD is a square where :

A $(2 , 4)$, B $(-3 , 0)$ and C $(-7 , 5)$, **find :**

1 The coordinates of D

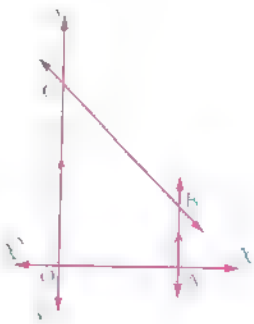
2 The area of the square ABCD

[b] In the opposite figure :

$\overleftrightarrow{AB}$ is parallel to y-axis

, $\overleftrightarrow{BC}$ has equation $y = -x + 3$

, the point B $(2 , 1)$



Find : 1 The length of $\overline{BC}$

2 The area of the figure OABC

3 $m(\angle OCB)$

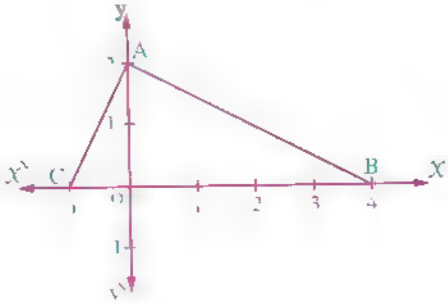
3 [a] **Prove that :** The points A $(-2 , 5)$, B $(3 , 3)$ and C $(-4 , 2)$ are not collinear.

[b] If A $(x , 3)$, B $(3 , 2)$ and C $(5 , 1)$ and AB = BC , **then find the value of :** x

4 [a] **In the opposite figure :**

A triangle ABC is drawn in the orthogonal Cartesian coordinates plane

Prove that : Δ ABC is a right-angled triangle , then find its area



[b] ABC is a triangle in which : A $(1 , 2)$, B $(5 , -2)$ and C $(3 , 4)$

Find the equation of the straight line passing through the vertex A and perpendicular to $\overleftrightarrow{BC}$

5 [a] ABCD is a quadrilateral in which : A $(0 , 6)$, B $(-1 , 3)$, C $(5 , 1)$ and D $(6 , 4)$

Prove by using the slope that the figure ABCD is a rectangle.

[b] If the axis of symmetry of $\overline{CD}$ is passing through the point A $(6 , m)$ where C $(3 , 1)$ and D $(-3 , 7)$, **then find the value of :** m

Wonders of numbers

Choose a number from 1 to 9 , multiply it by 3 , add 3 to the product , and multiply the result by 3 once again "use calculator" Find the sum of the digits of the product.

The answer is always 9.

A Research Project

On Unit Five

Project aims:

- Finding the distance between two points in the coordinates plane.
- Finding the coordinates of the midpoint of a line segment.
- Calculating the perimeter and area of a triangle.
- Finding the slope of a straight line.
- Finding the equation of a straight line.
- Associating geometry with history.

Do a research project on the following topic :

"Analytic geometry is a branch of mathematics that uses a coordinates system to study geometry".

Discuss the following points using available resources :

- Write a short note on the scientist , René Descartes who discovered the Cartesian coordinates and about his achievements in the field of mathematics.
- Draw the two axes of coordinates on a sheet of graph paper.
- On the coordinates plane , locate three points that represent the vertices of an isosceles triangle , then find :

1. The perimeter of this triangle.
2. The area of this triangle.
3. The slope of each side of this triangle.
4. The equation of the straight line that contains each side of this triangle.

Choose the correct answer from those given :

1 The number of diagonals of the hexagon is .... *(Qena 20)*

(a) 6 (b) 3 (c) 12 (d) 9

2 The two angles of base of an isosceles triangle are .... *(Alexandria 16 – North Sinai 17)*

(a) congruent. (b) supplementary.

(c) vertically opposite angles. (d) corresponding.

3 The measure of an exterior angle of an equilateral triangle is ....

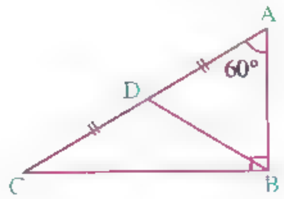
(Alex. 17 – Beni Suef 18 – Kafr El Sheikh 19 – Cairo 20)

(a) 60° (b) 150° (c) 120° (d) 30°

4 The number of axes of symmetry of the isosceles triangle equals .... *(Matrouh 17 – Alex. 18)*

(a) 0 (b) 1 (c) 2 (d) 3

5 **In the opposite figure :**



If m (∠ ABC) = 90° , m (∠ A) = 60°

and $\overline{BD}$ is a median in Δ ABC , then m (∠ DBC) = ...

(a) 20° (b) 30°

(c) 60° (d) 45°

6 The triangle whose side lengths are 5 cm. , 5 cm. , ... is an isosceles triangle. *(El Menia 17)*

(a) 9 cm. (b) 10 cm. (c) 11 cm. (d) 12 cm.

7 The triangle whose side lengths are 5 cm. , 12 cm. and 13 cm., its area = ... cm^2.

(Matrouh 18)

(a) 30 (b) 32.5 (c) 78 (d) 144

8 In any triangle , the sum of the lengths of any two sides is .... the length of the third side. (El Fayoum 18 , El-Menia 19)

(a) greater than (b) smaller than (c) equal to (d) half

9 The point of concurrence of the medians of the triangle divides the median in the ratio of .... from the base. (El-Fayoum 18)

(a) 1 : 3 (b) 2 : 1 (c) 3 : 1 (d) 1 : 2

10 The sum of the measures of the accumulative angles at a point equals .... (Assiut 18 , El-Fayoum 19)

(a) 90° (b) 180° (c) 270° (d) 360°

11 If ABCD is a square , then m (∠ CAB) = ... (Alex. 17 , El-Beheira 18)

(a) 90° (b) 45° (c) 60° (d) 30°

12 If the lengths of the diagonals of a rhombus are 6 cm. , 10 cm. , then its area equals .... cm^2 (Kafr El-Sheikh 17)

(a) 30 (b) 60 (c) 15 (d) 10

13 The image of the point (– 4 , 5) by the translation (2 , – 3) is ....

(a) (– 2 , – 2) (b) (2 , – 2) (c) (2 , 2) (d) (– 2 , 2)

14 The image of the point (– 2 , 5) by reflection in X-axis is .... (Ismailia 16)

(a) (– 2 , – 5) (b) (2 , 5) (c) (2 , – 5) (d) (5 , – 2)

15 The quadrilateral whose diagonals are equal in length and perpendicular is the .... (Beni Suef 20)

(a) square. (b) rhombus. (c) rectangle. (d) parallelogram.

16 The volume of the cuboid whose dimensions are $\sqrt{2}$, $\sqrt{3}$, $\sqrt{6}$ centimetres equals .... cm^3 (South Sinai 16)

(a) $2\sqrt{6}$ (b) $3\sqrt{6}$ (c) $3\sqrt{2}$ (d) 6

17 If 3 , 7 , ℓ are lengths of sides of a triangle , then ℓ may be equal to ....

(a) 3 (b) 4 (c) 7 (d) 10

18 Δ ABC is a triangle , (∠ B) = 3 m (∠ A) = 90° , then m (∠ C) = ...

(a) 30° (b) 45° (c) 60° (d) 90°

19 ABC is a triangle , if m (∠ B) > m (∠ C) , then .... (Suez 16)

(a) AC – AB < 0 (b) AC – AB ≤ 0 (c) BC ≤ AB (d) AC – AB > 0

20 The circumference of the circle with diameter length 14 cm. is .... cm. $\left(\text{where } \pi = \frac{22}{7}\right)$ *(El-Fayoum 17)*

(a) 7 (b) 22 (c) 44 (d) 14

21 If m (∠ X) = m (∠ Y) , ∠ X , ∠ Y are complementary

, then m (∠ X) = ... *(North Sinai 17)*

(a) 90° (b) 60° (c) 45° (d) 30°

22 If $\overleftrightarrow{XY}$ is the axis of symmetry of $\overline{AB}$, then XA .... XB *(Suez 20)*

(a) > (b) < (c) = (d) ⊥

23 ABCD is a parallelogram in which m (∠ A) + m (∠ C) = 200°

, then m (∠ B) = ... *(Ismailia 17 – Alex. 18 – Suez 19)*

(a) 50° (b) 80° (c) 100° (d) 160°

24 If ABCD is a parallelogram , then AB + CD = ... *(Suez 18)*

(a) 2 AC (b) 2 BC (c) 2 BD (d) 2 CD

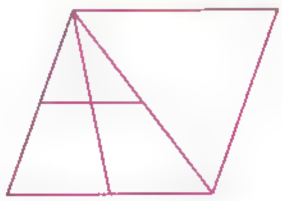
25 If $L_1 // L_2$, $L_3 \perp L_1$, $L_4 \perp L_2$, then .... *(El Beheira 17)*

(a) $L_2 // L_3$ (b) $L_1 // L_4$ (c) $L_3 // L_4$ (d) $L_3 \perp L_4$

26 The number of triangles in the opposite figure = ... triangles. *(New Valley 16)*

(a) 5 (b) 6

(c) 7 (d) 8

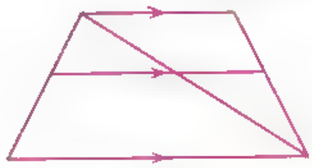


27 **In the opposite figure :** *(Luxor 17)*

The number of trapeziums = ...

(a) 2 (b) 3

(c) 4 (d) 5

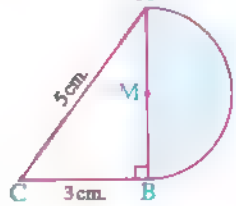


28 **In the opposite figure :** *(Seuz 16)*

$\overline{AB}$ is a diameter of a circle , then the surface area of the shaded shape = ... cm.^{2}

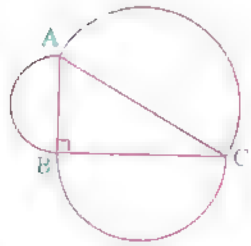
(a) 4 π (b) 16 π

(c) 2 π (d) 9 π



29 **In the opposite figure :**

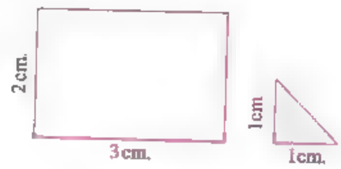
ABC is a right-angled triangle at B , what is the area of the semicircle drawn on the hypotenuse $\overline{AC}$ if the areas of the two semicircles drawn on $\overline{AB}$ and $\overline{BC}$ are 36 and 64 square units respectively ?



(a) 80 square units (b) 96 square units (c) 100 square units (d) 120 square units

30 **In the opposite figure :**

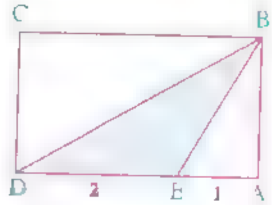
The number of the coloured right-angled triangles needed to cover the rectangle surface completely is ....



(a) 4 (b) 6 (c) 8 (d) 12

31 **In the opposite figure :**

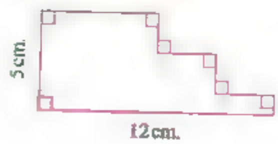
If AE : ED = 1 : 2 , then the ratio between the area of Δ BED and the rectangle ABCD is ....



(a) 1 : 2 (b) 1 : 3 (c) 2 : 3 (d) 2 : 5

32 **In the opposite figure :** *(Aswan 18)*

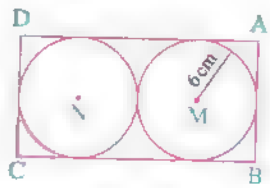
The perimeter of the figure = ... cm.



(a) 17 (b) 22
(c) 29 (d) 34

33 **In the opposite figure :**

Two circles M and N inside a rectangle , the radius length of each one is 6 cm. , then the area of the rectangle = ... cm^2

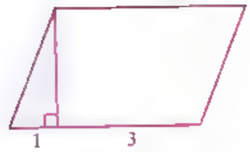


(a) 288 (b) 252 (c) 216 (d) 144

34 The opposite figure represents quarter a circle with radius 2 cm. long , then its perimeter = ... cm. *(Giza 19)*

(a) 2π (b) 5π
(c) $\pi + 4$ (d) $4\pi + 4$

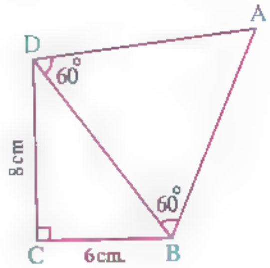
35 **In the opposite figure :**



If the base of the parallelogram is divided by the ratio 1 : 3 , then the ratio between the area of the coloured triangle and the area of the parallelogram is ....

(a) 1 : 3 (b) 1 : 6 (c) 1 : 8 (d) 1 : 9

36 **In the opposite figure :**

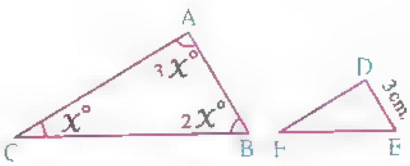


The perimeter of the figure = ... cm.

(a) 44 (b) 34

(c) 24 (d) 14

37 **In the opposite figure :**



If $\Delta ABC \sim \Delta DEF$

, DE = 3 cm.

, then EF = ... cm. *(Luxor 16)*

(a) 3 (b) 9 (c) 4 (d) 6

38 **In the opposite figure :**

If the side length of the square = 10 cm.

, then the area of the circle = ... cm^2

(a) 100π (b) 25π

(c) 50π (d) 40π

39 **In the opposite figure :**

If $A \in \overline{EF}$, $B \in \overline{EF}$, $m(\angle C) = 90°$

, then $x + y =$ ....

(a) 90° (b) 180°

(c) 270° (d) 360°

EL-MOASSER

By a group of supervisors

NOTEBOOK

- Accumulative Tests
- Final Revision
- Final Examinations

3rd PREP.

FIRST TERM

Maths

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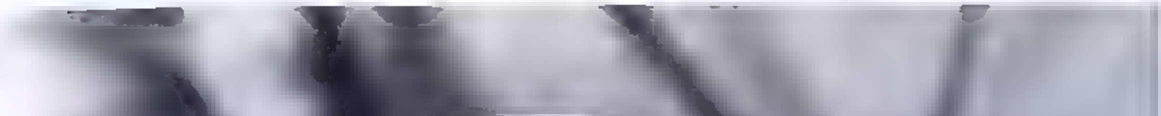
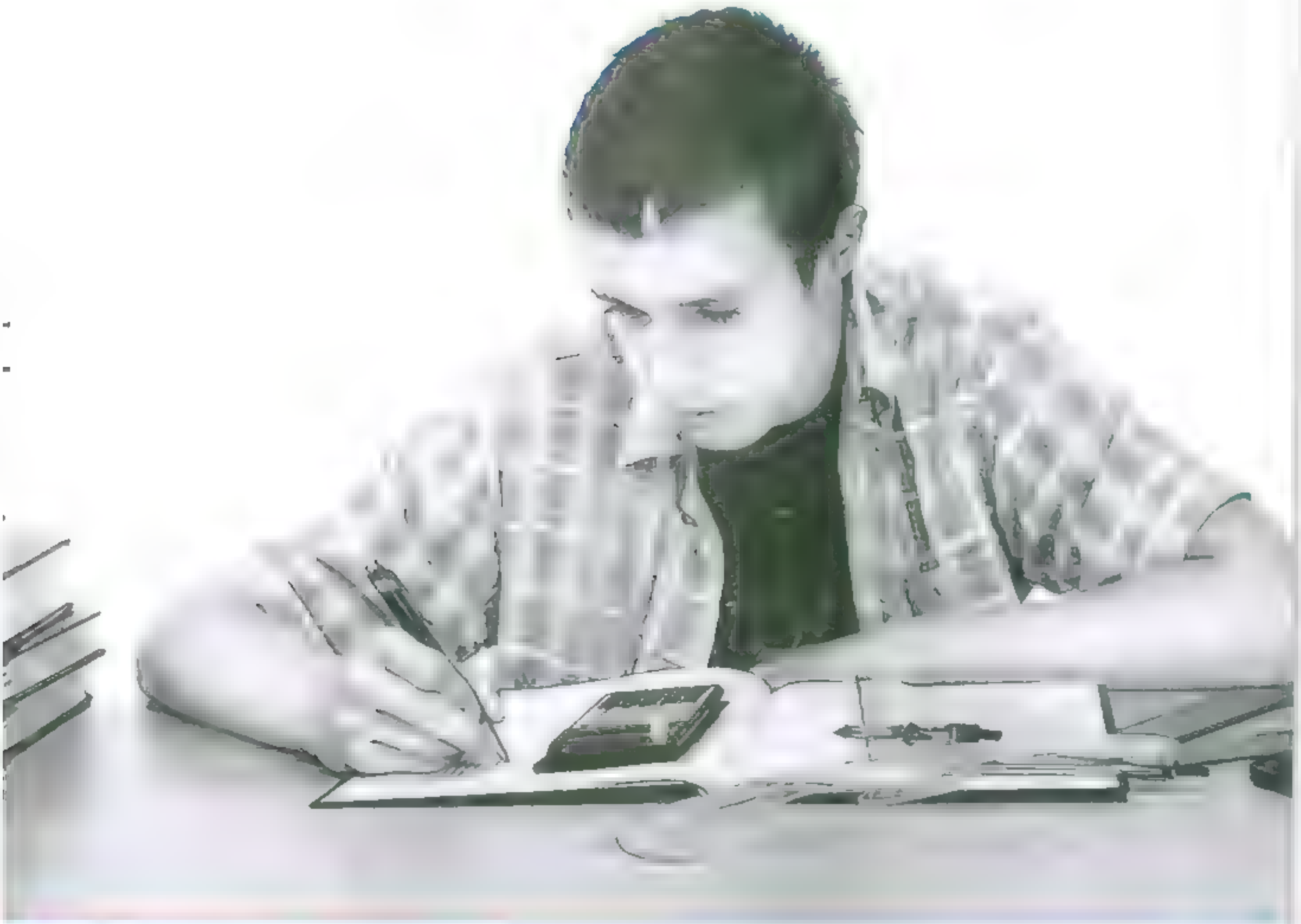


Accumulative Tests

on Algebra and Statistics



Accumulative test 1 on lesson 1 – unit 1

1 **Choose the correct answer from those given :**

1. If the point $(x , 7)$ lies on the y-axis , then $5x + 1 =$ .... « Port Said 17 »

(a) zero (b) 1 (c) 5 (d) 6

2. If $x \in \mathbb{R}_-$, then the point $\left(-x , \sqrt[3]{x}\right)$ lies in the .... quadrant. « El-Monofia 20 »

(a) first (b) second (c) third (d) fourth

3. If $n(X^2) = 4$, $n(X \times Y) = 6$, then $n(Y) =$ .... « South Sinai 17 »

(a) 2 (b) 3 (c) 4 (d) 6

4. If $(3 , 5) \in \{3 , 6\} \times \{m , 8\}$, then $m =$ .... « Kafr El-Sheikh 18 »

(a) 3 (b) 5 (c) 6 (d) 8

5. If $(2^x , 27) = (32 , y^3)$, then $\frac{x}{y} =$ .... « El-Gharbia 17 »

(a) $\frac{3}{5}$ (b) $\frac{5}{3}$ (c) $\frac{32}{27}$ (d) $\frac{27}{32}$

6. If $(x \quad 3 , 2 - x)$ lies in the fourth quadrant , then $x =$ .... « El-Dakahlia 20 »

(a) 4 (b) 3 (c) 2 (d) 1

7. If $X = \{1\}$, $Y = \{3\}$, then $n(X \times Y) =$ .... « Assiut 19 »

(a) $\{(1 , 3)\}$ (b) $\{(3 , 1)\}$ (c) 3 (d) 1

8. The volume of a sphere of radius length 3 cm. is .... cm^3 « Giza 18 »

(a) 4π (b) 36π (c) 36 (d) 27π

---

2 If $X = \{2\}$, $Y = \{3 , 4 , 5\}$, **find :**

1. $X \times Y$ 2. $n(Y^2)$ 3. X^2 « Cairo 20 »

---

3 If $(x - 1 , 29) = (4 , y^3 + 2)$, **then find the value of :** $x + 2y$ « Red Sea 17 »

Accumulative test 2 till lesson 2 – unit 1

1 Choose the correct answer from those given :

1 If the relation $\mathbb{R} = \{(4, 3), (1, 3), (2, 5)\}$, then $\mathbb{R}$ represents a function its range is .... « El-Kalyoubia 17 »

(a) $\{4, 1, 2\}$ (b) $\{4, 1, 2, 3, 5\}$ (c) $\{3, 5\}$ (d) $\mathbb{N}$

2 If $X = \{2\}$, then $X^2 =$ .... « El-Kalyoubia 20 »

(a) 4 (b) $\{4\}$ (c) $(2, 2)$ (d) $\{(2, 2)\}$

3 If $X = \{1, 2\}$, $Y = \{5, 6\}$, then $(5, 1) \in$ .... « El-Monofia 18 »

(a) $Y \times X$ (b) X^2 (c) $X \times Y$ (d) Y^2

4 The ordered pair (x^2, y^2), where $x \neq 0$, $y \neq 0$ lies in the .... quadrant. « Qena 20 »

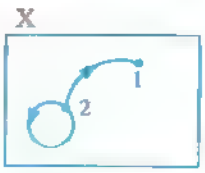
(a) first (b) second (c) third (d) fourth

5 If $a + b = ab = 5$, then $a^2 b + ab^2 =$ .... « Kafr El-Sheikh 18 »

(a) 25 (b) 20 (c) 15 (d) 10

6 If $X = \{1, 2\}$, then the arrow diagram that represents a function on X is .... « Luxor 17 »

(a)



(b)

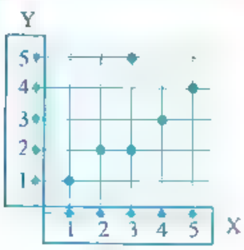
(c)

(d)

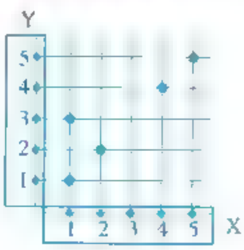
7 If $\mathbb{R}$ is a function on X where $X = \{1, 3, 5\}$, and $\mathbb{R} = \{(a, 3), (b, 1), (1, 5)\}$, then $a + b =$ .... « El-Dakahlia 18 »

(a) 4 (b) 6 (c) 8 (d) 2

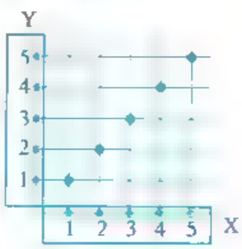
8 Which of the following relations is a function from X to Y ? « Port Said 17 »



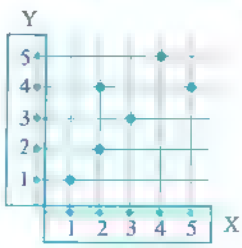
(a)



(b)



(c)



(d)

2 If $X = \{1, 2, 3, 4\}$, $Y = \{2, 3\}$, $Z = \{7, 2\}$, **find :** « El Sharkia 18 »

1. $(X \cap Y) \times Z$
2. $(X - Y) \times Z$

3 If $X = \{\frac{1}{2}, 1, 0, -\frac{1}{2}, -1\}$, $Y = \{1, 2, 0, -1, -2\}$ and R is a relation from X to Y , where "a R b" means "a is the multiplicative inverse of b" for each $a \in X$, $b \in Y$, write R and represent it by an arrow diagram and show if R is a function or not , and why ?

« El Sharkia 19 »

Accumulative test 3 till lesson 3 - unit 1

1 **Choose the correct answer from those given :**

1 The function d : d $(X) = X^2 - (X-3)^2$ is of the .... degree. « El Dakahlia 20 »

(a) zero (b) first (c) second (d) third

2 The following functions are polynomial functions of the first degree except $f : f(X) =$ .... « Ismailia 18 »

(a) $\frac{3}{5} X + 2$ (b) $\sqrt{2} X + 1$ (c) $X + (X + 5)$ (d) $X\left(\frac{1}{X} + 1\right)$

3 If $(X-3)^{zero} = 1$, then $X \in$ .... « El-Monofia 18 »

(a) $\mathbb{R}$ (b) $\mathbb{R} - \{3\}$ (c) $\mathbb{R} - \{4\}$ (d) $\mathbb{R} - \{1\}$

4 If $f : f(X) = (2a - 2) X^3 + 3 X^2 + X + 2$ is a polynomial function of the second degree , then a = ... « El-Sharkia 19 »

(a) zero (b) 2 (c) 3 (d) 1

5 If (5 , b – 7) lies on X-axis , then b = ... « Alexandria 18 »

(a) 2 (b) 5 (c) 7 (d) 12

6 The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = X(2X^3 + 5X)$ is polynomial of the .... degree. « Red Sea 16 »

(a) first (b) second (c) third (d) fourth

7 If the function $f : X \longrightarrow Y$, then the range of the function $f \subset$ .... « Cairo 17 »

(a) X × Y (b) X (c) Y × X (d) Y

8 If X = {3} , n (Y) = 5 , then n (X × Y) = ... « Cairo 19 »

(a) 1 (b) 5 (c) 8 (d) 15

---

2 If X = {3 , 5 , 7} , $Y = \{X : X \in \mathbb{N} , 8 < X < 30\}$ and the set of the function $f : X \longrightarrow Y$ is as follows $f = \{(3 , 9) , (5 , 15) , (7 , 21)\}$

1 Find the domain of the function f

2 Write the rule of the function f « El-Dakahlia 19 »

---

3 If $f(X) = 3X + b$, $f(4) = 13$, **find the value of :** b « Alexandria 17 »

Accumulative test 4 till lesson 4 – unit 1

1 **Choose the correct answer from those given :**

1 If f is a function such that $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(X) = 3$, then $\frac{f(6)}{f(\text{zero})} = \ldots\ldots$

« El-Dakahlia 17 »

(a) 6 (b) 1 (c) 3 (d) undefined

2 If $X - y = 5$, $X + y = 1$, then $X^2 - y^2 = \ldots\ldots$ « Red Sea 19 »

(a) $\frac{1}{25}$ (b) 1 (c) 5 (d) 25

3 If the straight line which represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = 2X + 3c^3$ passes through the origin point , then $c = \ldots\ldots$ « El Sharkia 18 »

(a) 3 (b) -3 (c) zero (d) $\frac{-3}{2}$

4 The linear function $f : f(X) = 2X - 1$ is represented by a straight line cutting the y-axis at the point $\ldots\ldots$ « Matrouh 20 »

(a) $(0 , 1)$ (b) $(0 , -1)$ (c) $(1 , 0)$ (d) $(-1 , 0)$

5 The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = aX + b$ represents a linear function on condition $a \in \ldots\ldots$ « El Gharbia 20 »

(a) $\mathbb{R}$ (b) $\mathbb{R}_+$ (c) $\mathbb{R} - \{0\}$ (d) $\mathbb{R}_-$

6 If $f(X) = 7X - 1$, then $f(1) = \ldots\ldots$ « El-Beheira 16 »

(a) 7 (b) 6 (c) -1 (d) 3

7 If the point $(b - 4 , 2 - b)$ lies in the third quadrant , then $b = \ldots\ldots$ « Ismailia 19 »

(a) 2 (b) 3 (c) 4 (d) 6

8 If the point $(a , 3)$ lies on the straight line which represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = 4X - 5$, then $a = \ldots\ldots$ « New Valley 20 »

(a) 2 (b) 3 (c) 4 (d) 5

---

2 Represent graphically the function $f : f(X) = 2 - X^2$, $X \in [\ 3 , 3]$ **and from the graph deduce :**

1 The coordinates of the vertex of the curve. 2 The equation of the axis of symmetry.

3 The maximum value of the function. « Souhag 20 »

---

3 If $X = \{1 , 5 , 6\}$, $Y = \{5\}$, $Z = \{2 , 3\}$, **find :**

1 $n(X \times Z)$ 2 $(Y \cap X) \times (X - Y)$ « Cairo 17 »

Accumulative test 5 till lesson 1 – unit 2

1 **Choose the correct answer from those given :**

1 If 3 a = 8 b , then a : b = ... « Assiut 20 »

(a) – 8 : 3 (b) 8 : 3 (c) 3 : 8 (d) – 3 : 8

2 If a , X , b , 2 X are proportional , then a : b = ... « Damietta 16 »

(a) 2 : 1 (b) 1 : 2 (c) 1 : 3 (d) 1 : 4

3 If $\frac{9}{a^2} = \frac{4}{b^2}$ (where a ≠ 0 , b ≠ 0) , then $\frac{a}{b}$ = ... « Port Said 17 »

(a) $\frac{2}{3}$ (b) $\pm \frac{3}{2}$ (c) $\pm \frac{2}{3}$ (d) $\pm \frac{4}{9}$

4 If f (3 X) = 6 , then f (– 2) = ... « El-Fayoum 19 »

(a) – 12 (b) – 3 (c) 6 (d) – 18

5 If X = {5} , Y = {3} , then n (X × Y) = ... « Suez 16 »

(a) 8 (b) 15 (c) 2 (d) 1

6 If the quantities 2 , 3 , 6 , X – 1 are proportional , then X = ... « El Monofia 18 »

(a) 18 (b) 9 (c) 20 (d) 10

7 If $\frac{a}{3} = \frac{b}{4}$, then 4 a – 3 b = ... « Suez 18 »

(a) zero (b) 3 (c) 5 (d) 7

8 If $X^2 + y^2 = 6$, $X y = 5$, then $(X + y)^2$ = ... « El Gharbia 20 »

(a) 16 (b) ± 16 (c) 11 (d) ± 11

2 If $f(X) = X^2 - \sqrt{2}\, X$, $g(X) = X + 1$

1 **Find :** $f(3) + 3\, g\left(\sqrt{2}\right)$

2 **Prove that :** $f\left(\sqrt{2}\right) = g(-1)$ « Beni Suef 20 »

3 Find the number which if it is added to the two terms of the ratio 3 : 5 , it will be 1 : 2

« Cairo 19 »

Accumulative test 6 till lesson 2 – unit 2

1 **Choose the correct answer from those given :**

1 If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = m$ (where $m \in \mathbb{R}^*$) , then $\frac{a\,c\,e}{b\,d\,f} = \cdots\cdots$ «El Kalyoubia 18»

(a) m (b) 3 m (c) m^3 (d) $3\,m^3$

2 If $\frac{a}{5} = \frac{b}{3} = \frac{c}{4} = \frac{a+b-c}{X}$, then the value of $X = \cdots\cdots$ «Suez 17»

(a) 3 (b) 4 (c) 5 (d) 6

3 If $\frac{a}{2} = \frac{b}{3} = \frac{c}{5}$, then each ratio is equal to $\cdots\cdots$ «El-Fayoum 19»

(a) $\frac{a+b+c}{3}$ (b) $\frac{a+2\,b-c}{3}$ (c) $\frac{a\quad b+c}{10}$ (d) $\frac{a-b}{5}$

4 If $\frac{a}{3} = \frac{b}{5}$, then $5\,a - 3\,b + 4 = \cdots\cdots$ «El-Monofia 19»

(a) 3 (b) 4 (c) 5 (d) 6

5 The ratio between the area of a square of side length ℓ and the area of a square of side length $3\,\ell$ equals $\cdots\cdots$ «Qena 20»

(a) 1 : 3 (b) 3 : 1 (c) 1 : 9 (d) 9 : 1

6 If $\frac{a}{b} = \frac{c}{d} = \frac{h}{m}$, then $\frac{a+c+h}{b+d+m} = \cdots\cdots$ «El-Sharkia 20»

(a) $\frac{a}{b} + \frac{c}{d} + \frac{h}{m}$ (b) $\frac{c}{h}$ (c) $\frac{c}{a}$ (d) $\frac{c}{d}$

7 If $2\,a + 2\,b + c = 36$ and $a + b = 15$, then the value of $c = \cdots\cdots$ «Ismailia 16»

(a) 3 (b) 6 (c) 10 (d) 21

8 If $X \times Y = \{(1 , 2) , (1 , 3) , (1 , 4)\}$, then $n(X) + n(Y^2) = \cdots\cdots$ «El Dakahlia 17»

(a) 3 (b) 4 (c) 6 (d) 10

2 If $\frac{a}{2\,X+y} = \frac{b}{3\,y-X} = \frac{c}{4\,X+5\,y}$

, **prove that :** $\frac{a+2\,b}{7} = \frac{4\,b+c}{17}$ «El-Kalyoubia 19»

3 If $\frac{a}{4} = \frac{b}{3}$, **find the value of :** $\frac{a\,b+a^2}{a\,b-b^2}$ «El-Sharkia 20»

Accumulative test 7 till lesson 3 unit 2

1 **Choose the correct answer from those given :**

1) If a , 2 , 4 , b are in continued proportion , then a + b = ... « El-Dakahlia 20 »

(a) 2 (b) 4 (c) 6 (d) 9

2) The middle proportional between 3 and 27 is .... « Ismailia 20 »

(a) 9 (b) – 9 (c) ± 9 (d) 1

3) If 7 , X , $\frac{1}{y}$ are in continued proportion , then X^2 y = ... « Port Said 18 »

(a) 5 (b) 9 (c) 7 (d) 12

4) If 3 , 6 , X are proportional , then X = ... « Kafr El-Sheikh 18 »

(a) $\frac{1}{2}$ (b) 2 (c) 9 (d) 12

5) If the point (2 , a – 1) lies on the straight line which represents the function $f : f(X) = 4X - 5$, then a = ... « El Gharbia 17 »

(a) 4 (b) 1 (c) 3 (d) 2

6) If 2 , 6 , X + 15 are proportional quantities , then X = ... « Luxor 16 »

(a) 1 (b) 2 (c) 3 (d) 4

7) [2 , 7] –]2 , 7[= .... « Beni Suef 18 »

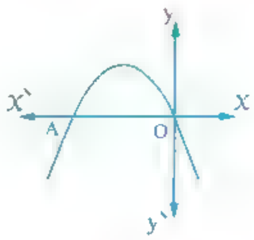
(a) ∅ (b) {2} (c) {7} (d) {2 , 7}

8) **In the opposite figure :**

The curve of a quadratic function , A (– 4 , 0)

, then the equation of the axis

of symmetry is X = ...



(a) 1 (b) – 1

(c) – 2 (d) 0 « El-Dakahlia 19 »

2 If a , b , c , d are in continued proportion

, **prove that :** $\frac{ab - cd}{b^2 - c^2} = \frac{a + c}{b}$ « El Monofia 20 »

3 If a : b : c = 4 : 5 : 3

, **prove that :** $\frac{a - b + c}{a + b - c} = \frac{1}{3}$ « Kafr El-Sheikh 16 »

1 Choose the correct answer from those given :

1 If $X y = 7$, then $y \propto$ .... « El-Menia 18 »

(a) X (b) $\frac{7}{y}$ (c) $\frac{1}{X}$ (d) $\frac{1}{y}$

2 If y varies inversely with X and $X = \sqrt{3}$ when $y = \frac{2}{\sqrt{3}}$, then the constant proportional equals .... « New Valley 20 »

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) 2 (d) 6

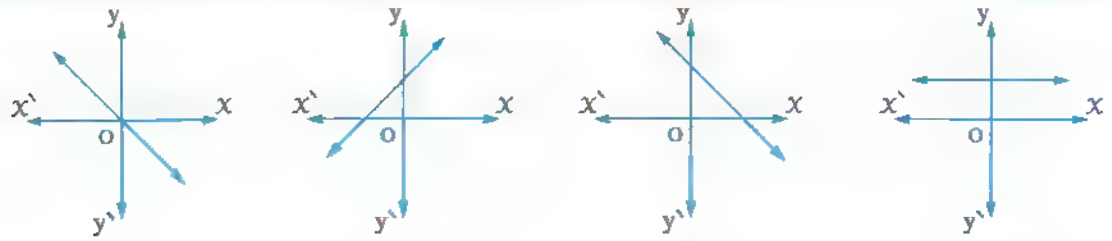
3 If $X^2 - 4 X y^2 + 4 y^4 = 0$, then $X \propto$ .... « El-Sharkia 17 »

(a) y (b) y^2 (c) $\frac{1}{y}$ (d) $\frac{1}{y^2}$

4 If $y = m X$ where m is a constant $\neq 0$, which of the following statements is false ? « El Sharkia 19 »

(a) $y \propto X$ (b) $X \propto y$ (c) $X = \frac{1}{m} y$ (d) $X \propto \frac{1}{y}$

5 The graph representing the direct variation between X and y is .... « Giza 17 »



(a) (b) (c) (d)

6 If $1 < X < 3$, $X \in \mathbb{R}$, then $(3 X - 1) \in$ .... « El-Monofia 20 »

(a) $[2 , 8[$ (b) $[2 , 8]$ (c) $]2 , 8[$ (d) $\{2 , 8\}$

7 If $f(X) = 3$, then $f(3) + f(-3) =$ .... « Giza 19 »

(a) zero (b) 1 (c) -6 (d) 6

8 If (X , y) lies in the second quadrant , then $X y$ .... zero « Suez 18 »

(a) $=$ (b) $>$ (c) $<$ (d) $\geq$

2 If $y = 3 - k$ where $k \propto \frac{1}{X}$ and $y = 5$ when $X = 1$, find the relation between X and y and calculate the value of y when $X = 3$ « El-Monofia 18 »

3 If $\frac{3a - 2c}{3b - 2d} = \frac{a}{b}$, prove that :

a , b , c , d are proportional quantities. « El-Sharkia 18 »

1 Choose the correct answer from those given :

1 The difference between the greatest value and the smallest value in a set of individuals is called .... « El-Sharkia 18 »

(a) the range. (b) the arithmetic mean.
(c) the median. (d) the standard deviation.

2 The relation which represents a direct variation between X and y is .... « El-Gharbia 20 »

(a) $X y = 5$ (b) $y = 3 - X$ (c) $\frac{X}{3} = \frac{y}{5}$ (d) $\frac{X}{3} = \frac{4}{y}$

3 If $\sum(X - X)^2 = 48$ of a set of values and the number of these values = 12 , then σ = ... « Cairo 17 »

(a) – 4 (b) – 2 (c) 2 (d) 4

4 If 18 is the greatest value of a set of individuals and the range = 6 , then the smallest value of this set is .... « El-Monofia 17 »

(a) 8 (b) 12 (c) 14 (d) 36

5 The most common measure of dispersion and the most accurate is .... « Damietta 19 »

(a) the median. (b) the arithmetic mean.
(c) the mode. (d) the standard deviation.

6 If $17 X + 8 = 11$, then $17 X + 11$ = ... « Ismailia 19 »

(a) 8 (b) 11 (c) 14 (d) 17

7 If $\frac{a + 2b}{a - b} = \frac{2}{3}$, then $\frac{b}{a}$ = ... « El-Monofia 20 »

(a) $\frac{1}{8}$ (b) 8 (c) $\frac{-1}{8}$ (d) – 8

8 If all individuals are equal in values , then .... « El-Sharkia 16 »

(a) $\overline{X} = 0$ (b) $\sigma = 0$ (c) $X - \overline{X} > 0$ (d) $X - \overline{X} < 0$

2 The following frequency distribution shows the ages of 20 persons :

Ages in years	15	20	22	23	25	30	Total
Number of persons	2	3	5	5	1	4	20

Calculate the mean and the standard deviation of ages. « Damietta 17 »

3 If a , b , c , d are proportional quantities

, **prove that :** $\frac{a + b}{b} = \frac{c + d}{d}$ « El-Gharbia 19 »

Final Revision

on Algebra and Statistics

Revision for the important rules of Algebra and Statistics

First Algebra

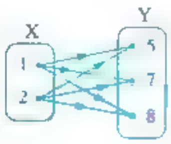
Remember: The Cartesian product of two finite sets and representing it

If $X = \{1, 2\}$, $Y = \{5, 7, 8\}$, then :

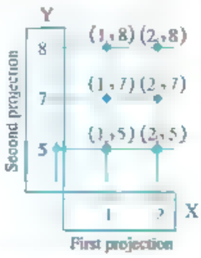
$X \times Y$

is the set of all ordered pairs whose first projection of each of them belongs to X and the second projection of each of them belongs to Y

i.e. $X \times Y = \{(1, 5), (1, 7), (1, 8), (2, 5), (2, 7), (2, 8)\}$



The arrow diagram

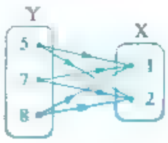


The graphical diagram (The Cartesian diagram)

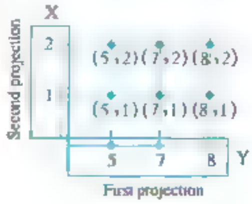
$Y \times X$

is the set of all ordered pairs whose first projection of each of them belongs to Y and the second projection of each of them belongs to X

i.e. $Y \times X = \{(5, 1), (5, 2), (7, 1), (7, 2), (8, 1), (8, 2)\}$



The arrow diagram

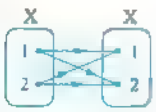


The graphical diagram (The Cartesian diagram)

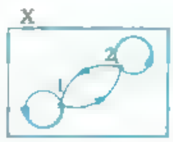
$X \times X$

is the set of all ordered pairs whose first projections and second projections belong to X

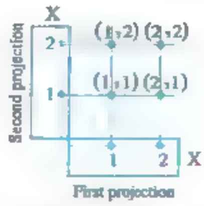
i.e. $X \times X = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$



The arrow diagram



The arrow diagram



The graphical diagram (The Cartesian diagram)

Remarks

(1) $X \times Y \neq Y \times X$, where $X \neq Y$

(2) $n(X \times Y) = n(Y \times X) = n(X) \times n(Y)$ where n is the number of elements

(3) $n(X \times X) = n(X^2) = [n(X)]^2$

(4) $X \times \varnothing = \varnothing \times X = \varnothing$

Remember **The relation and its representing**

- The relation from the set X to the set Y is a connecting joining some or all the elements of X with some or all the elements of Y
- If R is a relation from the set X to the set Y , then :

1. R is a set of ordered pairs where the first projection of each belongs to X and the second projection belongs to Y
2. $R \subset X \times Y$
3. The relation can be represented by an arrow diagram or by a Cartesian diagram (graphically)

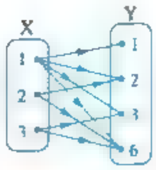
- If R is a relation from X to X , then R is a relation on X and $R \subset X \times X$

Example

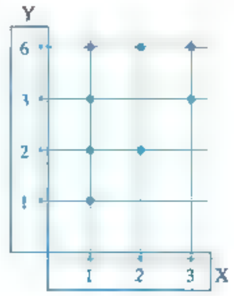
If $X = \{1, 2, 3\}$, $Y = \{1, 2, 3, 6\}$ and R is a relation from X to Y where "a R b" means "a is a factor of b" for each $a \in X$, $b \in Y$, then write R and represent it by an arrow diagram and a Cartesian diagram.

Solution

$R = \{(1, 1), (1, 2), (1, 3), (1, 6), (2, 2), (2, 6), (3, 3), (3, 6)\}$



The arrow diagram



The Cartesian diagram

Remember **The function**

- **A relation from X to Y is said to be a function if :**

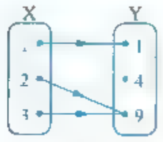
1. Each element of the set X appears only once as a first projection in one of the ordered pairs of the relation.
2. Each element of the set X has one and only one arrow going out of it to one element of Y in the arrow diagram which represents the relation.
3. Each vertical line has one and only one point lying on it of the points which represent the relation , in the Cartesian diagram which represents the relation.

- **If f is a function from the set X to the set Y is written as $f : X \longrightarrow Y$, then :**

1. X is called the domain of the function f
2. Y is called the codomain of the function f
3. The set of images of the elements of the set X by the function f is called the range of the function f which is a subset of the codomain Y

For example :

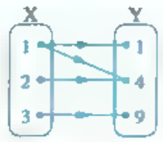
If $X = \{1, 2, 3\}$, $Y = \{1, 4, 9\}$, then the following diagrams show some of the relations from X to Y and we note which of the following relations represent a function from X to Y and which does not represent :



Note : Going out only one arrow from each element of the elements of X

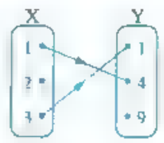
Then : The relation is a function from X to Y

- The domain $= \{1, 2, 3\}$
- The range $= \{1, 9\}$



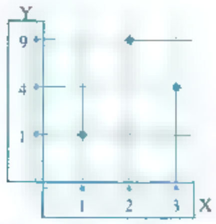
Note : Going out two arrows from the element 1 in X

Then : The relation is not a function from X to Y



Note : There are not arrows going out from the element 2 in X

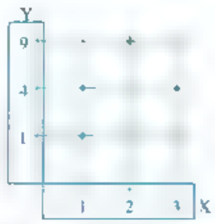
Then : The relation is not a function from X to Y



Note : Each vertical line has only one point lying on it

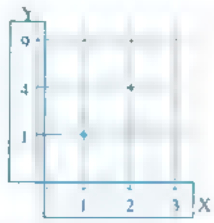
Then : The relation is a function from X to Y

- The domain $= \{1, 2, 3\}$
- The range $= \{1, 4, 9\}$



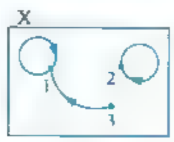
Note : There are two points lying on the vertical line at the element 1 in X

Then : The relation is not a function from X to Y



Note : There is not a point lying on the vertical line at the element 3 in X

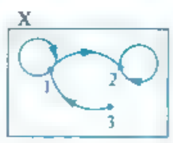
Then : The relation is not a function from X to Y



Note : Going out only one arrow from each element of the elements of X

Then : The relation is a function on X

- The domain $= \{1, 2, 3\}$
- The range $= \{1, 2\}$



Note : Going out two arrows from the element 1 in X

Then : The relation is not a function on X

Note : There are not arrows going out from the element 3 in X

Then : The relation is not a function on X

Remember **The polynomial functions**

The polynomial function is a function whose rule is a term or an algebraic expression in condition that the following should be identified :

1. Each of the domain and the codomain of the function is the set of real numbers.
2. The power (The index) of the variable X in any of its terms is a natural number with noticing that : the degree of the function is the highest power of the variable X

For example :

- The function $f : f(X) = 3$ is a polynomial function of zero degree.
- The function $f : f(X) = 2X + 1$ is a polynomial function of the first degree.
- The function $f : f(X) = X^3 - 5X^2 + 1$ is a polynomial function of the third degree.

While :

The function $f : f(X) = \frac{1}{X^2} + X^2$ is not a polynomial function because : $\frac{1}{X^2} = X^{-2}$

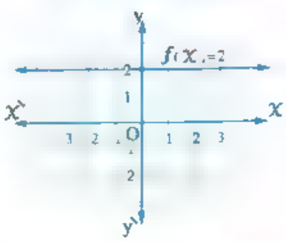
i.e. The index of the symbol X is not a natural number.

Remember **The graphical representation of the polynomial function**

1 The constant function

The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = b$, $b \in \mathbb{R}$ is represented by a straight line parallel to X-axis and intersects y-axis at the point $(0 , b)$

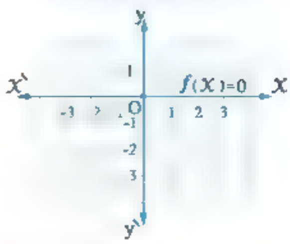
$\boxed{f : f(X) = 2}$



The straight line is above X-axis and passes through the point $(0 , 2)$

(is of zero degree)

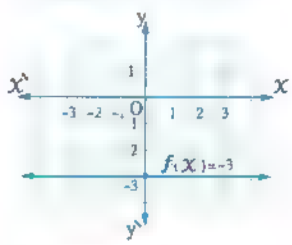
$\boxed{f : f(X) = 0}$



The straight line is coincident with X-axis and passes through the point $(0 , 0)$

(has not degree)

$\boxed{f : f(X) = -3}$



The straight line is below X-axis and passes through the point $(0 , -3)$

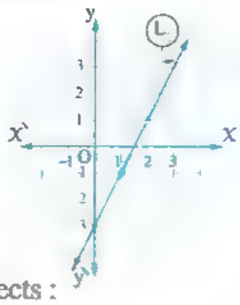
(is of zero degree)

2 The linear function

The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = aX + b$, $a \in \mathbb{R} - \{0\}$, $b \in \mathbb{R}$ is called a linear function (function of the first degree) and is represented by a straight line intersecting y-axis at (0 , b) and X-axis at $\left(\frac{-b}{a}, 0\right)$

$\boxed{f : f(X) = 2X - 3}$

X	0	1	2
$f(X)$	−3	−1	1

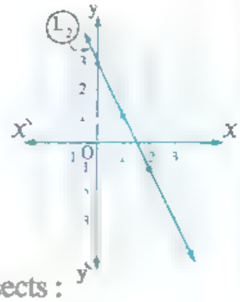


The straight line L_1 intersects :

- X-axis at $\left(1\frac{1}{2}, 0\right)$
- y-axis at (0 , − 3)

$\boxed{f : f(X) = 3 - 2X}$

X	0	1	2
$f(X)$	3	1	−1



The straight line L_2 intersects :

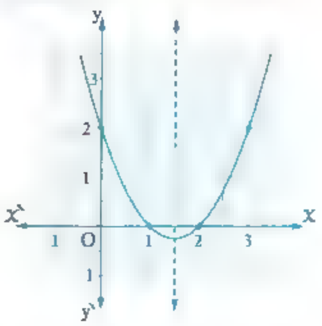
- X-axis at $\left(1\frac{1}{2}, 0\right)$
- y-axis at (0 , 3)

3 The quadratic function

The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = aX^2 + bX + c$, a , b and $c \in \mathbb{R}$, $a \neq 0$ is called a quadratic function and it is a polynomial function of the second degree and it is represented by a curve whose vertex is $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

$\boxed{f : f(X) = X^2 - 3X + 2 , X \in [0 , 3]}$

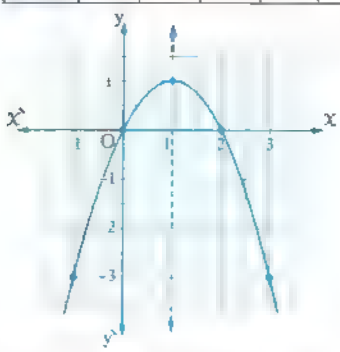
X	0	1	2	3
$f(X)$	2	0	0	2



- The vertex of the curve = $\left(\frac{3}{2}, -\frac{1}{4}\right)$
- The minimum value of the function = $-\frac{1}{4}$
- The equation of line of symmetry : $X = \frac{3}{2}$

$\boxed{f : f(X) = 2X - X^2 , X \in [-1 , 3]}$

X	−1	0	1	2	3
$f(X)$	−3	0	1	0	−3



- The vertex of the curve = (1 , 1)
- The maximum value of the function = 1
- The equation of line of symmetry : $X = 1$

Remember **The ratio and its properties**

- The ratio between the two real numbers a and b is written as a : b or $\frac{a}{b}$ and a is called the antecedent of the ratio , b is called the consequent and a , b are called the two terms of the ratio.
- The value of the ratio **does not change** if each of its terms is multiplied or divided by the same non-zero real number.
- The value of the ratio **changes** if we add or subtract (to or from) each of its two terms the same non-zero real number.
- If the ratio between two numbers is a : b , then : $\boxed{\text{The first number} = am}$, $\boxed{\text{The second number} = bm}$, $m \neq 0$

Example

Two numbers , their sum is 28 and the ratio between them is 3 : 4 , what are the two numbers ?

Solution

Let the two numbers be 3 m , 4 m $\quad \therefore 3\text{ m} + 4\text{ m} = 28 \quad \therefore 7\text{ m} = 28 \quad \therefore \text{m} = \frac{28}{7} = 4$

$\therefore$ The two numbers are : 3×4 and 4×4 $\quad$ *i.e.* 12 and 16

Remember **The proportion**

- The proportion is the equality of two ratios or more.
- If $\frac{a}{b} = \frac{c}{d}$, then a , b , c and d are proportional quantities.
- If a , b , c and d are proportional quantities , then $\frac{a}{b} = \frac{c}{d}$

Remember **The properties of the proportion**

Property 1

If $\frac{a}{b} = \frac{c}{d}$, then $a \times d = b \times c$

i.e. the product of the extremes = the product of the means.

Example

Find the fourth proportional of the quantities : 3 , 4 and 27

Solution

Let the fourth proportional be X $\quad \therefore$ The quantities : 3 , 4 , 27 and X are proportional

$\therefore \frac{3}{4} = \frac{27}{X} \quad \therefore 3 \times X = 4 \times 27 \quad \therefore X = \frac{4 \times 27}{3} = 36 \quad \therefore$ The fourth proportional = 36

Property 2

If $a \times d = b \times c$, then $\frac{a}{b} = \frac{c}{d}$

Also , each of the following proportions is correct : $\frac{a}{c} = \frac{b}{d}$, $\frac{d}{b} = \frac{c}{a}$, $\frac{b}{a} = \frac{d}{c}$

Example

If $\frac{x+3y}{2x-y} = \frac{4}{3}$, then find the ratio $x : y$

Solution

$\because \frac{x+3y}{2x-y} = \frac{4}{3}$ $\therefore 3(x+3y) = 4(2x-y)$ $\therefore 3x + 9y = 8x - 4y$

$\therefore 13y = 5x$ $\therefore x : y = 13 : 5$

Property 3

If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$

i.e. $\frac{\text{The antecedent of the first ratio}}{\text{The antecedent of the second ratio}} = \frac{\text{The consequent of the first ratio}}{\text{The consequent of the second ratio}}$

For example : If $\frac{a}{4} = \frac{b}{3}$, then $\frac{a}{b} = \frac{4}{3}$ or $\frac{b}{a} = \frac{3}{4}$

Property 4

If $\frac{a}{b} = \frac{c}{d}$, then $a = cm$, $b = dm$ where m is a constant $\neq 0$

Example

If $a : b = 3 : 5$, then find the ratio $20a - 7b : 15a + b$

Solution

$\because \frac{a}{b} = \frac{3}{5}$

$\therefore a = 3m$, $b = 5m$ where $m \neq 0$

Substituting by a and b in terms of m :

$\therefore \frac{20a - 7b}{15a + b} = \frac{60m - 35m}{45m + 5m} = \frac{25m}{50m} = \frac{1}{2}$

Remark

If a , b , c and d are proportional quantities and we assume that : $\frac{a}{b} = \frac{c}{d} = m$

, then $a = bm$, $c = dm$

For example : If $\frac{a}{b} = \frac{c}{d} = \frac{3}{4}$, then $a = \frac{3}{4}b$, $c = \frac{3}{4}d$

- **Generally :** If a , b , c , d , e , f , ... are proportional quantities and we assume that :

$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = ... = m$, then $a = bm$, $c = dm$, $e = fm$, ...

Example If a , b , c and d are proportional quantities , **prove that :**

1. $\frac{2a+3c}{7a-5c} = \frac{2b+3d}{7b-5d}$
2. $\frac{a+c}{b+d} = \frac{a^2+c^2}{ab+cd}$

Solution Let $\frac{a}{b} = \frac{c}{d} = m$ $\therefore a = bm$, $c = dm$

1. L.H.S. $= \frac{2bm+3dm}{7bm-5dm} = \frac{m(2b+3d)}{m(7b-5d)} = \frac{2b+3d}{7b-5d}$ = R.H.S

2. $\because \frac{a+c}{b+d} = \frac{bm+dm}{b+d} = \frac{m(b+d)}{(b+d)} = m$ (1)

, $\frac{a^2+c^2}{ab+cd} = \frac{(bm)^2+(dm)^2}{bm \times b + dm \times d} = \frac{b^2m^2+d^2m^2}{b^2m+d^2m} = \frac{m^2(b^2+d^2)}{m(b^2+d^2)} = m$ (2)

From (1) and (2) , we deduce that : $\frac{a+c}{b+d} = \frac{a^2+c^2}{ab+cd}$

Property 5

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = ...$ and m_1 , m_2 , m_3 , ... are non-zero real numbers ,

then $\frac{m_1 a + m_2 c + m_3 e + ...}{m_1 b + m_2 d + m_3 f + ...}$ = one of the given ratios.

Example If $\frac{a+3b}{x+5y} = \frac{3b+5c}{5y+7z} = \frac{5c+a}{7z+x}$, **prove that :** $\frac{a}{3b} = \frac{x}{5y}$

Solution

Multiplying the two terms of 2nd ratio by (−1) and adding the antecedents and consequents

of the three ratios : $\therefore \frac{a+3b-3b-5c+5c+a}{x+5y-5y-7z+7z+x} = \frac{2a}{2x} = \frac{a}{x}$ = one of the given ratios. (1)

Multiplying the two terms of 3rd ratio by (−1) and adding the antecedents and consequents

of the three ratios : $\therefore \frac{a+3b+3b+5c-5c-a}{x+5y+5y+7z-7z-x} = \frac{6b}{10y} = \frac{3b}{5y}$ = one of the given ratios (2)

From (1) and (2) , we deduce that : $\frac{a}{x} = \frac{3b}{5y}$ $\therefore \frac{a}{3b} = \frac{x}{5y}$

Remember **The continued proportion**

- The quantities a , b and c are said to be in continued proportion if $\frac{a}{b} = \frac{b}{c}$

a is called the **first proportional** , c is called the **third proportional** and

b is called the **middle proportional (proportional mean)**

- $\because \frac{a}{b} = \frac{b}{c}$ $\quad \therefore b^2 = ac$ $\quad \therefore b = \pm\sqrt{ac}$

i.e.

The middle proportional between two quantities $= \pm\sqrt{\text{the product of the two quantities}}$

Notice that :

The two quantities a and c should be either positive together or negative together.

- If $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$, then $\begin{cases} c = dm \\ b = dm^2 \\ a = dm^3 \end{cases}$

Example

If a , b , c and d are in continued proportion , **then prove that :** $\frac{2a+3c}{2b+3d} = \frac{a-c}{b-d}$

Solution

$\because$ a , b , c , d are in continued proportion

$\therefore \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = dm$, $b = dm^2$, $a = dm^3$

$\therefore \frac{2a+3c}{2b+3d} = \frac{2\,dm^3 + 3\,dm}{2\,dm^2 + 3\,d} = \frac{dm\,(2\,m^2+3)}{d\,(2\,m^2+3)} = m$ (1)

, $\frac{a-c}{b-d} = \frac{dm^3 - dm}{dm^2 - d} = \frac{dm\,(m^2-1)}{d\,(m^2-1)} = m$ (2)

From (1) and (2) , we deduce that : $\frac{2a+3c}{2b+3d} = \frac{a-c}{b-d}$

Remember **The direct variation and inverse variation**

Direct variation

- If y varies directly as x and is written as $y \propto x$, then :

1. $y = m x \left(i.e. \frac{y}{x} = m\right)$ where m is a constant $\neq 0$
2. $\frac{y_1}{y_2} = \frac{x_1}{x_2}$
3. The relation between x and y is represented graphically by a straight line passing through the origin point.

- To prove that $y \propto x$, we prove that : $y = m x$ where m is a constant $\neq 0$

For example :

If $y = 5x$, then $y \propto x$

Inverse variation

- If y varies inversely as x and is written as $y \propto \frac{1}{x}$, then :

1. $y = \frac{m}{x} \left(i.e. \ x y = m\right)$ where m is a constant $\neq 0$
2. $\frac{y_1}{y_2} = \frac{x_2}{x_1}$
3. The relation between x and y is not a linear relation.

- To prove that $y \propto \frac{1}{x}$, we prove that : $x y = m$ where m is a constant $\neq 0$

For example :

If $y = \frac{7}{x}$, then $x y = 7$, and then $y \propto \frac{1}{x}$

Example on direct variation

1. If $a \propto b$, $a = 5$ when $b = 2$, **find :** a when $b = 3$
2. If $a^2 + 4 b^2 = 4 a b$, **prove that :** $a \propto b$

Solution

1. $\because a \propto b$ $\quad \therefore \frac{a_1}{a_2} = \frac{b_1}{b_2}$

$\therefore \frac{5}{a_2} = \frac{2}{3}$ $\quad \therefore a_2 = 7.5$

2. $\because a^2 + 4 b^2 = 4 a b$ $\quad \therefore a^2 - 4 a b + 4 b^2 = 0$

$\therefore (a - 2 b)^2 = 0$ $\quad \therefore a - 2 b = 0$

$\therefore a = 2 b$ $\quad \therefore a \propto b$

Example on inverse variation

If x and y are two real variables where : $x^2 y^2 + 25 = 10 x y$, **prove that :** x varies inversely as y

Solution

$\because x^2 y^2 - 10 x y + 25 = 0$

$\therefore (x y - 5)^2 = 0$ $\quad \therefore x y - 5 = 0$

$\therefore x y = 5$ $\quad \therefore x \propto \frac{1}{y}$

Second Statistics

Remember The resources of collecting data

Primary resources (field resources)	Secondary resources (historical resources)
• These are the resources from which we get data directly.	• These are the resources from which we get data that previously collected.
Examples	**Examples**
* Questionnaires and survey. * Observing and measuring. * The personal interview.	* Central agency for public mobilization and statistics. * Mass-media. * Internet.

Remember The methods of collecting data

	Method of mass population	Method of samples
Definition	It is based on collecting the data related to the phenomenon under study from all individuals of the statistical society.	It is based on collecting the data related to the phenomenon under study from a representative sample of the society (Choosing a sample represented to the whole society)
Usages	• Elections • Census • Setting up a data base of all employees in an organization	• A sample of a patient's blood to make some clinical check up. • A sample of some products of a factory of find out if it matches the standard specifications.
Advantages	• Accuracy • Inclusiveness • Representing all the society individuals	• Saving time , effort and money. • It is the only method for collecting data about large unlimited societies. • It is the only method for collecting data about some limited societies in which mass population method leads to a great loss in it.
Disadvantages	• Sometimes it needs long time , great effort and a great cost.	• The results sometimes are not accurate specially if the sample doesn't represent the statistical society authentically.

Remember **The concept of the sample and the methods of collecting it**

The sample :
It is a small part from a large society that looks like the society and represents it well.

The methods of collecting the sample and its types

Biased selection

The type of the sample :
Not a random sample (deliberate sample)

Its usage :
It is used to select the individuals of the sample in a way to satisfy the objectives of the research.

Randomly selection

The type of the sample :
Simple random sample.

Its usage :
It is used for the homogeneous societies which are not naturally divided into groups or classes.

The type of the sample :
Layer random sample.

Its usage :
It is used in the statistical societies which are heterogeneous or made up of qualitative sets that are different in characteristics.

The number of individuals of the layer in the sample

$$= \frac{\text{the total number of individuals in the layer}}{\text{the total number of individuals in the society}} \times \text{the number of individuals of the sample}$$

«approximating the result to the nearest unit»

Example

At a faculty , there are 4 000 university students in the first grade , 3 000 in the second grade , 2 000 in the third grade and 1 000 in the fourth grade. If we want to draw a layer sample of 500 students , where each layer is represented in this sample according to its size.

, calculate the number of students in each layer in the sample.

Solution

The total number of students = 4 000 + 3 000 + 2 000 + 1 000 = 10 000 students.

The number of the individuals of the first layer in the sample $= \frac{4\,000}{10\,000} \times 500 = 200$ students.

The number of the individuals of the second layer in the sample $= \frac{3\,000}{10\,000} \times 500 = 150$ students.

The number of the individuals of the third layer in the sample $= \frac{2\,000}{10\,000} \times 500 = 100$ students.

The number of the individuals of the fourth layer in the sample $= \frac{1\,000}{10\,000} \times 500 = 50$ students.

Remember The dispersion and its measurements

The dispersion :

It is a measure that expresses how much the sets are homogeneous.

Dispersion measurements

1 The range (the simplest measure of dispersion)

It is the difference between the greatest value and the smallest value in the set.

i.e. The range = the greatest value – the smallest value

For example :

The values of set X are : 55 , 53 , 57 , 56 and 54 , then the range = 57 – 53 = 4

The values of set Y are : 67 , 73 , 41 , 60 and 34 , then the range = 73 – 34 = 39

So the set Y is more divergent than the set X

2 The standard deviation

It is the most important , common and accurate measure of dispersion. We can calculate it by calculating the positive square root of the average of squares of deviations of the values from their mean.

The standard deviation of a set of values

The standard deviation $\sigma = \sqrt{\dfrac{\sum (x-\overline{x})^2}{n}}$

Where :

x denotes a value of the values ,

$\overline{x}$ denotes the mean of the values ,

n denotes the number of the values ,

$\sum$ denotes the summation operation.

The standard deviation of a frequency distribution

The standard deviation $\sigma = \sqrt{\dfrac{\sum (x-\overline{x})^2 k}{\sum k}}$

Where :

x represents the value or the centre of the set ,

k represents the frequence of the value or the set ,

$\sum k$ is the sum of frequences

and $\overline{x}$ (the mean) $= \dfrac{\sum (x \times k)}{\sum k}$

Example on the standard deviation of a set of values

Calculate the standard deviation of the values : 55 , 53 , 57 , 56 and 54

Solution

❶ We find the mean of the values $(\overline{x}) = \frac{\sum x}{n}$

$= \frac{55 + 53 + 57 + 56 + 54}{5} = 55$

❷ We form the opposite table.

x	$x - \overline{x}$	$(x - \overline{x})^2$
55	55 55 = 0	0
53	53 − 55 = − 2	4
57	57 − 55 = 2	4
56	56 − 55 = 1	1
54	54 − 55 = − 1	1
Total		10

❸ We calculate standard deviation by substituting in the law :

The standard deviation $(\sigma) = \sqrt{\frac{\sum (x \quad \overline{x})^2}{n}} = \sqrt{\frac{10}{5}} = \sqrt{2} \simeq 1.4$

Example on the standard deviation of a simple frequency distributionmial function

The following table shows the distribution of wages of 20 persons in pounds :

The wage	20	25	30	35	40	45	Total
Number of persons	2	3	5	5	1	4	20

Find the standard deviation of the wages.

Solution

❶ **We find the mean of the wages $(\overline{x})$ by using the opposite table :**

$\therefore$ The mean $(\overline{x}) = \frac{\sum (x \times k)}{\sum k}$

$= \frac{660}{20} = 33$ pounds.

The wage (x)	Number of persons (k)	$x \times k$
20	2	40
25	3	75
30	5	150
35	5	175
40	1	40
45	4	180
Total	20	660

❷ **We form the opposite table :**

x	k	$x - \overline{x}$	$(x - \overline{x})^2$	$(x - \overline{x})^2 \times k$
20	2	20 − 33 = − 13	169	338
25	3	25 − 33 = − 8	64	192
30	5	30 − 33 = − 3	9	45
35	5	35 33 = 2	4	20
40	1	40 − 33 = 7	49	49
45	4	45 − 33 = 12	144	576
Total	20			1220

❸ **We calculate the standard deviation from the law :**

The standard deviation $(\sigma) = \sqrt{\frac{\sum (x - \overline{x})^2 \times k}{\sum k}} = \sqrt{\frac{1220}{20}} = \sqrt{61} \simeq 7.8$ pounds.

Example on the standard deviation of a frequency distribution of sets

The following is the frequency distribution of weekly incentives of 100 workers in a factory :

Incentives in pounds	35 –	45 –	55 –	65 –	75 –	85 –	Total
Number of workers	10	14	20	28	20	8	100

Find the standard deviation of this distribution.

Solution

Remember that

The centre of the set $= \dfrac{\text{lower limit} + \text{upper limit}}{2}$

1 **We find the mean ($\overline{X}$) by using the following table :**

Sets	Centres of sets (X)	Frequency (k)	$X \times k$
35 –	40	10	400
45 –	50	14	700
55 –	60	20	1200
65 –	70	28	1960
75 –	80	20	1600
85 –	90	8	720
Total		100	6580

$\therefore$ The mean ($\overline{X}$) $= \dfrac{\sum (X \times k)}{\sum k} = \dfrac{6580}{100} = 65.8$ pounds.

2 **We form the following table :**

X	k	$X - \overline{X}$	$(X - \overline{X})^2$	$(X - \overline{X})^2 \times k$
40	10	$40 - 65.8 = -25.8$	665.64	6656.4
50	14	$50 - 65.8 = -15.8$	249.64	3494.96
60	20	$60 - 65.8 = -5.8$	33.64	672.8
70	28	$70 - 65.8 = 4.2$	17.64	493.92
80	20	$80 - 65.8 = 14.2$	201.64	4032.8
90	8	$90 - 65.8 = 24.2$	585.64	4685.12
Total	100			20036

3 **We calculate the standard deviation by using the law :**

The standard deviation $(\sigma) = \sqrt{\dfrac{\sum (X - \overline{X})^2 \times k}{\sum k}} = \sqrt{\dfrac{20036}{100}} \simeq 14.15$ pounds.

Notice that :

- The values which are more homogeneous have less dispersion and their standard deviation is small.
- If the standard deviation equals zero that means the all values are equal , it is the perfect homogeneous case (the vanished dispersion)

2022

Final Examinations

on Algebra and Statistics

Model 1

Answer the following questions :

1 **Choose the correct answer from those given :**

1. The point (– 3 , 4) lies in the .... quadrant.

   (a) first (b) second (c) third (d) fourth

2. The positive square root of mean of the squares of deviations of values from its arithmetic mean is called ....

   (a) the range. (b) the arithmetic mean.

   (c) the standard deviation. (d) the mode.

3. If $3a = 4b$, then $a : b =$ ....

   (a) 3 : 4 (b) 4 : 3 (c) 3 : 7 (d) 4 : 7

4. If $n(X) = 2$, $n(Y^2) = 9$, then $n(X \times Y) =$ ....

   (a) 6 (b) 18 (c) 11 (d) 7

5. The range of the set of the values : 7 , 3 , 6 , 9 and 5 is ....

   (a) 3 (b) 4 (c) 6 (d) 12

6. If $y \propto X$ and $y = 2$ when $X = 8$, then $y = 3$ when $X =$ ....

   (a) 16 (b) 12 (c) 24 (d) 6

---

2 [a] If $X \times Y = \{(2, 2), (2, 5), (2, 7)\}$

, **find :** 1 Y 2 $Y \times X$

[b] If a , b , c and d are proportional , **prove that :** $\frac{a}{b - a} = \frac{c}{d - c}$

---

3 [a] If $X = \{2, 3, 5\}$, $Y = \{4, 6, 8, 10\}$ and R is a relation from X to Y where "a R b" means "2 a = b" for all $a \in X$, $b \in Y$

1 Write R and represent it by an arrow diagram.

2 Show that R is a function.

[b] Find the number that if we add it to each term of the ratio 7 : 11 , it becomes 2 : 3

4 [a] If $X = \{1, 3, 5\}$ and R is a function on X, where $R = \{(a, 3), (b, 1), (1, 5)\}$

, **find :**

1 The range of the function.

2 The value of $a + b$

[b] If $y \propto \frac{1}{x}$ and $y = 3$ when $x = 2$

, **find :**

1 The relation between x and y

2 The value of y when $x = 1.5$

5 [a] Represent graphically the function $f : f(x) = (x - 3)^2$, $x \in [0, 6]$, from the graph deduce the vertex of the curve, the minimum value of the function and the equation of the axis of symmetry.

[b] **Calculate the arithmetic mean and the standard deviation of the set of values :**

8, 9, 7, 6 and 5

Model 2

Answer the following questions :

1 **Choose the correct answer from those given :**

1 The point (3, 4) lies in the .... quadrant.

(a) first (b) second (c) third (d) fourth

2 .... is one of the measures of the dispersion.

(a) The median (b) The arithmetic mean

(c) The standard deviation (d) The mode

3 The third proportional of the two numbers 3 and 6 is ....

(a) $\frac{1}{2}$ (b) 9 (c) 2 (d) 12

4 If $n(X) = 2$, $n(Y \times X) = 6$, then $n(Y^2) =$ ....

(a) 4 (b) 9 (c) 16 (d) 12

5 The range of the set of the values : 7, 3, 6, 9 and 5 is ....

(a) 3 (b) 4 (c) 6 (d) 12

6 If $xy = 7$, then $y \propto$ ....

(a) $\frac{1}{x}$ (b) $x - 7$ (c) x (d) $x + 7$

2 [a] If $X = \{2, 5\}$, $Y = \{1, 2\}$, $Z = \{3\}$

, **find :** 1 $n(X \times Z)$

2 $(Y \cap X) \times Z$

[b] If b is the middle proportional between a and c , **prove that :** $\frac{a}{a} \frac{b}{c} = \frac{b}{b + c}$

3 [a] If $X = \{1, 3, 4, 5\}$, $Y = \{1, 2, 3, 4, 5, 6\}$ and R is a relation from X to Y where "a R b" means "$a + b = 7$" for all $a \in X$, $b \in Y$

1 Write R and represent it by an arrow diagram.

2 Show that R is a function.

[b] If $5a = 3b$, **find the value of :** $\frac{7a + 9b}{4a + 2b}$

4 [a] If $f(x) = 4x + b$ and $f(3) = 15$, **find the value of :** b

[b] If $y \propto x$, $y = 6$ when $x = 3$, **find :**

1 The relation between x and y

2 The value of y when $x = 5$

5 [a] Represent graphically the function $f : f(x) = 4 - x^2$, $x \in [-3, 3]$, from the graph deduce the vertex of the curve , the maximum value of the function and the equation of the axis of symmetry.

[b] **The following frequency distribution shows the number of children of some families in a new city :**

Number of children	0	1	2	3	4	Total
Number of families	6	15	40	25	14	100

Calculate the mean and the standard deviation of the number of children.

Model for the merge students

Answer the following questions :

1 Complete :

1 The point (5 , 3) lies in .... quadrant.

2 $n : n(X) = X^3 + 8$ is called a polynomial function of .... degree.

3 The range of the set of the values : 4 , 14 , 25 and 34 is ....

4 If $y = 2X$, then $y \propto$ ....

5 If $X = \{2 , 4 , 6\}$, then $n(X^2) =$ ....

6 If (a , 3) = (6 , b) , then a + b = ...

2 Choose the correct answer from those given :

1 If $X y = 7$, then $y \propto$ ....

(a) $\frac{1}{X}$ (b) $X - 7$ (c) X (d) $X + 7$

2 If 2 , 3 , 6 and X are proportional , then $X =$ ....

(a) 9 (b) 18 (c) 12 (d) 3

3 If 2 a = 5 b , then $\frac{a}{b} =$ ....

(a) $\frac{-5}{2}$ (b) $\frac{-2}{5}$ (c) $\frac{2}{5}$ (d) $\frac{5}{2}$

4 .... is one of the measures of the dispersion.

(a) The arithmetic mean (b) The range

(c) The mode (d) The median

5 If n (X) = 5 , n (X × Y) = 10 , then n (Y) = ...

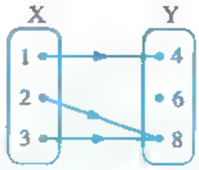
(a) 4 (b) 3 (c) 2 (d) 1

6 If $X = \{1\}$, then $X^2 =$ ....

(a) 1 (b) (1 , 1) (c) $\{(1 , 1)\}$ (d) $\{1\}$

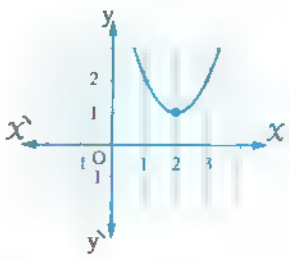
3 **Put (✓) or (✗) :**

1. If the function $f = \{(1 , 3) , (2 , 4) , (3 , 3)\}$
, then the domain of the function is $\{1 , 2 , 3\}$ ( )
2. If $y \propto X$ and $y = 6$ when $X = 3$, then $y = 2$ when $X = 4$ ( )
3. If $\sum \left(X - \overline{X}\right)^2 = 36$ for a set of values whose number equals 9 , then $\sigma = 4$ ( )
4. The intersection point of the straight line $f(X) = X + 2$
with X-axis is the point $(-2 , 0)$ ( )
5. If $f : X \longrightarrow Y$, then X is called the domain of this function. ( )
6. The arrow diagram from X to Y
represents a function.



( )

4 **Join from column (A) to column (B) :**

(A)	(B)
1. If $(1 , 4) \in \{2 , X\} \times \{1 , 4\}$, then $X =$	• 6
2. If the function f where $f(X) = X - 4$ is represented graphically by a straight line passing through the point $(a , 2)$, then $a =$	• 1
3. $\frac{1}{2} = \frac{3}{6} = \frac{4}{8} = \frac{\cdots\cdots}{16}$	• 10
4. If $f(X) = 5$, then $f(5) + f(-5) =$	• ± 6
5. The middle proportional of the two numbers 4 and 9 is	• 2
6. **In the opposite figure :** The equation of the line of symmetry is $X =$ .... 	• 8

1 Cairo Governorate

Answer the following questions : *(Calculator is allowed)*

1 Choose the correct answer from those given :

1 If $(a+3, b-1) = (-2, 4)$, then $a + b = \dots\dots$

(a) 0 (b) 2 (c) 5 (d) 10

2 If $X - y = 5$, then $6X - 6y = \dots\dots$

(a) 30 (b) 11 (c) 1 (d) -1

3 If X, 3, 4 and 6 are proportional, then $X = \dots\dots$

(a) 0 (b) 1 (c) 2 (d) 3

4 $\{3\} \cup]3, 5] = \dots\dots$

(a) $\varnothing$ (b) $\{3\}$ (c) $]3, 5]$ (d) $[3, 5]$

5 The positive square root of mean of the squares of deviations of the values from their arithmetic mean is called ……

(a) the range. (b) the standard deviation.

(c) the median. (d) the mean.

6 If $X^2 = 25$, where $X \in \mathbb{Z}$, then $X = \dots\dots$

(a) 5 (b) -5 (c) ± 5 (d) -25

---

2 [a] If $X = \{2\}$, $Y = \{3, 4, 5\}$, **find :**

1 $X \times Y$ 2 $n(Y^2)$ 3 X^2

[b] If $\frac{a}{b} = \frac{3}{5}$, **then find the value of :** $\frac{7a + 9b}{4a + 2b}$ in the simplest form.

---

3 [a] If $y \propto \frac{1}{X}$ and $y = 3$, when $X = 2$, **find :**

1 The relation between y and X 2 The value of y when $X = 1.5$

[b] If $X = \{1, 3, 4, 5\}$, $Y = \{2, 3, 4, 5, 6\}$ and R is a relation from X to Y where "a R b" means "a + b = 7" for all $a \in X$, $b \in Y$ write R and represent it by an arrow diagram. Is R a function ? Why ?

---

4 [a] **The following frequency distribution shows the ages of 10 children :**

Ages in years	5	8	9	10	12	Total
Number of children	1	2	3	3	1	10

Calculate the standard deviation to ages in years.

[b] Graph the curve of the function $f : f(X) = X^2 + 2X - 4$, where $X \in [-4, 2]$

From the graph find :

1. The vertex of the curve.
2. The equation of the axis of symmetry.

5 [a] If b is the middle proportional between a and c , **prove that :** $\frac{a^2 + b^2}{b^2 + c^2} = \frac{a}{c}$

[b] If $f(X) = X^2 - 2X$, $g(X) = X - 2$

1. **Prove that :** $f(2) = g(2)$
2. If $g(k) = 7$, **find :** the value of k

2 Giza Governorate

Answer the following questions :

1 **Choose the correct answer :**

1. If $X \in \mathbb{R}$ and $1 < X < 3$, then $(3X - 1) \in$ ....

(a) $]2, 8[$ (b) $[2, 8]$ (c) $]2, 8]$ (d) $\{2, 8\}$

2. The range of the set of the values : 7 , 3 , 6 , 5 , 9 is ....

(a) 3 (b) 4 (c) 6 (d) 12

3. Half of the number 4^{20} = ...

(a) 2^{20} (b) 2^{39} (c) 2^{29} (d) 4^{19}

4. If X , Y are two non empty sets and $n(X^2) = 4$, $n(X \times Y) = 6$, then $n(Y^2)$ = ...

(a) 4 (b) 9 (c) 16 (d) 12

5. If $a \times \frac{b}{3} = \frac{a}{3}$, then b = ...

(a) $-a$ (b) 1 (c) $\frac{a}{3}$ (d) a

6. If $X y = 7$, then $y \propto$ ....

(a) $\frac{1}{X}$ (b) $X - 7$ (c) X (d) $X + 7$

2 [a] If $(X + 3, 9) = (5, y^2)$, **then find :** the value of each of X and y

[b] If $y \propto \frac{1}{X}$ and $y = 4$ when $X = 2$, **then find :**

1. The relation between X and y
2. The value of y when $X = 8$

3 [a] If $X = \{0, 1, 2, 3, 4, 5, 6\}$ and R is a relation on X "where a R b" means "a double b" for all $a \in X$, $b \in X$

1. Write R as a set of ordered pairs and show if it is a function or not.
2. Is 2 R 4 ?
3. Find the value of X if 6 R X

[b] If b is the middle proportional between a and c , **then prove that :** $\frac{a - b}{a - c} = \frac{b}{b + c}$

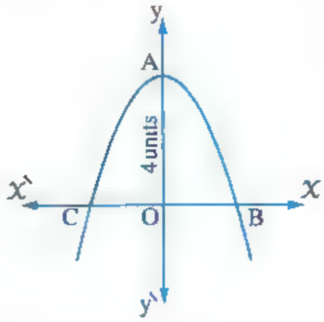
4 [a] The opposite figure represents the curve of the function $f : f(X) = m - X^2$

If OA = 4 units , **then find :**

1 The value of m

2 The coordinates of the two points B and C

3 The area of the triangle whose vertices are A , B , C



[b] If the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = 2X + a$ and $f(3) = 9$, **then find :**

1 The value of a

2 The coordinates of the intersection point of the straight line representing the function with X-axis

---

5 [a] If $\frac{X}{2} = \frac{y}{3} = \frac{z}{4} = \frac{2X - y + 5z}{3m}$, **then find :** the value of m

[b] **Find the standard deviation of the values :** 4 , 8 , 12 , 10 , 6

3 Alexandria Governorate

Answer the following questions : *(Calculators are allowed)*

1 **Choose the correct answer from those given :**

1 If $(3 , 5) \in \{3 , 6\} \times \{X , 8\}$, then $X =$ ....

(a) 8 (b) 6 (c) 5 (d) 3

2 A quarter of the number 2^8 is ....

(a) 2^6 (b) 2^{10} (c) $\left(\frac{1}{2}\right)^8$ (d) $\left(\frac{1}{2}\right)^6$

3 If $\frac{3a}{5b} = \frac{1}{2}$, then $\frac{a}{b} =$ ....

(a) $\frac{6}{5}$ (b) $\frac{5}{6}$ (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

4 If X is an odd number , then the next odd number directly is ....

(a) X^2 (b) $X^2 + 1$ (c) $X + 1$ (d) $X + 2$

5 $\frac{\text{Sum of the values}}{\text{Their number}}$ is ....

(a) the range. (b) the standard deviation.

(c) the mode. (d) the arithmetic mean.

6 If $3 > X > 1$, $X \in \mathbb{R}$, then $(3X - 1) \in$ ....

(a) $\{2 , 8\}$ (b) $]2 , 8[$ (c) $[2 , 8]$ (d) $[2 , 8[$

2 [a] If $X = \{2 , 5\}$, $Y = \{1 , 2\}$, $Z = \{3\}$

, **find :** 1 $n(X \times Z)$ 2 $(Y \cap X) \times Z$

[b] Find the number which if its square is added to each of the two terms of the ratio 5 : 11 it becomes 3 : 5

---

3 [a] If $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$, **find the value of :** $\frac{2y - z}{3x - 2y + z}$

[b] If $X = \{1 , 3 , 4 , 5\}$, $Y = \{1 , 2 , 3 , 4 , 5 , 6\}$, and R is a relation from X to Y where "a R b" means "a + b = 7" for all $a \in X$, $b \in Y$, write the relation R and represent it by an arrow diagram. Is R a function ? and why ?

---

4 [a] If y varies inversely with x , y = 2 when $x = 4$, **find :**

1 The relation between y and x 2 The value of y when $x = 16$

[b] **The following frequency distribution shows the ages of 20 persons :**

Ages in years	15	20	22	23	25	30	Total
Number of persons	2	3	5	5	1	4	20

Calculate the standard deviation to ages.

---

5 [a] Represent graphically the function f where $f(x) = 4 - x^2$ taking $x \in [-3 , 3]$

and from the drawing deduce :

1 The coordinates of the vertex of the curve.

2 The maximum or the minimum value of the function.

3 The equation of the symmetry axis

[b] If $f(x) = 5x - a$, $r(x) = x - 2a$ and $f(1) + r(3) = -7$, **find :** the value of a

4 El-Kalyoubia Governorate

Answer the following questions :

1 **Choose the correct answer from the given answers :**

1 If the point (5 , b – 7) lies on the x-axis , then b = ...

(a) 2   (b) 5   (c) 7   (d) 12

2 If $f(x) = 7$, then $f(7) + f(-7) =$ ....

(a) 7   (b) – 7   (c) – 14   (d) 14

3 If $\sqrt[3]{-27} = -\sqrt{x}$, then $x =$ ....

(a) 9 (b) – 9 (c) 3 (d) – 3

4 If $\frac{a}{3} = \frac{b}{4}$, then $8a - 6b + 4 =$ ....

(a) 3 (b) 4 (c) 5 (d) 6

5 If $X = \{2\}$, then $X^2 =$ ....

(a) 4 (b) $\{4\}$ (c) $(2, 2)$ (d) $\{(2, 2)\}$

6 The positive square root of the average of squares of deviations of the values from their mean is called ....

(a) the mean. (b) the range.
(c) the standard deviation. (d) the mode.

---

2 [a] If y varies inversely as x and $y = 3$ as $x = 2$

1 Find the relation between x and y 2 Find the value of y when $x = \frac{3}{2}$

[b] If $\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a - b + 5c}{3x}$, **find :** the value of x

---

3 [a] If $X = \{1, 3, 5\}$, $Y = \{2, 3, 4, 5, 6\}$ and R is a relation from X to Y, where "a R b" means "a + b = 7" for each $a \in X$, $b \in Y$

1 Write R and represent it by an arrow diagram. 2 Is R a function ? and why ?

[b] If b is the middle proportional between a and c, **prove that :** $\frac{a - c}{a - b} = \frac{b + c}{b}$

---

4 [a] If $X \times Y = \{(1, 1), (1, 3), (1, 5)\}$

, **find :** 1 X, Y 2 $Y \times X$

[b] Represent graphically the function $f : f(x) = 2 - x^2$ since $x \in [-3, 3]$

and find from the drawing deduce :

1 The coordinates of the vertex of the curve. 2 The maximum value of the function.

3 The equation of the symmetry axis.

---

5 [a] If $X = \{1, 3, 5\}$ and R is a function on X where $R = \{(a, 3), (b, 1), (1, 5)\}$, **find :**

1 The range of the function. 2 The numerical value of a + b

[b] **Find the mean and the standard deviation for the following frequency distribution :**

Set	zero –	2 –	4 –	6 –	8 – 10	Total
Frequency	1	3	6	5	5	20

5 El-Monofia Governorate

Answer the following questions : *(Using calculator is permitted)*

1 Choose the correct answer :

1 If $\frac{5}{4} + \frac{5}{x} = \frac{5}{2}$, then $x =$ ....

(a) 2 (b) 4 (c) 5 (d) $\frac{5}{2}$

2 If $x + y = x y = 5$, then $x^2 y + x y^2 =$ ....

(a) 10 (b) 15 (c) 20 (d) 25

3 If $1 < x < 3$, $x \in \mathbb{R}$, then $(3x - 1) \in$ ....

(a) $[2 , 8[$ (b) $[2 , 8]$ (c) $]2 , 8[$ (d) $\{2 , 8\}$

4 If $\frac{a + 2b}{a - b} = \frac{2}{3}$, then $\frac{b}{a} =$ ....

(a) $\frac{1}{8}$ (b) 8 (c) $\frac{-1}{8}$ (d) -8

5 Which of the following values of the number x makes the range of the set of the values x , 15 , 20 , 24 equal to 14 ?

(a) 30 (b) 25 (c) 19 (d) 10

6 If $x \in \mathbb{R}_-$, then the point $\left(-x , \sqrt[3]{x}\right)$ lies in the .... quadrant.

(a) first (b) second (c) third (d) fourth

2 [a] If $X = \{4 , 3\}$, $Y = \{5 , 4\}$ and $Z = \{5 , 6\}$, **find :**

1 $X \times (Y \cap Z)$ 2 $(X - Y) \times Z$ 3 $n(Z^2)$

[b] If a , b , c and d are in continued proportion , **prove that :** $\frac{ab - cd}{b^2 - c^2} = \frac{a + c}{b}$

3 [a] If $X = \{-2 , -1 , 1 , 2\}$, $Y = \left\{8 , \frac{1}{3} , -1 , 1 , -8\right\}$ and R is a relation from X to Y where "a R b" means "$b = a^3$" for each $a \in X$, $b \in Y$

1 Write R and represent it by an arrow diagram.

2 Show that R is a function and find its range.

[b] If the straight line that represents the function f where $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = a x + b$ cuts y-axis at the point (0 , 3) and $f(2) = 7$,

find : the values of a and b

4 [a] Find the number that if its square is added to the terms of the ratio 7 : 11 , then it will become 4 : 5

[b] If y varies inversely as X^2 and $X = 3$ when $y = 4$, **find :**

1 The relation between X and y 2 The value of X when $y = 9$

5 [a] Draw the curve of the function f where $f(X) = 1 - X^2$ taking $X \in [-3, 3]$ **and from the graph find :**

1 The coordinates of the vertex of the curve.

2 The equation of the axis of symmetry.

3 The area of the triangle whose vertices are the intersection points of the curve with the two axes.

[b] **The following frequency distribution shows the number of children of some families in a new city :**

Number of children	zero	1	2	3	4
Number of families	8	16	50	20	6

Calculate the mean and the standard deviation of the number of children.

6 El-Gharbia Governorate

Answer the following questions : ***(Calculators are allowed)***

1 **Choose the correct answer :**

1 The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = a X + b$ represents a linear function on condition $a \in$ ....

(a) $\mathbb{R}$ (b) $\mathbb{R}_+$ (c) $\mathbb{R} - \{0\}$ (d) $\mathbb{R}_-$

2 The fourth proportional of the numbers : 4 , 12 , 16 is ....

(a) 24 (b) ± 24 (c) 48 (d) ± 48

3 If the weekly wages in pounds of a set of workers in a factory are 170 , 180 , 180 , 230 and 240 , then the median of wages equals ....

(a) 200 (b) 70 (c) 180 (d) 205

4 If $X^2 + y^2 = 6$, $X y = 5$, then $(X + y)^2 =$ ....

(a) 16 (b) ± 16 (c) 11 (d) ± 11

5 The relation which represents the direct variation between y and X is ....

(a) $X y = 5$ (b) $y = 3 - X$ (c) $\frac{X}{3} = \frac{y}{5}$ (d) $\frac{X}{3} = \frac{4}{y}$

6 If $X = \{1, 3, 5\}$ and R is a function on X where $R = \{(a, 3), (b, 1), (1, 5)\}$, then the numerical value of $a + b = \cdots\cdots$

(a) 4 (b) 6 (c) 8 (d) other.

---

2 [a] If $X = \{-1, \text{zero}, 2, 3\}$, $Y = \{1, \text{zero}, \frac{1}{2}, \frac{1}{3}\}$ and R is a relation from X to Y where "a R b" means "The number a is the multiplicative inverse of the number b" for each $a \in X$, $b \in Y$, write R, and represent it by an arrow diagram and show if R is a function or not ? And why ?

[b] **From the data of the following table answer the following questions :**

x	2	4	6
y	6	3	2

1 Show the kind of variation between x and y

2 Find the constant proportional.

3 Find the value of y when $x = 3$

---

3 [a] If a, b, c and d are in continued proportion, **prove that :** $\frac{a}{b+d} = \frac{c^3}{c^2 d + d^3}$

[b] If $X = \{6\}$, $Y = \{2, 3\}$ and $Z = \{2, 5, 6\}$, **find :**

1 $n(X^2)$ 2 $(Z - Y) \times (X \cap Z)$

---

4 [a] Two integers the ratio between them is 2 : 3, if you add to the first 7 and subtract from the second 12, the ratio between them becomes 5 : 3, find the two integers.

[b] If the function $f : f(x) = 3x - 6$ represents a straight line passing through the point $(a, 2a)$, find the value of a, and find the intersection point of the straight line with y-axis.

---

5 [a] **Calculate the standard deviation for the following data :**

16, 32, 5, 20, 27 rounding the result to one decimal place.

[b] Represent graphically the function $f : f(x) = (x - 2)^2$, taking $x \in [-1, 5]$

and from the graph deduce :

1 The equation of the axis of symmetry.

2 The maximum value or the minimum value of the function.

7 El-Dakahlia Governorate

Answer the following questions : *(Calculator is permitted)*

1 [a] **Choose the correct answer :**

1 The point $(X - 3 , 2 - X)$ lies in the fourth quadrant , then $X =$ ....

(a) 4 (b) 3 (c) 2 (d) 1

2 If $d(X) = c X + 8$, $d(2) = 0$, then $c =$ ....

(a) 8 (b) 6 (c) 4 (d) -4

3 If a , 2 , 4 , b are in continued proportion , then $a + b =$ ....

(a) 2 (b) 4 (c) 6 (d) 9

[b] If b is the middle proportional between a and c , **prove that :** $\frac{2 c^2 \quad 3 b^2}{2 b^2 - 3 a^2} = \frac{c}{a}$

---

2 [a] **Choose the correct answer :**

1 If $y \propto X$, $y \propto \frac{1}{d}$, then $y \propto$ ....

(a) $X d$ (b) $\frac{d}{X}$ (c) $\frac{X}{d}$ (d) $X^2 d$

2 The standard deviation of the values 5 , 5 , 5 , 5 equals ....

(a) zero (b) 5 (c) 6 (d) 2

3 The function $d : d(X) = X^2 - (X - 3)^2$ is of the .... degree.

(a) zero (b) first (c) second (d) third

[b] If $(-1 , 2)$ is the point of the vertex of the curve of the function $d : d(X) = a X^2 - 6 X + c$, **find :** the value of c

---

3 [a] If $3 a = 4 b = 6 c$, **find :** a : b : c , and the value of : $\frac{3 a + 2 b}{a + 4 c}$

[b] If $X = \{-2 , -1 , 0 , 1 , 2\}$, R is a relation on the set X where "a R b" means "a is the additive inverse of the number b" for every $a \in X$ and $b \in X$, state R , then represent it by an arrow diagram , and mention giving reasons if R represents a function or not.

---

4 [a] If $X = z + 8$ where z varies inversely as y and z = 2 when y = 3 , find the relation between y and X , **then find :** y when $X = 3$

[b] If $d(X) = 2 X + 5$, $r(X) = X - 6$, **prove that :** $d(2) + 3 r(3) = 0$

---

5 [a] **Calculate the mean and the standard deviation of the following data :** 5 , 7 , 8 , 9 , 6

[b] If $(X - 2 , 3^{y-1}) = (3 , 1)$, **find :** X , y

8 Ismailia Governorate

Answer the following questions : *(Calculators are allowed)*

1 **Choose the correct answer from those given :**

1) $\sqrt{36}+\sqrt{16}=$ ....

(a) 10 (b) 24 (c) 52 (d) 100

2) The middle proportional between 3 , 27 is ....

(a) 9 (b) -9 (c) ± 9 (d) 1

3) If $f(x)=2$, then $f(2)+f(-2)=$ ....

(a) zero (b) 4 (c) -4 (d) 1

4) The positive number which twice its square equals 50 is ....

(a) 5 (b) 10 (c) 25 (d) 100

5) If $x+y=xy=5$, then $x^2y+y^2x=$ ....

(a) 10 (b) 15 (c) 20 (d) 25

6) The simplest and easiest method of measuring dispersion is ....

(a) the range. (b) the standard deviation.

(c) the arithmetic mean. (d) the mode.

---

2 [a] If $X=\{2 , 3 , 5\}$, $Y=\{4 , 6 , 8 , 10\}$ and R is a relation from X to Y where "a R b" means "2 a = b" for all $a \in X$, $b \in Y$

1) Write R and represent it by an arrow diagram.

2) Is the relation R a function ? Why ? and if it's a function , find its range.

[b] The ratio between two integers is 3 : 7 If 5 is subtracted from each of them , then the ratio becomes 1 : 3 , find the two integers.

---

3 [a] As Yousef was reading a book , he found out after 3 hours 50 pages remained , after 6 hours 20 pages remained. If there was a relation between the time (t) and the number of pages (y) Is a linear relation.

1) Represent the relation between (t) and (y) , then find the algebraic relation between them.

2) How much time did Yousef takes to finish reading the book ?

3) How many pages left when Yousef started reading ?

[b] If x , y , z and ℓ are proportional quantities , prove that : $\frac{y \quad x}{x}=\frac{\ell \quad z}{z}$

4 [a] If $y \propto X$ and $y = 40$ at $X = 14$, find the relation between X and y ,
then find the value of X when $y = 80$

[b] If $X \times Y = \{(1 , 2) , (1 , 3) , (2 , 2) , (2 , 3)\}$
, **find :** 1 $X \cup Y$ 2 $n(Y^2)$

5 [a] Represent graphically the function $f : f(X) = (X - 2)^2$, taking $X \in [-1 , 5]$
And from the graph find :
1 The coordinates of the vertex of the curve. 2 The equation of the line of symmetry.
3 The maximum or the minimum value of the function.

[b] **Find the standard deviation for the following set of values :** 13 , 14 , 17 , 19 , 22

9 Suez Governorate

Answer the following questions : *(Calculators are allowed)*

1 Choose the correct answer from those given :

1 If $(2 , 3) \in \{2 , 5\} \times \{X , 6\}$, then $X =$ ....
(a) 6 (b) 5 (c) 3 (d) 2

2 $(\sqrt{5} - 3)(\sqrt{5} + 3) =$ ....
(a) 8 (b) 2 (c) 4 (d) -4

3 The positive square root of the mean of the squares of deviations of the values from their arithmetic mean is called ....
(a) the range. (b) the arithmetic mean.
(c) the standard deviation. (d) the mode.

4 If the number $\frac{3}{b} + 1 = 4$, then $b =$ .... where $b \neq 0$
(a) 1 (b) 2 (c) 3 (d) 4

5 $\mathbb{Z}_- \cup \mathbb{N} =$ ....
(a) $\varnothing$ (b) $\mathbb{Z}$ (c) $\mathbb{N}$ (d) $\mathbb{R}$

6 If $\frac{a}{b} = \frac{c}{d} = m$ (where $m \in \mathbb{R}^*$) , then $\frac{ac}{bd} =$ ....
(a) m (b) m^2 (c) 2 m (d) $2 m^2$

2 [a] If $X = \{1 , 2 , 3\}$, $Y = \{1 , 4 , 6 , 9\}$ and R is a relation from X to Y where "a R b" means "$a = \sqrt{b}$" for all $a \in X$, $b \in Y$
1 Find the relation R 2 Represent the relation R by an arrow diagram.
3 Is R a function ? Why ?

[b] If b is the middle proportional between a and c , **prove that :** $\frac{2 c^2 - 3 b^2}{2 b^2 - 3 a^2} = \frac{c}{a}$

3 [a] If $(2X, 4) = (8, y+1)$, **find :** $\sqrt{X^2+y^2}$

[b] If $y \propto X$ and $y = 2$ when $X = 8$, **find :**

1 The relation between y and X

2 The value of y when $X = 12$

4 [a] Draw the curve of the function $f : f(X) = X^2 + 1$, taking $X \in [-2, 2]$

and from the graph find :

1 The coordinates of the vertex of the curve.

2 The equation of the axis of symmetry.

3 The minimum value.

[b] If $\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a - b + 5c}{3X}$, **find :** X

5 [a] If $X = \{1, 3, 5\}$ and R is a function on X where $R = \{(a, 3), (b, 1), (1, 5)\}$, **find :**

1 The range of the function.

2 The numerical value of the expression a + b

[b] **Calculate the standard deviation for the values :** 8, 9, 7, 6, 5

10 Port Said Governorate

Answer the following questions :

1 **Choose the correct answer from those given :**

1 $[1, 3] - \{0, 1\} =$ ....

(a) $]1, 3[$ (b) $]1, 3]$ (c) $[1, 3[$ (d) $\{3\}$

2 If $2^X = 2^6$, then $X =$ ....

(a) 3 (b) 4 (c) 6 (d) 64

3 20% from 10 pounds = ... pounds.

(a) 2 (b) 2.5 (c) 5 (d) 20

4 If $n(X) = 3$, $n(X \times Y) = 12$, then $n(Y) =$ ....

(a) 4 (b) 9 (c) 15 (d) 36

5 If $3a = 4b$, then $a : b =$ ....

(a) 3 : 4 (b) 4 : 7 (c) 3 : 7 (d) 4 : 3

6 The range of the set of the values 7, 3, 6, 9 and 5 equals ....

(a) 3 (b) 4 (c) 6 (d) 12

2 [a] If $X = \{2, 3, 4\}$, $Y = \{2, 3, 4, 5, 6, 7, 8\}$, R is a relation from X to Y where "a R b" means "$a = \frac{1}{2}$ b" for each $a \in X$, $b \in Y$, write R and represent it by an arrow diagram. Show that R is a function from X to Y and find its range.

[b] If $f(X) = 4X + b$ and $f(3) = 15$, **find :** the value of b

3 [a] If $f(X) = X^2 - 3X$, $g(X) = X - 3$

1 **Find :** $f(\sqrt{2}) + 3g(\sqrt{2})$ 2 **Prove that :** $f(3) = g(3) = 0$

[b] Represent graphically the quadratic function f where $f(X) = X^2$, $X \in \mathbb{R}$, consider $X \in [-3, 3]$, from the graph deduce the vertex of the curve , the minimum value of the function , the equation of the axis of symmetry.

4 [a] If b is the middle proportional between a and c , **prove that :** $\frac{a^2 + b^2}{b^2 + c^2} = \frac{a}{c}$

[b] If $y \propto X$, where $y = 14$ when $X = 42$, **then find :**

1 The relation between X and y 2 The value of y when $X = 60$

5 [a] **Calculate the standard deviation for the values :** 16 , 32 , 5 , 20 , 27

[b] If the height of a right constant cylinder (constant volume) is (h) varies inversely as the square of its radius length r and h = 27 cm. when r = 10.5 cm. , find h when r = 15.75 cm.

11 Damietta Governorate

Answer the following questions : (Calculators are allowed)

1 **Choose the correct answer from the given ones :**

1 If $n(X) = 3$, $n(Y^2) = 4$, then $n(X \times Y) =$ ....

(a) 6 (b) 12 (c) 18 (d) 36

2 The range of the set of the values 7 , 4 , 6 , 9 and 5 equals ....

(a) 3 (b) 4 (c) 5 (d) 6

3 If $\frac{y}{X} = 5$, then $y \propto$ ....

(a) X (b) $\frac{1}{X}$ (c) $X - 5$ (d) $X + 5$

4 If $\frac{3}{4} + \frac{3}{X} = \frac{3}{2}$, then $X =$ ....

(a) $\frac{3}{2}$ (b) 2 (c) 3 (d) 4

5 The third proportional of the two numbers 3 and 6 is ....

(a) $\frac{1}{2}$ (b) 2 (c) 9 (d) 12

6 The solution set of the equation $(X-1)^2 = 9$ in $\mathbb{R}$ is ....

(a) $\{4\}$ (b) $\{-2\}$ (c) $\{4, -2\}$ (d) $\{3\}$

2 [a] If $X = \{1, 9, 6\}$, $Y = \{3, 4, 5, 6\}$, $Z = \{4\}$, **then find :** $(X - Y) \times Z$

[b] If b is the middle proportional between a and c , **prove that :** $\frac{a^2 + b^2}{b^2 + c^2} = \frac{a}{c}$

3 [a] If $y \propto \frac{1}{X}$ and $y = 3$ when $X = 2$

1 Find the relation between X and y

2 Find the value of y when $X = 1.5$

[b] If $\frac{y}{X - z} = \frac{X}{y} = \frac{X + y}{z}$, **prove that :**

1 Each ratio is equal to 2 (unless $X + y = 0$)

2 $3y = 2z$

4 [a] If $(X^3, y + 1) = (8, 3)$, **find the value of :** $\sqrt[3]{X + 3y}$

[b] If $X = \{-1, 0, 2, 3\}$, $Y = \{0, 1, 4, 6, 9\}$ and R is a relation from X to Y where "a R b" means "$a^2 = b$" for each $a \in X$, $b \in Y$

1 Write R and represent it by an arrow diagram

2 Show that R is a function from X to Y and find its range.

5 [a] **Calculate the arithmetic mean and the standard deviation of the set of values :**

72 , 53 , 61 , 70 , 59

[b] Represent graphically the function $f : f(X) = X^2 - 2$, $X \in [-3, 3]$

From the graph deduce : 1 The vertex of the curve.

2 The equation of the axis of symmetry.

12 Kafr El-Sheikh Governorate

Answer the following questions : *(Calculators are permitted)*

1 **Choose the correct answer from those given :**

1 If $X =]-\infty, 0[$, then $X^\backprime =$ ....

(a) $\mathbb{R}_+$ (b) $[0, \infty[$ (c) $]-\infty, 0]$ (d) $\mathbb{R}_-$

2 The function $f : f(X) = (X-2)^2 - X^2$ is of the .... degree.

(a) first (b) second (c) third (d) fourth

3 If $\sum (X - \overline{X})^2 = 36$ of a set of values and the number of these values = 9 ,
then σ = ...

(a) 2 (b) 18 (c) 27 (d) 4

4 The middle proportional between $3 X^3$ and $27 X$ is ....

(a) $9 X^2$ (b) $\pm 9 X^4$ (c) $\pm 9 X^2$ (d) $9 X^4$

5 If $y^2 + 4 X^2 = 4 X y$, then ....

(a) $y \propto X$ (b) $y \propto X^2$ (c) $y \propto \frac{1}{X}$ (d) $y \propto \frac{1}{X^2}$

6 If $\{2\} \times \{X , y\} = \{(2 , 4) , (2 , 3)\}$, then $X - y$ = ...

(a) 1 (b) -1 (c) ± 1 (d) zero

---

2 [a] If $X = \{-1 , 1 , 2\}$, $Y = \{2 , 4 , 6 , 8\}$ and R is a relation from X to Y , where "a R b" means "b = 2 a + 4" for each $a \in X$, $b \in Y$, write R and represent it by an arrow diagram , and show if R is a function or not ? If R is a function mention its range.

[b] If $\frac{21 X - y}{7 X - z} = \frac{y}{z}$, **prove that :** $y \propto z$

---

3 [a] Represent graphically the function $f : f(X) = X^2 - 2 X$, $X \in [-2 , 4]$

and from the graph deduce :

1 The equation of the line of symmetry.

2 The maximum or the minimum value of the function.

[b] If a , b , c and d are in continued proportion , **prove that :** $\frac{a}{b + d} = \frac{c^3}{c^2 d + d^3}$

---

4 [a] If $\frac{X + y}{3} = \frac{y + z}{8} = \frac{z + X}{6}$, **prove that :** $\frac{X + y + z}{2 X + 3 y + 3 z} = \frac{17}{50}$

[b] If the point (a , 4) is one of the points of the function $g : \mathbb{R} \longrightarrow \mathbb{R}$ where $g(X) = 2 X + b$, **then find the value of :** 6 a + 3 b

---

5 [a] **The following table represents the daily wages of a set of workers in a factory :**

Set of wages	20 –	30 –	40 –	50 –	60 –	70 –
Number of workers	10	12	8	6	3	1

Find the mean and the standard deviation of the wages.

[b] If the straight line which represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(X) = a X + b$ cuts from the positive part of y-axis 3 length units and passes through the point (1 , 5) ,
find : the value of each of a , b

13 El-Beheira Governorate

Answer the following questions : (Calculator is permitted)

1 **Choose the correct answer from the given ones :**

1 If $3^{X} = 9^2$, then $X =$ ....

(a) 3 (b) 4 (c) 6 (d) 64

2 The range of the set of the values 7 , 3 , 6 , 8 and 5 equals ....

(a) 3 (b) 8 (c) 11 (d) 5

3 If the point $(X - 4 , 2 - X)$ where $X \in \mathbb{Z}$ is located in the third quadrant , then $X =$ ....

(a) 2 (b) 3 (c) 4 (d) 6

4 The relation which represents the direct variation between the two variables X and y is ....

(a) $X y = 7$ (b) $y = X + 5$ (c) $\frac{X}{3} = \frac{7}{y}$ (d) $\frac{X}{2} = \frac{y}{5}$

5 The solution set of the equation $X^2 - 25 = 0$ in $\mathbb{R}$ is ....

(a) $\{5 , -5\}$ (b) $[-5 , 5]$ (c) 5 (d) -5

6 If $(3 , 5) \in \{3 , 6\} \times \{y , 8\}$, then $y =$ ....

(a) 8 (b) 6 (c) 5 (d) 3

2 [a] If $X = \{2 , 3 , 5\}$, $Y = \{4 , 6 , 8 , 10\}$ and R is a relation from X to Y where "a R b" means "2 a = b" for all $a \in X$, $b \in Y$

1 Write R 2 Show that R is a function and find its range.

[b] If b is the middle proportional between a and c , **then prove that :** $\frac{2 c^2 - 3 b^2}{2 b^2 - 3 a^2} = \frac{c}{a}$

3 [a] If $y \propto \frac{1}{X}$ and $y = 9$ when $X = 2$, **find :**

1 The relation between y and X 2 The value of y when $X = 3$

[b] If $f(X) = 5 X + a$ and $f(2) = 12$, **find :** the value of a

4 [a] If $X = \{3 , 4\}$, $Y = \{4 , 5\}$, $Z = \{6 , 5\}$, **find :**

1 $(X - Y) \times Z$ 2 $n(X \times Y)$

[b] Find the number that if subtracted thrice of it from each of the two terms of the ratio $\frac{49}{69}$ the ratio becomes $\frac{2}{3}$

5 [a] **Calculate the mean and the standard deviation of the following data :**

$8 , 13 , 20 , 16 , 18 , 21$

[b] Represent graphically the function f where $f(X) = 3 - X^2$, where $X \in [-3 , 3]$ **and from the graph deduce :**

1 The equation of the symmetry axis.
2 The maximum value of the function.

14 El-Fayoum Governorate

Answer the following questions : (Using calculators is allowed)

1 **Choose the correct answer :**

1 If $\left(X + 1 , \sqrt[3]{27}\right) = (-1 , y)$, then the point (X , y) lies in the .... quadrant.

(a) first (b) second (c) third (d) fourth

2 If $\frac{3}{4} + \frac{3}{X} = \frac{3}{2}$, then $X =$ ....

(a) 2 (b) 4 (c) 3 (d) $\frac{3}{2}$

3 Twice of the number 2^8 is ....

(a) 2^{10} (b) 2^{16} (c) 2^4 (d) 2^9

4 If $X y = 12$, then y varies directly as ....

(a) $\frac{1}{X}$ (b) $X - 12$ (c) X (d) $X + 12$

5 Omar bought 4 notebooks and 3 pens , he paid 50 pounds for them. If the price of a pen is twice the price of a notebook , then the price of a notebook is .... pounds.

(a) 4 (b) 5 (c) 10 (d) 20

6 If the range of the set of the values 7 , X , 8 , 9 and 5 is 6 , then $X =$ ....

(a) 3 (b) 4 (c) 6 (d) 12

2 [a] If $X = \{2 , 5\}$, $Y = \{1 , 2\}$, $Z = \{3\}$, **find :**

1 $n(X \times Y)$
2 $(Y \cap X) \times Z$

[b] If $a = 2b$, **find the value of :** $\frac{8a + 5b}{7a - 2b}$

3 [a] If $X = \{1 , 2 , 3\}$, $Y = \left\{1 , \frac{1}{2} , \frac{1}{3} , \frac{1}{4}\right\}$ and $R : X \longrightarrow Y$, where "a R b" means "a is the multiplicative inverse of b" for all $a \in X$, $b \in Y$

1 Write R and represent it by an arrow diagram
2 Is R a function ? Write its range.

[b] If $f(X) = 4X + a$, $f\left(\frac{1}{4}\right) = 12$, **find :** the value of a

4 [a] If a , b , c and d are in continued proportion , **prove that :** $\frac{a}{b+d} = \frac{c^3}{c^2 d + d^3}$

[b] If y varies inversely as X , and y = 3 when $X = 2$

1 Find the relation between X and y 2 Find the value of y when $X = 3$

5 [a] Graph the function f where $f(X) = 4 - X^2$, $X \in [-3 , 3]$, **from the graph determine :**

1 The coordinates of the vertex of the curve.

2 The equation of the symmetry axis of this function.

[b] **Calculate the mean and the standard deviation of the following data :**

3 , 6 , 7 , 9 , 15

15 Beni Suef Governorate

Answer the following questions : *(Calculator is allowed)*

1 **Choose the correct answer from those given :**

1 The middle proportional between a and c equals ....

(a) $\sqrt{a+c}$ (b) $\frac{a+c}{2}$ (c) $\pm\sqrt{ac}$ (d) a c

2 The difference between the greatest value and the smallest value of a set of data is called ....

(a) the range. (b) the arithmetic mean.

(c) the mode. (d) the standard deviation.

3 $|\sqrt[3]{-8}| =$ ....

(a) 4 (b) 2 (c) -2 (d) ± 2

4 $\frac{7}{X}$ is a rational number if $X \neq$ ....

(a) 7 (b) -7 (c) 1 (d) zero

5 If the point (a , 3 – a) lies on the X-axis , then a = ...

(a) zero (b) 3 (c) -3 (d) 5

6 If $-X > 3$, then $X \in$ ....

(a) $\{-3\}$ (b) $]3 , \infty[$ (c) $]-\infty , 3[$ (d) $]-\infty , -3[$

2 [a] If $X = \{2 , 5\}$, $Y = \{3 , 2\}$, $Z = \{3\}$, **find :**

1 $X \times Z$ 2 Y^2 3 $(X \cap Y) \times Z$

[b] Find the positive number which if we add its square to each of the two terms of the ratio 5 : 11 it becomes 3 : 5

3 [a] If $f(x) = x^2 - \sqrt{2}\,x$, $g(x) = x + 1$

1 **Find :** $f(3) + 3\,g\left(\sqrt{2}\right)$

2 **Prove that :** $f\left(\sqrt{2}\right) = g(-1)$

[b] If y varies inversely with x and $y = 3$ when $x = 2$, **find :**

1 The relation between x and y

2 The value of y when $x = 1.5$

---

4 [a] If $X = \{1 , 2 , 3\}$, $Y = \{6 , 7 , 8\}$ and R is a relation from X to Y where "a R b" means "a + b = 8" for all $a \in X$ and $b \in Y$, write R and represent it by an arrow diagram. Is R a function ? Why ?

[b] If $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$, **prove that :** $\frac{2y - z}{3x - 2y + z} = \frac{1}{2}$

---

5 [a] **Calculate the arithmetic mean and the standard deviation of the following values :**

7 , 16 , 13 , 9 , 5

[b] Represent graphically the function $f : f(x) = x^2 - 2x$ where $x \in [-1 , 3]$ and from the drawing deduce the equation of the axis of symmetry and the maximum or minimum value of the function.

10 El-Menia Governorate

Answer the following questions : (Calculators are allowed)

1 **Choose the correct answer from the given ones :**

1 $\left(\sqrt{7} - \sqrt{5}\right)\left(\sqrt{7} + \sqrt{5}\right) =$ ....

(a) 2 (b) 12 (c) $2\sqrt{7}$ (d) $-2\sqrt{5}$

2 If $x\,y = 3$, then $y \propto$ ....

(a) x (b) $x - 3$ (c) $\frac{1}{x}$ (d) $x + 3$

3 $[1 , 3] - \{0 , 1\} =$ ....

(a) $]1 , 3[$ (b) $]1 , 3]$ (c) $[1 , 3[$ (d) $\{3\}$

4 The arithmetic mean of the set of values 8 , 9 , 7 , 6 and 5 equals ....

(a) 5 (b) 2 (c) 3 (d) 7

5 20% of 10 pounds = ... pounds.

(a) 2 (b) 2.5 (c) 5 (d) 20

6 If the point $(x - 4 , 2 - x)$ where $x \in \mathbb{Z}$ is located in the third quadrant , then $x =$ ....

(a) 2 (b) 3 (c) 4 (d) 6

2 [a] **Find the standard deviation of the values :** 6 , 8 , 10 , 12 and 14

[b] If $X = \{2, 5\}$, $Y = \{1, 2\}$, $Z = \{3\}$, **find :**

1 $n(X \times Z)$ 2 $(Y \cap X) \times Z$

3 [a] If $y \propto \frac{1}{x}$ and $y = 3$ when $x = 2$, **find :**

1 The relation between x and y 2 The value of y when $x = 1.5$

[b] If $X = \{1, 3, 4, 5\}$, $Y = \{1, 2, 3, 4, 5, 6\}$ and R is a relation from X to Y where "a R b" means "a + b = 7" for each $a \in X$, $b \in Y$, write R and represent it by an arrow diagram. Show if R is a function from X to Y or not. Give the reason.

4 [a] If $\frac{x}{y} = \frac{2}{3}$, **find the value of the ratio :** $\frac{3x + 2y}{6y - x}$

[b] If the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = 4x - a$ is represented graphically by a straight line intersecting the x-axis at the point (2 , b) , **find :** a , b

5 [a] If b is the middle proportional between a and c , **prove that :** $\frac{a^2 + b^2}{b^2 + c^2} = \frac{a}{c}$

[b] Represent graphically the following function and from the drawing deduce the coordinates of the curve , and the equation of the symmetry axis and the minimum or the maximum value of the function $f : f(x) = x^2 - 2$, where $x \in [-3, 3]$

17 Assiut Governorate

Answer the following questions : (Calculator is allowed)

1 **Choose the correct answer :**

1 $]2, 5] \cup \{2\} = \ldots\ldots$

(a) $]2, 5[$ (b) $[2, 5[$ (c) $]-\infty, \infty[$ (d) $[2, 5]$

2 $\sqrt{10^2 - 8^2} = \ldots\ldots\ldots\ldots$

(a) 8 (b) 6 (c) 4 (d) 2

3 The solution set of the equation : $x(x - 1) = 0$ in $\mathbb{R}$ is $\ldots\ldots\ldots\ldots$

(a) $\{0\}$ (b) $\{1\}$ (c) $\{0, 1\}$ (d) $\varnothing$

4 If $3a = 8b$, then $a : b = \ldots\ldots\ldots\ldots$

(a) $-8 : 3$ (b) $8 : 3$ (c) $3 : 8$ (d) $-3 : 8$

5 If $xy = 5$, then $y \propto \ldots\ldots$

(a) $\frac{1}{x}$ (b) $x - 5$ (c) x (d) $\frac{1}{y}$

6 If a regular die is thrown once , then the probability of appearance of an odd number is ....

(a) zero (b) $\varnothing$ (c) 1 (d) $\frac{1}{2}$

2 [a] If $X = \{1 , 5 , 6\}$, $Y = \{2 , 4 , 5\}$, find $X \times Y$ and represent it by an arrow diagram.

[b] Represent graphically the quadratic function $f : f(X) = X^2 - 1$, $X \in [-2 , 2]$, **from the graph deduce :**

1 The equation of the axis of symmetry.

2 The maximum value or the minimum value of the function.

3 [a] If $f(X) = 4X + m$, $f(3) = 15$, **find :** the value of m

[b] If $\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a - b + 5c}{3X}$, **then find :** the value of X

4 [a] If $y \propto X$, $y = 3$ when $X = 2$, **find :**

1 The relation between y , X

2 The value of y when $X = \frac{1}{3}$

[b] If b is the middle proportional between a and c , **then prove that :** $\frac{a^2 + b^2}{b^2 + c^2} = \frac{a}{c}$

5 [a] If $X = \{1 , 3 , 5\}$, $Y = \{2 , 3 , 4 , 6\}$ and R is a relation from X to Y where "a R b" means "a + b = 7" for each $a \in X$, $b \in Y$

1 Write R and represent it by an arrow diagram.

2 Show if R is a function or not. If R is a function , find its range.

[b] **Calculate the mean and the standard deviation for the values :** 8 , 9 , 7 , 6 , 5

18 Souhag Governorate

Answer the following questions : ***(Calculators are allowed)***

1 Choose the correct answer :

1 The simplest dispersion measure is ....

(a) the mean. (b) the median. (c) the range. (d) the mode.

2 20% from 100 pounds = ... pounds.

(a) 5 (b) 10 (c) 15 (d) 20

3 $[3 , 7] - \{3 , 7\} =$ ....

(a) $[3 , 7[$ (b) $]3 , 7]$ (c) $]3 , 7[$ (d) $[3 , 7]$

4 The solution set of the equation : $x^2 - 9 = 0$ in $\mathbb{R}$ is ....

(a) $\{-3\}$ (b) $\{3\}$ (c) $\{-3 , 3\}$ (d) $\varnothing$

5 If n (X) = 5 , n (X × Y) = 10 , then n (Y) = ...

(a) 4 (b) 3 (c) 2 (d) 1

6 The relation representing the direct variation between the two variables y and x is ....

(a) $x y = 5$ (b) $y = x + 3$ (c) $\frac{x}{3} = \frac{4}{y}$ (d) $\frac{x}{5} = \frac{y}{2}$

2 [a] If $\frac{x}{y} = \frac{3}{4}$, **find the value of :** $\frac{3x + y}{x + 5y}$

[b] If X = $\{1 , 2 , 3\}$, Y = $\left\{1 , \frac{1}{2} , \frac{1}{3} , \frac{1}{5}\right\}$ and R is a relation from X to Y where "a R b" means "a is the multiplicative inverse of b" for all a ∈ X , b ∈ Y , write R and represent it by an arrow diagram. Is R a function ? Why ?

3 [a] If X = $\{4 , 5 , 7\}$, R is a function on X and R = $\{(a , 5) , (b , 5) , (4 , 7)\}$, **find :**

1 The value of a + b 2 The range of the function.

[b] Represent graphically the function $f : f(x) = 2 - x^2 , x \in [-3 , 3]$,

from the graph deduce :

1 The coordinates of the vertex point of the curve.

2 The equation of the axis of symmetry.

3 The maximum value of the function.

4 [a] If b is the middle proportional between a and c , **prove that :** $\frac{a^2 + b^2}{b^2 + c^2} = \frac{a}{c}$

[b] **From the data of the following table , answer the following questions :**

x	2	4	6
y	6	3	2

1 Show the kind of variation between y and x

2 Find the constant proportional.

3 Find the value of y when $x = 2\frac{2}{5}$

5 [a] If the point (a , 3) is located on the straight line which represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = 4x - 5$, **find :** the value of a

[b] **Find the standard deviation of the set of the values :** 15 , 19 , 20 , 21 , 25

19 Qena Governorate

Answer the following questions : (Calculators are permitted)

1 Choose the correct answer from those given :

1 The ordered pair (x^2, y^2), where $x \neq 0$, $y \neq 0$ lies in the .... quadrant.

(a) first (b) second (c) third (d) fourth

2 The positive square root of mean of the squares of deviations of the values from their arithmetic mean is called ....

(a) the range. (b) the median.

(c) the standard deviation. (d) the mode.

3 If x and $x + 17$ are two prime numbers, then $x =$ ....

(a) 1 (b) 2 (c) 3 (d) 5

4 If $x y = 5$, then $y \propto$ ....

(a) x (b) $\frac{1}{x}$ (c) x^2 (d) $\frac{1}{x^2}$

5 If $X = \{3\}$, then $n(X^2) =$ ....

(a) 1 (b) 9 (c) $\{(3, 3)\}$ (d) 3

6 The ratio between the area of a square of side length ℓ and the area of a square of side length 3ℓ equals ....

(a) 1 : 3 (b) 3 : 1 (c) 1 : 9 (d) 9 : 1

2 [a] If $X = \{1, 2, 3\}$, $Y = \{0, 1, 2, 3, 4\}$ and R is a relation from X to Y where "a R b" means "b − a = 1" for all $a \in X$, $b \in Y$, write R and represent it by an arrow diagram. Show that R is a function and write its range.

[b] If $\frac{a}{4} = \frac{b}{5} = \frac{c}{3}$, **prove that :** $\frac{a - b + c}{a + b - c} = \frac{1}{3}$

3 [a] If $y \propto x$, $y = \frac{5}{6}$ when $x = \frac{1}{6}$, write the relation between y and x, then find the value of x when $y = 15$

[b] If the point $(a, -a)$ lies on the straight line that represents the function $f : f(x) = x - 6$, **find :** the value of a

4 [a] If y is the middle proportional between x and z, **prove that :** $\frac{x z}{y (y + z)} = \frac{x}{x + y}$

[b] If $X = \{2, 3\}$, $Y = \{5\}$, $Z = \{4, 5\}$, **find :**

1 $(X - Y) \times Z$ 2 $X \times (Y \cap Z)$

5 [a] Represent graphically the function $f : f(X) = (X - 3)^2$, $X \in [0 , 6]$,

from the graph find :

1 The vertex of the curve. 2 The maximum or minimum value of the function.

[b] **Calculate the arithmetic mean and the standard deviation for the following data :**

73 , 54 , 62 , 71 , 60

20 Luxor Governorate

Answer the following questions :

1 Choose the correct answer :

1 $\frac{1}{3}$ of the number 3^4 =

(a) 3 (b) 3^2 (c) 3^3 (d) 2^3

2 If $n(X^2) = 4$, $n(X \times Y) = 6$, then $n(Y^2) =$ ....

(a) 12 (b) 9 (c) 6 (d) 3

3 $]4 , 6[\cap \{4 , 6\} =$ ....

(a) $\{5\}$ (b) $[4 , 6]$ (c) $\{4 , 6\}$ (d) $\varnothing$

4 If X , y , z are in continued proportion , then $X =$ ....

(a) $\pm\sqrt{y\,z}$ (b) yz (c) $\frac{y^2}{z}$ (d) $\frac{y}{z}$

5 $\sqrt[3]{64} = \sqrt{\cdots\cdots}$

(a) 2 (b) 16 (c) 8 (d) 4

6 If all the values are equal , then ....

(a) $X - \overline{X} > 0$ (b) $X - \overline{X} < 0$ (c) $\overline{X} = 0$ (d) $\sigma = 0$

2 [a] If $X = \{2 , 1\}$, $Y = \{4 , 0\}$, $Z = \{4 , 5 , -2\}$, **find :**

1 $X \times Y$ 2 $(Y \cap Z) \times X$ 3 $n(Y^2)$

[b] Find the number which if subtracted from the first term of the ratio 15 : 13 and added to the second term , then it becomes 3 : 4

3 [a] If $f(X) = 2X + a$, $g(X) = X^2 + a$ and if $f(2) + g(-4) = 30$, **find :** the value of a

[b] If a , b , c and d are proportional quantities , **prove that :** $\frac{a+c}{b+d} = \frac{a^2+c^2}{a\,b+c\,d}$

4 [a] If $X = \{0 , 1 , 2 , \frac{1}{2}\}$ and R is a relation on X where "a R b" means "a is the multiplicative inverse of b" for each $a \in X$, $b \in X$, write R and represent it by an arrow diagram. Is R a function or not ?

[b] If $y \propto x^3$ and $y = 64$ when $x = 2$, find :

1 The relation between x and y

2 The value of y when $x = \frac{1}{2}$

5 [a] **Calculate the mean and the standard deviation for the values :** 22 , 20 , 20 , 20 , 18

[b] Represent graphically the function $f : f(x) = x^2 - 4x + 5$ where $x \in [0 , 4]$

, then from the graph find :

1 The equation of the axis of symmetry.

2 The maximum or the minimum value of the function.

21 Aswan Governorate

Answer the following questions : *(Calculator is allowed)*

1 **Choose the correct answer :**

1 If $n(X^2) = 9$, $n(X \times Y) = 6$, then $n(Y) =$ ....

(a) 2 (b) 3 (c) 4 (d) 6

2 If $x y = 3$, then $y \propto$ ....

(a) $3x$ (b) $\frac{3}{x}$ (c) $\frac{1}{x}$ (d) $\frac{x}{3}$

3 $[2 , 5] - \{2 , 5\} =$ ....

(a) $[1 , 6]$ (b) $\varnothing$ (c) $]2 , 5[$ (d) $\{0\}$

4 $\sqrt{50} - \sqrt{8} =$ ....

(a) $\sqrt{200}$ (b) $\sqrt{98}$ (c) $\sqrt{42}$ (d) $\sqrt{18}$

5 If $\sum (x - \overline{x})^2 = 48$ of a set of values and the number of these values = 12 , then $\sigma =$ ....

(a) -2 (b) 2 (c) 4 (d) 6

6 If $x - y = 5$, $x + y = \frac{1}{5}$, then $x^2 - y^2 =$ ....

(a) $\frac{1}{25}$ (b) 1 (c) 5 (d) 25

2 [a] If $X = \{1 , 3 , 4 , 5\}$, $Y = \{1 , 2 , 3 , 4 , 5 , 6\}$ and R is a relation from X to Y where "a R b" means "a + b = 7" for each $a \in X$, $b \in Y$

1 Write R and represent it by an arrow diagram.

2 Is R a function ? and why ?

[b] If $y \propto x$ and $y = 6$ when $x = 3$, find :

1 The relation between x and y

2 The value of y when $x = 5$

3 **[a]** Represent graphically the function $f : f(X) = 4 - X^2$, taking $X \in [-3, 3]$ and from the graph deduce : the coordinates of the vertex point of the curve, the maximum value of the function and the equation of line of symmetry.

[b] Find the positive number which if its square is added to the antecedent of the ratio 29 : 46 and subtracted its square from its consequent the ratio becomes 3 : 2

4 **[a]** If the straight line which represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(X) = 6X - a$ intersects the y-axis at the point (b , 2), **find :** the value of each of a and b

[b] The following frequency distribution shows the marks of a number of students in an exam :

Marks	0	1	2	3	4	5	6
Number of students	3	4	6	9	5	3	4

Find the standard deviation of the marks.

5 **[a]** If $X = \{1, 3, 5\}$ and R is a function on X where $R = \{(a, 3), (b, 1), (1, 5)\}$, **find :**

1. The range of the function.
2. The value of a + b

[b] If a , b , c and d are proportional quantities, **prove that :** $\frac{a}{b-a} = \frac{c}{d-c}$

22 New Valley Governorate

Answer the following questions : (Calculator is allowed)

1 **Choose the correct answer from those given :**

1. The next in the pattern : $\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}$ is ....
   (a) $\sqrt{50}$ (b) $\sqrt{75}$ (c) $\sqrt{60}$ (d) $\sqrt{90}$
2. The point (– 3 , 4) lies in the .... quadrant.
   (a) first (b) second (c) third (d) fourth
3. If y varies inversely with X, and $X = \sqrt{3}$ when $y = \frac{2}{\sqrt{3}}$, then the constant proportional equals ....
   (a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) 2 (d) 6
4. If the point (a , 3) is located on the straight line which represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = 4X - 5$, then a = ...
   (a) 2 (b) 3 (c) 4 (d) 5
5. .... is one of the measures of the dispersion.
   (a) The median (b) The arithmetic mean
   (c) The standard deviation (d) The mode

6 If $(x+1)^2$ is one of the linear factors of the expression $(x^2-1)^2$, then the other factor is ……………

(a) $(x-1)^2$ (b) $(x-1)$ (c) (x^2+1) (d) (x^2-1)

2 [a] If $X=\{2 , 3 , 5\}$, $Y=\{4 , 6 , 8 , 10\}$ and R is a relation from X to Y where "a R b" means "2 a = b" for all $a \in X$, $b \in Y$

1 Write R and represent it by an arrow diagram. 2 Show that R is a function.

[b] If $\frac{x}{y}=\frac{2}{3}$, **find the value of the ratio :** $\frac{3x+2y}{6y-x}$

3 [a] If $X \times Y=\{(1 , 1) , (1 , 3) , (1 , 5)\}$, **then find :**

1 X , Y 2 $Y \times X$ 3 Y^2

[b] If $\frac{21x-y}{7x-z}=\frac{y}{z}$, **then prove that :** $y \propto z$

4 [a] If $f(x)=4x+b$ and $\frac{1}{3} f(3)=5$, **find :** the value of b

[b] If a , b , c and d are in continued proportion , **then prove that :** $\frac{a^2-3c^2}{b^2-3d^2}=\frac{b}{d}$

5 [a] **Calculate the standard deviation for the values :** 12 , 13 , 16 , 18 , 21

[b] Represent graphically the function $f : f(x)=(x-3)^2$, $x \in [0 , 6]$, from the graph deduce the vertex of the curve , the minimum value of the function , the equation of the axis of symmetry.

23 South Sinai Governorate

Answer the following questions :

1 **Choose the correct answer from the given answers :**

1 If $(2 , 3) \in \{2 , 5\} \times \{x , 4\}$, then $x=$ ……………

(a) 2 (b) 3 (c) 4 (d) 5

2 If $x y=5$, then $y \propto$ ……………

(a) $\frac{1}{x}$ (b) x (c) $x-5$ (d) $x+5$

3 …………… is one of the measures of the dispersion.

(a) The arithmetic mean (b) The median

(c) The mode (d) The standard deviation

4 The mean of the values 1 , 2 , 3 , 4 and 5 equals ……………

(a) 5 (b) 4 (c) 3 (d) 2

5 $\sqrt[3]{x^6} = \sqrt{\quad}$

(a) x^4 (b) x^3 (c) x^2 (d) x

6 If $\frac{5}{4} + \frac{5}{a} = \frac{5}{2}$, then a = ...

(a) $\frac{5}{2}$ (b) $-\frac{5}{2}$ (c) 4 (d) -4

2 [a] If $X = \{1\}$, $Y = \{2, 3\}$, $Z = \{2, 5, 6\}$,

find : 1 $X \times (Y \cap Z)$ 2 $n(X \times Y)$ 3 $Z - Y$

[b] Represent graphically the function $f : f(x) = x^2 - 4$, $x \in [-3, 3]$, from the graph deduce the vertex of the curve , the minimum value of the function.

3 If $X = \{1, 3, 4, 5\}$, $Y = \{1, 2, 3, 4, 5, 6\}$ and R is a relation from X to Y where "a R b" means "a + b = 7" for all $a \in X$, $b \in Y$, write R , and represent it by an arrow diagram and also by a Cartesian diagram. Is R a function ? and why ?

4 [a] If $y \propto x$, $y = 6$ when $x = 3$, find the value of y when $x = 5$

[b] Find the positive number which if we add its square to each of the two terms of the ratio 5 : 11 it becomes 3 : 5

5 [a] If a , b , c and d are in continued proportion , then prove that : $\frac{c^2 - d^2}{a - c} = \frac{b\,d}{a}$

[b] The following frequency distribution shows the ages of 10 children :

Ages in years	5	8	9	10	12	Total
Number of children	1	2	3	3	1	10

Calculate the standard deviation to the ages in years.

24 North Sinai Governorate

Answer the following questions : *(Calculators are allowed)*

1 Choose the correct answer from those given :

1 $\sqrt{16} + \sqrt[3]{-8} =$ ....

(a) -4 (b) -2 (c) 2 (d) 4

2 If $(9, 4) \in \{9, 7\} \times \{x, 5\}$, then $x =$ ....

(a) 9 (b) 4 (c) 7 (d) 5

3 If $x^2 - y^2 = 12$, $x + y = 4$, then $x - y =$ ....

(a) -3 (b) 3 (c) 4 (d) 12

4 The fourth proportional of the quantities 2 , 3 , 6 equals ....

(a) 9 (b) 3 (c) 12 (d) 18

5 If $\frac{3}{4} + \frac{3}{X} = \frac{3}{2}$, then $X =$ ....

(a) 2 (b) 3 (c) $\frac{3}{2}$ (d) 4

6 The range of the set of the values 3 , 5 , 6 , 7 , 9 equals ....

(a) 3 (b) 4 (c) 6 (d) 12

---

2 [a] If $X = \{2 , 3 , 4\}$, $Y = \{2 , 3 , 4 , 5 , 6 , 7 , 8\}$ and R is a relation from X to Y where "a R b" means "$a = \frac{1}{2}$ b" for all $a \in X$, $b \in Y$

1 Write R and represent it by an arrow diagram.

2 Show that R is a function from X to Y and find its range.

[b] If $y \propto X$ and y = 2 when $X = 8$, find the value of y when $X = 12$

---

3 [a] If $f(X) = 4X + b$, $f(3) = 15$, **find :** the value of b

[b] If $\frac{X}{y} = \frac{2}{3}$, **then find the value of :** $\frac{3X + 2y}{6y - X}$

---

4 [a] If $(6 , b - 3) = (2 - a , - 1)$, **find :** the value of a + b

[b] If b is the middle proportional between a and c , **prove that :** $\frac{a^2 + b^2}{b^2 + c^2} = \frac{a}{c}$

---

5 [a] **Calculate the arithmetic mean and the standard deviation of the set of the values :**
23 , 12 , 17 , 13 , 15

[b] Graph the function $f : f(X) = 4 - X^2$ where $X \in [-3 , 3]$ **and from the graph find :**

1 The vertex of the curve. 2 The equation of the axis of symmetry.

3 The maximum value of the function.

25 Red Sea Governorate

Answer the following questions : *(Calculators are allowed)*

1 **Choose the correct answer from those given :**

1 The range of the set of the values 7 , 3 , 6 , 9 , 5 equals ....

(a) 3 (b) 4 (c) 6 (d) 12

2 If $X = 3$, $y = 5$, then $y^X =$ ....

(a) 243 (b) 125 (c) 15 (d) 8

3 The relation which represents the direct variation between the two variables X and y is ....

(a) $X y = 5$ (b) $y = X + 3$ (c) $\frac{X}{3} = \frac{4}{y}$ (d) $\frac{X}{5} = \frac{y}{2}$

4 If $X - y = 5$, $X + y = 1$, then $X^2 - y^2 =$ ....

(a) 5 (b) 4 (c) 25 (d) $\frac{1}{25}$

5 If $n(X^2) = 9$, then $n(X) =$ ....

(a) 1 (b) 3 (c) 6 (d) 9

6 $[3 , 5] -]3 , 5] =$ ....

(a) $[3 , 5[$ (b) $\{3 , 5\}$ (c) $\{3\}$ (d) $\{5\}$

2 [a] If $X \times Y = \{(2 , 2) , (2 , 5) , (2 , 7)\}$, **find :**

1 Y 2 X^2

[b] If $5a = 3b$, **find the value of :** $\frac{7a + 9b}{4a + 2b}$

3 [a] If $y \propto \frac{1}{X}$ and $y = 3$ when $X = 2$, **then find :**

1 The relation between X and y 2 The value of y when $X = 1.5$

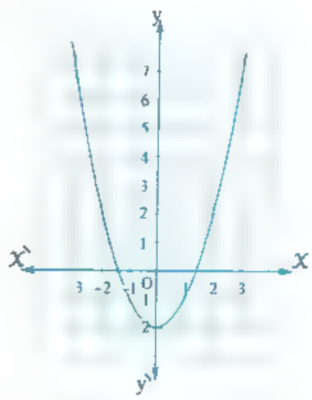
[b] The opposite figure represents the function $f : f(X) = X^2 - 2$

Find :

1 The point of the vertex of the curve.

2 The equation of the line of symmetry.

3 The maximum or minimum value of the function.



4 [a] If $X = \{-2 , -1 , 0 , 1 , 2\}$, R is a relation on X where "a R b" means "a is the additive inverse of b" for each $a \in X$, $b \in X$, write R and represent it by an arrow diagram.

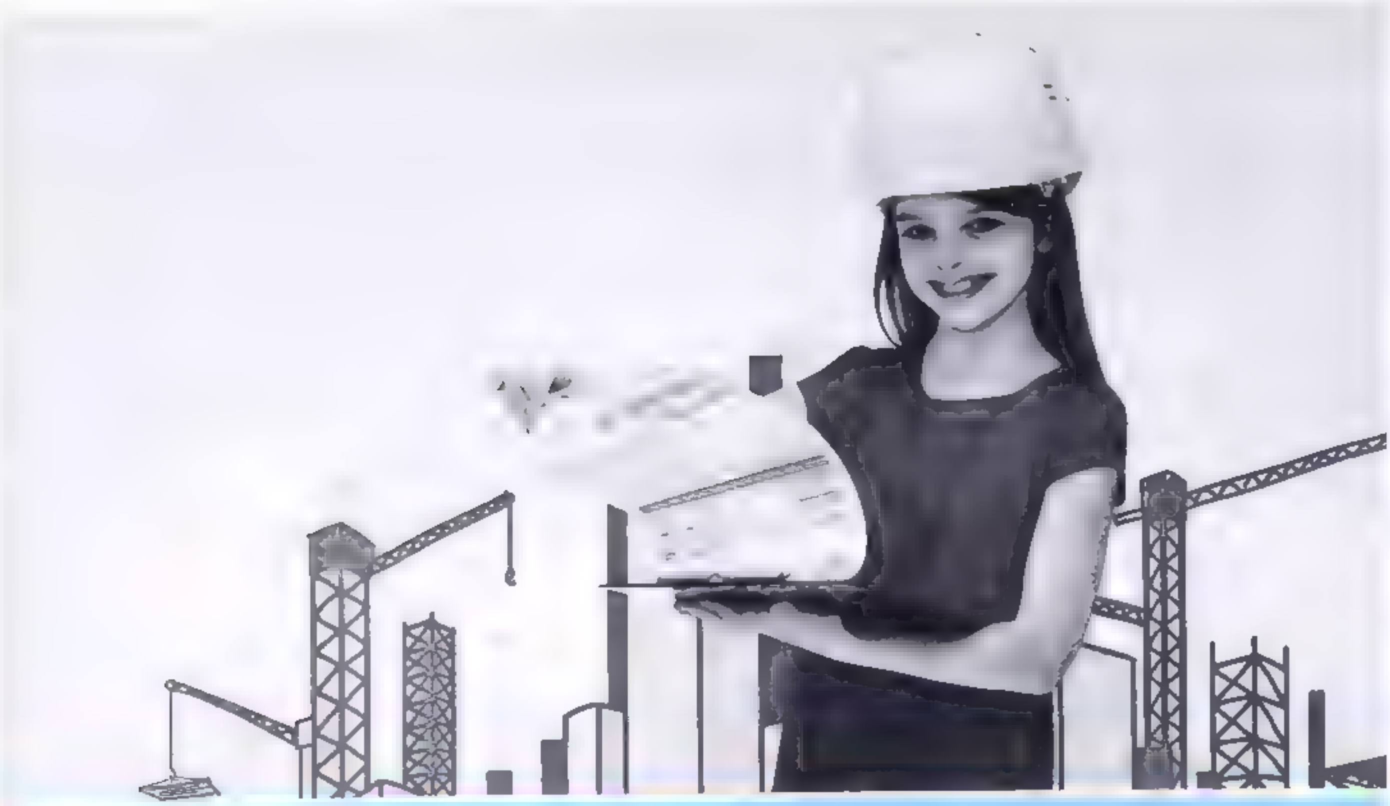
[b] If b is the middle proportional between a and c , **prove that :** $\frac{a - b}{a - c} = \frac{b}{b + c}$

5 [a] Represent graphically $f : f(X) = X - 3$, then find the points of intersection with X-axis and y-axis.

[b] **Calculate the standard deviation for the values :** 8 , 9 , 7 , 6 , 5

Second

Trigonometry and Geometry



Accumulative Tests

on Trigonometry and Geometry

Accumulative test 1 on lesson 1 – unit 4

1 Choose the correct answer from those given :

1 If X , y are the measures of two complementary angles and $\sin X = \frac{3}{5}$, then cos y = ... « El-Beheira 18 »

(a) $\frac{4}{5}$ (b) $\frac{3}{5}$ (c) $\frac{3}{4}$ (d) $\frac{5}{3}$

2 For any two acute angles A and B if m (∠ A) + m (∠ B) = 90° , m (∠ A) ≠ m (∠ B) , then .... « Alexandria 19 »

(a) sin A = cos B (b) sin A = sin B (c) tan A = tan B (d) cos A = cos B

3 If $\sin X = \cos X$, then X = ...° (X is the measure of an acute angle) « Souhag 19 »

(a) 30 (b) 45 (c) 60 (d) 90

4 For any angle A , $\frac{\sin A}{\cos A}$ = ... « New Valley 19 »

(a) sin A (b) cos A (c) tan A (d) 1

5 Δ ABC is a right-angled triangle at B , then sin C + cos C .... 1 « Damietta 16 »

(a) = (b) > (c) < (d) ≤

6 ABC is a right-angled triangle at B , and $2\,AB = \sqrt{3}\,AC$, then cos C = ... « New Valley 17 »

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{3}$ (d) 1

7 The surface area of a square is 25 cm^2 , then the length of its diagonal is .... cm. « El-Monofia 20 »

(a) 5 (b) 10 (c) $5\sqrt{2}$ (d) $10\sqrt{2}$

8 Δ ABC is a right-angled triangle at A , then cosine angle B : sine angle C equals .... « El-Sharkia 18 »

(a) $\frac{3}{5}$ (b) $\frac{4}{3}$ (c) $\frac{3}{4}$ (d) 1

2 In the opposite figure :

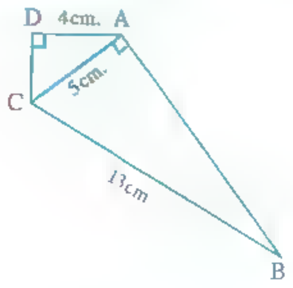
ABC is an isosceles triangle where

AB = AC = 10 cm. , BC = 12 cm.

Find : 1 sin B

2 The area of the triangle ABC « El-Sharkia 19 »

3 **In the opposite figure :**



m (∠ ADC) = 90° , m (∠ BAC) = 90°

, AD = 4 cm. , AC = 5 cm. , BC = 13 cm.

Find the value of each of :

1 tan (∠ ACB) + tan (∠ ACD)

2 sin (∠ B) cos (∠ CAD) + cos (∠ B) sin (∠ CAD)

« El Gharbia 17 »

Accumulative test 2 till lesson 2 – unit 4

1 **Choose the correct answer from those given :**

1 If $\cos 3X = \frac{1}{2}$ where $(3X)$ is the measure of an acute angle , then $X =$ .... « El Sharkia 17 »

(a) 15° (b) 20° (c) 30° (d) 45°

2 If $\tan \frac{3X}{2} = 1$ where X is the measure of an acute angle , then $X =$ .... « Qena 16 »

(a) 15° (b) 30° (c) 45° (d) 60°

3 If m (∠ A) = 75° , sin B = cos A , ∠ B is acute , then m (∠ B) = ... « El Dakahlia 20 »

(a) 45° (b) 75° (c) 15° (d) 105°

4 If $\sin X = \frac{1}{2}$, X is the measure of an acute angle , then $\sin 2X =$ .... « Damietta 20 »

(a) $\frac{\sqrt{3}}{3}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{\sqrt{2}}{2}$ (d) 1

5 If $\tan (X + 10°) = \sqrt{3}$ where X is the measure of an acute angle , then $X =$ .... « El-Fayoum 19 »

(a) 60° (b) 30° (c) 50° (d) 70°

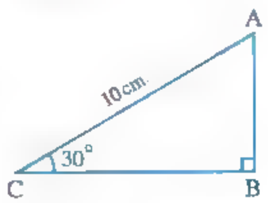
6 If ABCD is a square , then m (∠ CAB) = ... « Kafr El-Sheikh 19 »

(a) 90° (b) 45° (c) 60° (d) 30°

7 If 4 cos 60° sin 30° = tan X , then $X =$ .... where X is the measure of an acute angle. « Qena 18 »

(a) 45° (b) 30° (c) 60° (d) 80°

8 **In the opposite figure :**



AB = ... cm.

(a) 5 (b) 15

(c) 20 (d) 40

« Assiut 20 »

2 **ABC is a right-angled triangle at B**

1 **Prove that :** $\sin^2 A + \cos^2 A = 1$

2 If AB = 5 cm. , AC = 13 cm.

, **find :** m (∠ C) to the nearest minute. « El-Dakahlia 19 »

3 **Find the value of X if :** $4X = \cos^2 30° \tan^2 30° \tan^2 45°$ « Alexandria 17 »

Accumulative test 3 till lesson 1 – unit 5

1 Choose the correct answer from those given :

1 The distance between the two points (2 , 0) and (5 , 0) is .... length unit.

« Cairo 17 »

(a) 7 (b) $\sqrt{29}$ (c) $3\frac{1}{2}$ (d) 3

2 The distance between the point (– 6 , 8) and y-axis is .... length unit.

« Damietta 16 »

(a) 6 (b) – 6 (c) 8 (d) – 8

3 The distance between the point A $(\sqrt{2}, 4)$ and the origin point is .... length unit.

« El Sharkia 19 »

(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $3\sqrt{2}$ (d) $4\sqrt{2}$

4 The number of axes of symmetry of any isosceles triangle is « Ben Suef 18 »

(a) 0 (b) 1 (c) 2 (d) 3

5 A circle its centre is the origin and its radius length is 2 length unit , then the point .... belongs to it.

« Alexandria 19 »

(a) (1 , – 2) (b) $(-2, \sqrt{5})$ (c) (0 , 1) (d) $(\sqrt{3}, 1)$

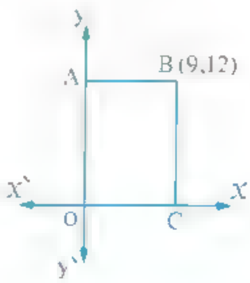
6 If $\cos \frac{X}{2} = \frac{\sqrt{3}}{2}$ where $\frac{X}{2}$ is the measure of an acute angle , then $\tan (X - 15°) =$ ....

« El-Monofia 20 »

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) 1 (d) $\frac{\sqrt{3}}{2}$

7 **In the opposite figure :**

OABC is a rectangle in the Cartesian coordinates plane , then AC = ... length unit.



(a) 12 (b) 9

(c) 15 (d) 25

« El Monofia 17 »

8 2 sin 30° cos 30° = ...

« Suez 16 »

(a) $\frac{1}{2}$ (b) 1 (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{3}}$

2 ABCD is a quadrilateral where :

A $(2 , 4)$, B $(-3 , 0)$, C $(-7 , 5)$ and D $(-2 , 9)$

Prove that : ABCD is a square. « North Sinai 20 »

---

3 Find m $(\angle X)$ where X is an acute angle if : $3 \tan^2 X = 4 \sin^2 30° + 8 \cos^2 60°$

« El-Fayoum 18 »

Accumulative test 4 till lesson 2 – unit 5

1 **Choose the correct answer from those given :**

1 If A $(-1, 2)$, B $(5, -10)$, then the midpoint of $\overline{AB}$ is .... « El-Gharbia 17 »

(a) $(-4, -2)$ (b) $(-2, 4)$ (c) $(2, -4)$ (d) $(2, 4)$

2 If the origin point is the midpoint of $\overline{AB}$, where A $(5, -2)$, then the point B is .... « Port Said 19 »

(a) $(2, 5)$ (b) $(5, -2)$ (c) $(-2, -5)$ (d) $(-5, 2)$

3 If $(3, -1)$ is the midpoint of $\overline{AB}$ where A $(X, 2)$, B $(-1, -4)$, then $X =$ .... « El-Kalyoubia 16 »

(a) 17 (b) 6 (c) 13 (d) 7

4 ABC is a right-angled triangle at B, then $\sin A + 2 \cos C =$ .... « El-Gharbia 20 »

(a) $2 \sin C$ (b) $3 \sin A$ (c) $2 \sin A$ (d) $3 \cos A$

5 If the side lengths of a triangle are 5 cm., 12 cm. and 13 cm., then its area equals .... cm^2. « Matrouh 18 »

(a) 30 (b) 32.5 (c) 78 (d) 144

6 If $\cos 4X = \frac{1}{2}$ where $4X$ is the measure of an acute angle, then $X =$ .... « Ismailia 18 »

(a) 30° (b) 45° (c) 60° (d) 15°

7 If $\overline{AB}$ is a diameter in a circle of centre M, where A $(2, 4)$ and B $(-2, 0)$, then M = ... « Beni Suef 20 »

(a) $(0, 2)$ (b) $(2, 0)$ (c) $(0, 0)$ (d) $(2, 2)$

8 If the X-axis bisects $\overline{AB}$ such that A $(3, 2)$ and B $(-2, y)$, then y = ... « El-Dakahlia 17 »

(a) 3 (b) 2 (c) -2 (d) 4

---

2 ABCD is a trapezium, $\overline{AD} \parallel \overline{BC}$, m $(\angle B) = 90°$, AB = 3 cm., BC = 6 cm., AD = 2 cm. Find the length of $\overline{DC}$ and the value of $\cos (\angle BCD)$ « El-Beheira 19 »

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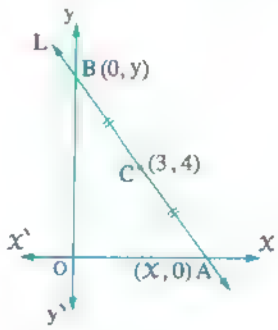
3 **In the opposite figure :**

The point C is the midpoint of $\overline{AB}$ where C $(3, 4)$

Find the perimeter of the triangle AOB



« El-Kalyoubia 20 »

Accumulative test 5 till lesson 3 – unit 5

1 **Choose the correct answer from those given :**

1 The slope of the straight line which makes with the positive direction of X-axis an angle whose positive measure is $X°$ equals .... « Giza 20 »

(a) $\sin X$ (b) $\cos X$ (c) $\frac{\sin X}{\cos X}$ (d) $\sin X + \cos X$

2 The slope of the straight line which passes through the two points (2 , 3) and (2 , – 3) is .... « El-Sharkia 18 »

(a) zero (b) $\frac{-3}{2}$ (c) $\frac{3}{2}$ (d) undefined

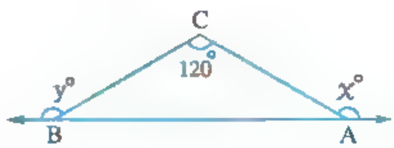
3 The slope of the straight line L_1 is $\frac{a}{5}$ and the slope of the straight line L_2 is $\frac{-b}{3}$ where $a \neq 0$, $b \neq 0$ and $L_1 \perp L_2$, then ab = ... « El-Sharkia 19 »

(a) $\frac{3}{5}$ (b) $\frac{-3}{5}$ (c) 15 (d) – 15

4 If the two straight lines whose slopes are $\frac{2}{3}$ and $\frac{k}{2}$ are parallel , then k = ... « Alexandria 17 »

(a) $\frac{-3}{4}$ (b) $\frac{1}{3}$ (c) 3 (d) $\frac{-4}{3}$

5 **In the opposite figure :**



If m (∠ C) = 120°

, then $X° + y°$ = ...

(a) 90° (b) 180°

(c) 300° (d) 360° « Matrouh 20 »

6 The slope of the perpendicular straight line to the straight line which passes through the two points (2 , 3) and (5 , 1) equals .... « Giza 17 »

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{-3}{2}$ (d) $\frac{-2}{3}$

7 The distance between the point (– 3 , – 4) and y-axis is .... length unit. « El-Sharkia 16 »

(a) 4 (b) – 4 (c) 3 (d) – 3

8 If A (5 , 7) and B (1 , – 1) , then the midpoint of $\overline{AB}$ is .... « El-Beheira 20 »

(a) (2 , 3) (b) (3 , 3) (c) (3 , 2) (d) (3 , 4)

2 ABCD is a quadrilateral , where A $(2 , 3)$, B $(6 , 2)$, C $(-2 , -2)$ and D $(-2 , 1)$

Prove that : ABCD is a trapezoid. « Damietta 18 »

3 If ABCD is a parallelogram in which A $(3 , 3)$, B $(2 , -2)$ and C $(5 , -1)$, **find :**

1. The coordinates of the point of intersection of its diagonals.
2. The coordinates of the point D « El Sharkia 19 »

Accumulative test 6 till lesson 4 – unit 5

1 Choose the correct answer from those given :

1 The equation of the straight line which passes through the origin point and makes with the positive direction of X-axis an angle of measure 60° is .... « El Sharkia 19 »

(a) $X = \sqrt{3}\, y$ (b) $y = \sqrt{3}\, X + 2$ (c) $y = 3\, X$ (d) $y = \sqrt{3}\, X$

2 The straight line whose equation is $3\, y = 4\, X - 12$ intercepts from the negative direction of y-axis a part of length .... length unit. « Giza 17 »

(a) $\frac{4}{3}$ (b) 3 (c) 4 (d) – 4

3 The perpendicular length between $X = 5$ and $X + 3 = 0$ equals .... length unit. « El-Kalyoubia 20 »

(a) 2 (b) 8 (c) – 8 (d) 5

4 In the square XYZL , if the slope of $\overleftrightarrow{XZ} = 1$, then the slope of $\overleftrightarrow{YL} =$ .... « El-Sharkia 20 »

(a) 1 (b) – 1 (c) ± 1 (d) 45°

5 The equation of the straight line which passes through the point (2 , – 3) and is parallel to X-axis is .... « North Sinai 16 »

(a) $X = 2$ (b) $y = -3$ (c) $X = -3$ (d) $y = 3$

6 If the lengths 3 , 7 , ℓ are lengths of sides of a triangle , then ℓ can be equal to .... « El-Gharbia 19 »

(a) 3 (b) 7 (c) 4 (d) 10

7 If the two straight lines : $X + y = 5$, $k\, X + 2\, y = 0$ are perpendicular , then k = ... « New Valley 20 »

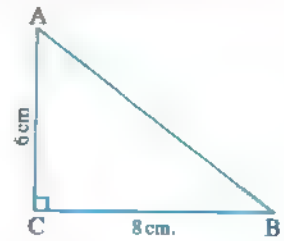
(a) 1 (b) – 1 (c) 2 (d) – 2

8 If $\tan (X + 20°) = \sqrt{3}$ where X is the measure of an acute angle , then $X =$ .... « El-Sharkia 18 »

(a) 20° (b) 30° (c) 40° (d) 50°

2 If A (– 2 , 3) , B (0 , 5) , C is the midpoint of $\overline{AB}$, find the equation of the straight line perpendicular to $\overleftrightarrow{AB}$ and passing thorugh the point C « El Monofia 18 »

3 **In the opposite figure :**



ABC is a right-angled triangle

at C , AC = 6 cm. , BC = 8 cm.

Find : 1 cos A cos B – sin A sin B

2 m (∠ B)

« El-Beheira 20 »

Final Revision

on Trigonometry and Geometry

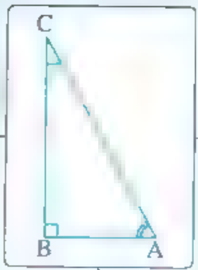
Revision for the important rules and laws of Trigonometry and Geometry

First Trigonometry

Remember: The main trigonometrical ratios of the acute angle and the important relations between them

The trigonometrical ratios of the angle A

- $\sin A = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{BC}{AC}$
- $\cos A = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{AB}{AC}$
- $\tan A = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{BC}{AB}$



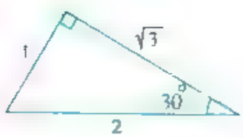
The trigonometrical ratios of the angle C

- $\sin C = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{AB}{AC}$
- $\cos C = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{BC}{AC}$
- $\tan C = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{AB}{BC}$

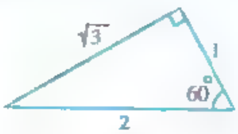
Some important relations

- $\tan A = \frac{\sin A}{\cos A}$
- If $m(\angle A) + m(\angle C) = 90°$, then $\sin A = \cos C$, $\cos A = \sin C$
- If $\sin A = \cos C$ or $\cos A = \sin C$, then $m(\angle A) + m(\angle C) = 90°$

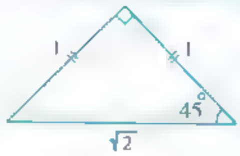
Remember: The trigonometrical ratios of some angles



- $\sin 30° = \frac{1}{2}$
- $\cos 30° = \frac{\sqrt{3}}{2}$
- $\tan 30° = \frac{1}{\sqrt{3}}$



- $\sin 60° = \frac{\sqrt{3}}{2}$
- $\cos 60° = \frac{1}{2}$
- $\tan 60° = \sqrt{3}$



- $\sin 45° = \frac{1}{\sqrt{2}}$
- $\cos 45° = \frac{1}{\sqrt{2}}$
- $\tan 45° = 1$

Notice that :

If $\cos \theta = 0.7152$, then we use the calculator to find θ by using the keys as the following sequence from left : SHIFT cos . 7 1 5 2 = °'''

Then $\theta \simeq 44° \, 20' \, 25''$

Second Analytical geometry

Remember The important laws

If $A(x_1, y_1)$, $B(x_2, y_2)$

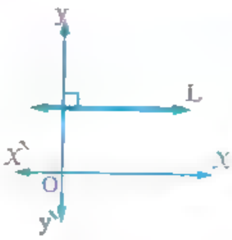
- The law of the distance between the two points A, B (the length of $\overline{AB}$):
  $AB = \sqrt{(\text{difference between } x\text{-coordinates})^2 + (\text{difference between y-coordinates})^2}$
- The law of finding the coordinates of the midpoint of $\overline{AB}$:
  The midpoint of $\overline{AB} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
- The law of finding the slope of the straight line $\overleftrightarrow{AB}$:
  $m = \frac{y_2 - y_1}{x_2 - x_1}$

Remember How to find the slope of the straight line

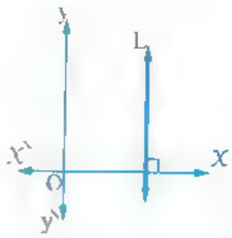
1	Given two points on the line as : $A(x_1, y_1)$, $B(x_2, y_2)$	$m = \frac{y_2 - y_1}{x_2 - x_1}$
2	Given the measure of the positive angle which the straight line makes with the positive direction of x-axis, say θ	$m = \tan\theta$
3	Given the equation of the straight line in the form : $y = b\,x + c$	$m = b$ where b is the coefficient of x
4	Given the equation of the straight line in the form : $a\,x + b\,y + c = 0$	$m = \frac{-\text{coefficient of } x}{\text{coefficient of y}} = \frac{-a}{b}$
5	Given the slope of the parallel straight line to it, say m_1	$m = m_1$ because the two slopes are equal.
6	Given the slope of the perpendicular straight line to it, say m_2	$m = \frac{-1}{m_2}$ because : $m \times m_2 = -1$

Important remarks on the slope of the straight line

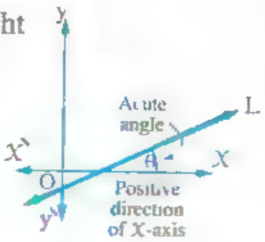
- The slope of X-axis equals 0
- The slope of the straight line parallel to X-axis equals 0



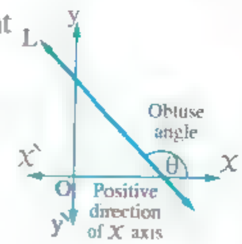
- The slope of y-axis is undefined.
- The slope of the straight line parallel to y-axis is undefined.



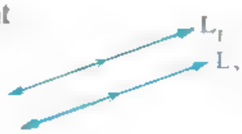
- The slope of the straight line which makes an acute angle with the positive direction of X-axis is positive.



- The slope of the straight line which makes an obtuse angle with the positive direction of X-axis is negative.

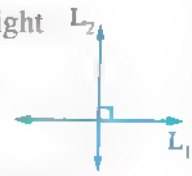


- The two parallel straight lines their slopes are equal.



i.e. If $L_1 // L_2$, then $m_1 = m_2$

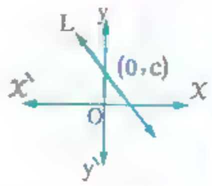
- The two perpendicular straight lines the product of their slopes equals -1



i.e. If $L_1 \perp L_2$, then $m_1 \times m_2 = -1$

Remember : The equation of the straight line

- The equation of the straight line whose slope = m and cuts y-axis at the point $(0 , c)$ is : $y = m X + c$



For example :

- The equation of the straight line whose slope is -2 and cuts from the positive part of y-axis 7 units is : $y = -2 X + 7$
- To find the equation of the straight line whose slope is 3 and passes through the point $(1 , -2)$:

$\because$ The slope = 3 $\therefore$ The equation of the straight line is : $y = 3 X + c$

, then substitute by the point $(1 , -2)$ to find the value of c as the following :

$-2 = 3 \times 1 + c$, then : $c = -5$

$\therefore$ The equation of the straight line is : $y = 3 X - 5$

Important remarks on the equation of the straight line

1. The equation of the straight line which passes through the origin point O (0 , 0) is :
 $y = m\,x$ where m is the slope.
2. The equation of x-axis is : $y = 0$ and the equation of y-axis is : $x = 0$
3. The equation of the straight line parallel to x-axis and cuts y-axis at the point (0 , c) is :
 $y = c$
4. The equation of the straight line parallel to y-axis and cuts x-axis at the point (a , 0) is :
 $x = a$

Remember **Some rules and remarks which help you to solve the exercises**

1 To prove that the points A , B and C are collinear

We will prove that :

- The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$

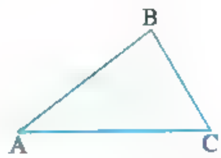
or • AB + BC = AC (where AC is the greatest length)

2 To prove that the points A , B and C are vertices of a triangle

We prove that :

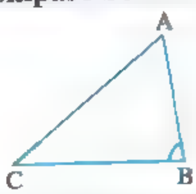
- The slope of $\overleftrightarrow{AB}$ ≠ the slope of $\overleftrightarrow{BC}$

or • AB + BC > AC (where AC is the greatest length)



3 To determine the type of the triangle ABC according to its angle measures

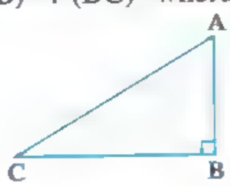
We compare between : $(AC)^2$, $(AB)^2 + (BC)^2$ where $\overline{AC}$ is the longest side , if :



$(AC)^2 < (AB)^2 + (BC)^2$

, then :

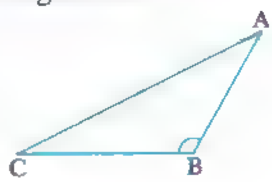
Δ ABC is acute-angled.



$(AC)^2 = (AB)^2 + (BC)^2$

, then :

Δ ABC is right-angled at B



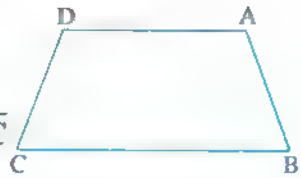
$(AC)^2 > (AB)^2 + (BC)^2$

, then :

Δ ABC is obtuse-angled at B

4 To prove that the quadrilateral ABCD is a trapezium

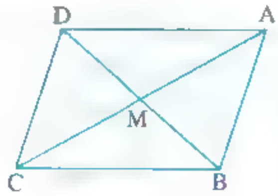
We prove that :



The slope of $\overleftrightarrow{AD}$ = the slope of $\overleftrightarrow{BC}$, then $\overline{AD} // \overline{BC}$

, the slope of $\overleftrightarrow{AB} \neq$ the slope of $\overleftrightarrow{DC}$, then $\overline{AB}$ is not parallel to $\overline{DC}$

5 To prove that the quadrilateral ABCD is a parallelogram



- **By using the slope , we prove that :**

  The slope of $\overleftrightarrow{AD}$ = the slope of $\overleftrightarrow{BC}$, then $\overline{AD} // \overline{BC}$

  , the slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{DC}$, then $\overline{AB} // \overline{DC}$

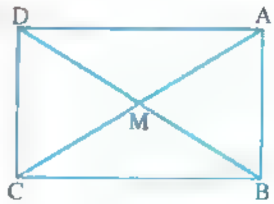
- **By using the distance between two points , we prove that :**

  The length of $\overline{AD}$ = the length of $\overline{BC}$, the length of $\overline{AB}$ = the length of $\overline{DC}$

- **By using the midpoint of a line segment , we prove that :**

  The midpoint of $\overline{AC}$ is the midpoint of $\overline{BD}$, then : $\overline{AC}$, $\overline{BD}$ bisect each other.

6 To prove that the quadrilateral ABCD is a rectangle



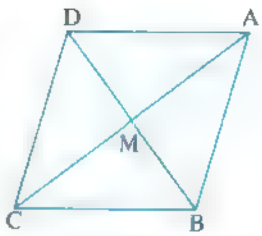
* **First we prove that :** The quadrilateral ABCD is a parallelogram by one of the previous methods

, then prove that :

- AC = BD (By using the distance between two points)

or • The slope of $\overleftrightarrow{AB}$ × the slope of $\overleftrightarrow{BC}$ = – 1 , then $\overline{AB} \perp \overline{BC}$

7 To prove that the quadrilateral ABCD is a rhombus



* **First we prove that :** The quadrilateral ABCD is a parallelogram

, then prove that :

- AB = BC (By using the distance between two points)

or • The slope of $\overleftrightarrow{AC}$ × the slope of $\overleftrightarrow{BD}$ = – 1 , then $\overline{AC} \perp \overline{BD}$

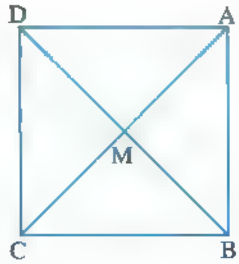
* We can prove that the quadrilateral ABCD is a rhombus directly by using the distance between two points

We prove that :

AB = BC = CD = DA

8 To prove that the quadrilateral ABCD is a square

* **First we prove that :** The quadrilateral ABCD is a parallelogram



, then prove that :

• AB = BC (By using the distance between two points)

and the slope of $\overleftrightarrow{AB}$ × the slope of $\overleftrightarrow{BC} = -1$, then $\overline{AB} \perp \overline{BC}$

or • AC = BD (By using the distance between two points)

and the slope of $\overleftrightarrow{AC}$ × the slope of $\overleftrightarrow{BD} = -1$, then $\overline{AC} \perp \overline{BD}$

* We can prove that the quadrilateral ABCD is a square by using the distance between two points

We prove that :

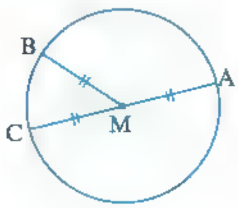
AB = BC = CD = DA , then the quadrilateral is a rhombus

, then prove that : AC = BD

9 To prove that the points A , B and C lie on one circle of centre M

By using the distance between two points

We prove that : MA = MB = MC



2022

Final Examinations

on Trigonometry and Geometry

Model 1

Answer the following questions :

1 Choose the correct answer from those given :

1. tan 45° = ...
   (a) 1 (b) $2\sqrt{2}$ (c) $\frac{1}{2}$ (d) $\sqrt{2}$
2. If $\sin X = \frac{1}{2}$, X is an acute angle , then m (∠ X) = ...
   (a) 45° (b) 60° (c) 30° (d) 90°
3. The distance between the two points (3 , 0) and (0 , – 4) equals .... length units.
   (a) 4 (b) 5 (c) 6 (d) 7
4. If $x + y = 5$, $k x + 2 y = 0$ are perpendicular , then k = ...
   (a) – 2 (b) – 1 (c) 1 (d) 2
5. If A (5 , 7) , B (1 , – 1) , then the midpoint of $\overline{AB}$ is ....
   (a) (2 , 3) (b) (3 , 3) (c) (3 , 2) (d) (3 , 4)
6. The equation of the straight line which passes through the point (3 , – 5) and parallel to y-axis is ....
   (a) $x = 3$ (b) $y = -5$ (c) $y = 2$ (d) $x = -5$

---

2 [a] **Without using calculator , prove that :** sin 60° = 2 sin 30° cos 30°

[b] Prove that the points A (– 3 , – 1) , B (6 , 5) and C (3 , 3) are collinear.

---

3 [a] If 4 cos 60° sin 30° = tan x , find the value of x , where x is the measure of an acute angle.

[b] If the midpoint of $\overline{AB}$ is C (6 , – 4) where A (5 , 3) , find the point B

---

4 [a] If the straight line L_1 passes through the points (3 , 1) , (2 , k) and the straight line L_2 makes with the positive direction of the x-axis an angle of measure 45° , find the value of k if $L_1 // L_2$

[b] ABC is a right-angled triangle at C , AC = 6 cm. , BC = 8 cm.

Find : 1. cos A cos B – sin A sin B

2. m (∠ B)

5 [a] Find the equation of the straight line whose slope is 2 and passes through the point $(1 , 0)$

[b] Prove that the points $A(3 , -1)$, $B(-4 , 6)$ and $C(2 , -2)$ which belongs to an orthogonal Cartesian coordinates plane lie on the circle whose centre is $M(-1 , 2)$

Find the circumference of the circle.

Model 2

Answer the following questions :

1 **Choose the correct answer from those given :**

1 $2 \sin 30° \tan 60° =$ ....

(a) $\sqrt{3}$ (b) 3 (c) $\frac{\sqrt{3}}{3}$ (d) $\frac{1}{2}$

2 The equation of the straight line which passes through the point $(-2 , -3)$ and parallel to X-axis is ......

(a) $X = -2$ (b) $X = -3$ (c) $y = -2$ (d) $y = -3$

3 If $\cos X = \frac{\sqrt{3}}{2}$, X is the measure of an acute angle , then $\sin 2X =$ ....

(a) 1 (b) $\frac{\sqrt{3}}{2}$ (c) -2 (d) $\frac{1}{\sqrt{3}}$

4 A circle of centre at the origin point and its radius length is 2 length units , which of the following points belongs to the circle ?

(a) $(1 , -2)$ (b) $\left(-2 , \sqrt{5}\right)$ (c) $\left(\sqrt{3} , 1\right)$ (d) $(0 , 1)$

5 The perpendicular distance between the two straight lines : $X - 2 = 0$, $X + 3 = 0$ equals .... length units.

(a) 1 (b) 5 (c) 2 (d) 3

6 If $\frac{-3}{2}$, $\frac{6}{k}$ are the slopes of two parallel straight lines , then $k =$ ....

(a) 6 (b) -4 (c) $\frac{3}{2}$ (d) 2

2 [a] If $\cos E \tan 30° = \cos^2 45°$, **find :** $m(\angle E)$, E is an acute angle.

[b] Show the type of the triangle whose vertices are $A(3 , 3)$, $B(1 , 5)$ and $C(1 , 3)$ due to its side lengths.

3 [a] Find the equation of the straight line which passes through the points $(1 , 3)$ and $(-1 , -3)$ and prove that it is passing through the origin point.

[b] If the point $(3 , 1)$ is the midpoint of $(1 , y)$, $(X , 3)$, find the point (X , y)

4 [a] Find the equation of the straight line which intercepts from the two axes two positive parts of lengths 1 and 4 for X and y axes respectively and find its slope.

[b] ABC is a right-angled triangle at B , AC = 10 cm. and BC = 8 cm.

Prove that : $\sin^2 A + 1 = 2\cos^2 C + \cos^2 A$

5 [a] Prove that the straight line which passes through the points (1 , 3) and (2 , 4) is parallel to the straight line : $3y - X - 1 = 0$

[b] ABCD is a trapezium , $\overline{AD} \parallel \overline{BC}$, $m(\angle B) = 90^\circ$, AB = 3 cm. , BC = 6 cm. and AD = 2 cm.

Find : The length of $\overline{DC}$ and the value of $\cos(\angle BCD)$

Model for the merge students

Answer the following questions :

1 Put (✓) or (✗) :

1. The distance between the points (9 , 0) , (4 , 0) equals 5 length units. ( )
2. If tan E = 1 , then m (∠ E) = 45° ( )
3. The straight line $y = 2X + 1$ intercepts a part of length -1 from y-axis ( )
4. If $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$, then the slope of $\overleftrightarrow{AB}$ × the slope of $\overleftrightarrow{CD} = 1$
   (both of $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ aren't parallel to any axis) ( )
5. $\tan 60° = \frac{1}{\sqrt{3}}$ ( )
6. If A (1 , 2) , B (3 , 4) , then the midpoint of $\overline{AB}$ is (2 , 3) ( )

2 Choose the correct answer from those given :

1. The distance between the point (4 , 3) and X-axis is .... length units.
   (a) -3 (b) 3 (c) 4 (d) -4
2. $4 \cos 30° \tan 60° =$ ....
   (a) 3 (b) $2\sqrt{3}$ (c) 6 (d) 12
3. If $X + y = 5$, $kX + 2y = 0$ are parallel , then k = ...
   (a) -2 (b) -1 (c) 1 (d) 2
4. The points (0 , 1) , (3 , 0) and (0 , 4) ....
   (a) form a right-angled triangle. (b) form an acute-angled triangle.
   (c) form an obtuse-angled triangle. (d) are collinear.
5. If $\overleftrightarrow{AB} // \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = \frac{2}{3}$, then the slope of $\overleftrightarrow{CD} =$ ....
   (a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{-2}{3}$ (d) $\frac{-3}{2}$
6. If $\sin X = \frac{1}{2}$, X is the measure of an acute angle , then $\sin 2X =$ ....
   (a) 1 (b) $\frac{1}{4}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{3}}$

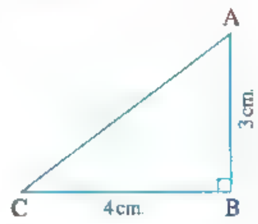
3 Join from column (A) to column (B) :

(A)	(B)
1 The slope of the straight line which is parallel to X-axis is	• 10
2 $\sin^2 30° + \cos^2 30° =$	• 0
3 If ABCD is a rectangle where A (1 , – 4) , C (5 , 4) , then the length of $\overline{BD}$ = ... length units.	• 1
4 The equation of the straight line which passes through the origin point and its slope is 2 is y = ... X	• – 3
5 The equation of the straight line which passes through the point (2 , – 3) and parallel to X-axis is y = ...	• 2
6 The value of : $\frac{2 \tan 30°}{1 + \tan^2 30°}$ = ...	• $\frac{\sqrt{3}}{2}$

4 Complete the following :

1 If $\overline{AB}$ // $\overline{CD}$ and the slope of $\overleftrightarrow{AB} = \frac{1}{2}$, then the slope of $\overleftrightarrow{CD}$ = ...

2 **In the opposite figure :**



ABC is a right-angled triangle at B

, AB = 3 cm. and BC = 4 cm.

, then sin C = ...

3 If the point (0 , a) belongs to the straight line : 3 X – 4 y = 12 , then a = ...

4 If X cos 60° = tan 45° , then X = ...

5 The distance between the point (4 , 3) and the origin point in the coordinates plane is ....

6 If the origin point is the midpoint of $\overline{AB}$ where A (5 , – 2)

, then B (............... , ...)

Governorates' Examinations on Trigonometry and Geometry

1 Cairo Governorate

Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

1 If $\sin X = \frac{1}{2}$, where X is the measure of an acute angle , then $X =$ ....°

(a) 30 (b) 45 (c) 60 (d) 90

2 The straight line whose equation is $y = 3X + 4$ intercepts from the positive part of y-axis a part of length .... length units.

(a) 3 (b) 4 (c) 5 (d) 7

3 The measure of the exterior angle of an equilateral triangle equals ....°

(a) 120 (b) 90 (c) 60 (d) 30

4 If $\Delta ABC \equiv \Delta XYZ$, then AB = ...

(a) BC (b) YZ (c) XZ (d) XY

5 The equation of the straight line whose slope equals 1 and passes through the origin point is ....

(a) $y = X + 1$ (b) $X = 1$ (c) $y = 1$ (d) $y = X$

6 The angle whose measure is 30° supplements an angle of measure ....°

(a) 60 (b) 120 (c) 150 (d) 180

2 [a] **Without using calculator , prove that :**

$4 \sin 45° \cos 45° = 2$ (showing the steps of the solution).

[b] Find the equation of the straight line which passes through the point (1 , 2) and is parallel to the straight line whose equation is $y = 3X + 5$

3 [a] **Find the value of X which satisfies that :**

$X \sin 30° = \sin 30° \cos 60° + \cos 30° \sin 60°$

[b] Prove that the straight line which passes through the points (0 , 5) , (3 , 2) is perpendicular to the straight line which makes an angle of measure 45° with the positive direction of X-axis.

4 [a] ABCD is a parallelogram , M is the point of intersection of its diagonals where , A (3 , – 1) , C (1 , 7) Find the coordinates of the point M

[b] If A (2 , 8) , B (– 1 , 4) and C (3 , 1) are the vertices of the triangle ABC

, **prove that :** 1 The triangle ABC is a right-angled triangle at B

2 The triangle ABC is an isosceles triangle.

---

5 [a] The triangle ABC is a right-angled triangle at B where AB = 7 cm. and BC = 24 cm.

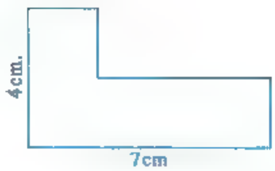
Find the value of : 1 3 tan A × tan C 2 $\sin^2 A + \sin^2 C$

[b] If the points (0 , 1) , (a , 3) and (2 , 5) are collinear , find the value of a

2 Giza Governorate

Answer the following questions :

1 **Choose the correct answer :**



1 The perimeter of the opposite figure equals .... cm.

(a) 44 (b) 22
(c) 18 (d) 11

2 If ∠ X , ∠ Y are two complementary angles and sin X = $\frac{3}{5}$, then cos Y = ...

(a) $\frac{4}{5}$ (b) $\frac{3}{5}$ (c) $\frac{3}{4}$ (d) $\frac{5}{3}$

3 ABCD is a parallelogram and m (∠ A) : m (∠ B) = 1 : 2 , then m (∠ B) = ... °

(a) 45 (b) 135 (c) 120 (d) 115

4 The straight line whose equation is : $y - 2X \quad 5 = 0$ cuts from the positive part of y-axis a part of length .... length units.

(a) 2 (b) 5 (c) 7 (d) 10

5 In Δ ABC , if the angles ∠ A , ∠ B are complementary , then m (∠ C) = ... °

(a) 45 (b) 30 (c) 90 (d) 60

6 The slope of the straight line which makes with the positive direction of X-axis an angle whose positive measure is $X°$ equals ....

(a) $\sin X$ (b) $\cos X$ (c) $\frac{\sin X}{\cos X}$ (d) $\sin X + \cos X$

---

2 [a] ABCD is a trapezoid in which $\overline{AD}$ // $\overline{BC}$, m (∠ B) = 90° If AB = 3 cm. , AD = 6 cm. , BC = 10 cm. , **then prove that :** cos (∠ DCB) tan (∠ ACB) = $\frac{1}{2}$

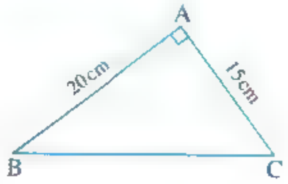
[b] If the straight line L_1 passes through the points (3 , 1) , (2 , k) and the straight line L_2 makes with the positive direction of X-axis an angle of measure 45° , then find the value of k which makes the two straight lines L_1 , L_2 parallel.

3 **[a] In the opposite figure :**

ABC is a triangle , m (∠ A) = 90° , AC = 15 cm.

, AB = 20 cm.

Prove that : cos C cos B – sin C sin B = 0



[b] ABCD is a parallelogram its diagonals intersect at M where :

A (3 , – 1) , B (6 , 2) , C (1 , 7)

Find the coordinates of the two points M and D

---

4 **[a] Without using calculator , find m (∠ X) which satisfies the equation :**

tan X = 4 sin 30° cos 60° where X is a positive acute angle.

[b] Find the equation of the straight line passing through the point (3 , 4) and perpendicular to the straight line $5X - 2y + 7 = 0$

---

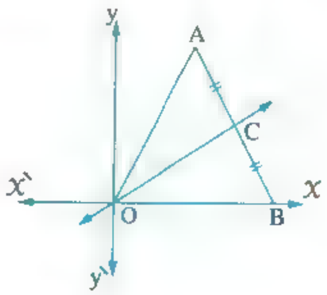
5 [a] If the distance between the point (a , 7) and the point (0 , 3) is equal to 5 length units , then find the value of a

[b] In the opposite figure :

AOB is an equilateral triangle

, C is the midpoint of $\overline{AB}$

Find the equation of $\overleftrightarrow{OC}$ where O is the origin point.



3 Alexandria Governorate

Answer the following questions : (Calculators are permitted)

1 **Choose the correct answer from those given :**

1 If C (6 , – 4) is the midpoint of $\overline{AB}$ where A (5 , – 3) , then B is ....

(a) (7 , – 5) (b) (– 5 , – 7) (c) (– 5 , 7) (d) (11 , – 7)

2 The measure of the angle that complements an angle of measure 60° is ....°

(a) 120 (b) zero (c) 30 (d) 90

3 If sin θ = 0.6 , then m (∠ θ) ≃ ....

(a) $51° \; 33' \; 35''$ (b) $36° \; 52' \; 12''$ (c) $47° \; 15' \; 48''$ (d) $45° \; 15' \; 6''$

4 The square whose area is 100 cm^2 , then its diagonal length is .... cm.

(a) 10 (b) 50 (c) $2\sqrt{10}$ (d) $10\sqrt{2}$

5 ABC is a right-angled triangle at B where A (1 , 4) , B (– 1 , – 2)

, then the slope of $\overleftrightarrow{BC}$ equals ....

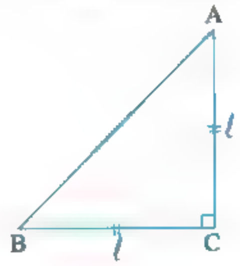
(a) $-\frac{1}{3}$ (b) 3 (c) $\frac{1}{3}$ (d) – 3

6 The sum of the lengths of any two sides of a triangle is .... the length of the third side.

(a) smaller than (b) equal to (c) greater than (d) twice

---

2 [a] **In the opposite figure :**



ABC is an isosceles triangle and right-angled at C

and the length of each of its legs is ℓ

Find : 1 The ratio among the lengths of the triangle sides AC : BC : AB

2 tan B , sin A

[b] If the distance between the two points (X , 5) , (6 , 1) equals $2\sqrt{5}$ length units , find the values of X

---

3 [a] If the points A (3 , 2) , B (4 , – 3) , C (– 1 , – 2) , D (– 2 , 3) are the vertices of a rhombus

, find : 1 The coordinates of the intersection point of its diagonals.

2 The area of the rhombus ABCD

[b] **Without using calculator , find the value of X (where X is the measure of an acute angle) which satisfies :** $2 \sin X = \sin 30° \cos 60° + \cos 30° \sin 60°$

---

4 [a] Find the equation of the straight line passing through the point (1 , 2) and perpendicular to the straight line passing through the two points A (2 , – 3) , B (5 , – 4)

[b] **Prove the following equality with indicating the steps :** $\tan 60° = \frac{2 \tan 30°}{1 - \tan^2 30°}$

---

5 [a] If the straight line L_1 passes through the two points (3 , 1) , (2 , k) and the straight line L_2 makes with the positive direction of the X-axis an angle of measure 45° , find the value of k , if $L_1 // L_2$

[b] Prove that the points A (– 2 , 5) , B (3 , 3) , C (– 4 , 2) are not collinear.

El-Kalyoubia Governorate

Answer the following questions :

1 **Choose the correct answer :**

1 If $\cos X = \frac{\sqrt{2}}{2}$ where X is the measure of an acute angle , then $\sin 2X =$ ......

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{-\sqrt{2}}{2}$ (c) 1 (d) $\frac{2}{\sqrt{2}}$

2 The number of the axes of symmetry of the circle equals ....

(a) zero (b) 1 (c) 2 (d) an infinite number.

3 If ABCD is a rectangle , A (– 4 , – 1) , C (4 , 5) , then the length of $\overline{BD}$ = ... length units.

(a) 10 (b) 6 (c) 5 (d) 4

4 The perpendicular length between $X = 5$, $X + 3 = 0$ equals .... length units.

(a) 2 (b) 8 (c) – 8 (d) 5

5 Δ ABC is an isosceles triangle and right-angled at C and the length of each leg is ℓ , then AB : BC : CA = ...

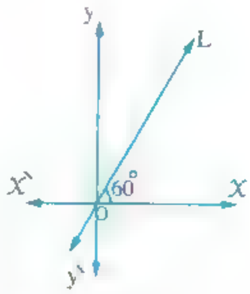
(a) $1 : 1 : \sqrt{2}$ (b) $1 : \sqrt{2} : 1$ (c) $\sqrt{2} : 1 : 2$ (d) $\sqrt{2} : 1 : 1$

6 **In the opposite figure :**

The equation of the straight line L is ....

(a) $X = \sqrt{3}\, y$ (b) $y = \sqrt{3}\, X$

(c) $X = y$ (d) $y = \sqrt{3}$



2 [a] Find the slope and the length of the y-intercept for the straight line : $\frac{X}{2} + \frac{y}{3} = 1$

[b] If $\sin X = \tan 30° \sin 60°$ where X is the measure of an acute angle , **find :** $4 \cos X \sin X$

3 [a] Find the equation of the straight line which passes through the point (2 , – 5) and is parallel to the straight line which passes through the two points (2 , 1) , (2 , 7)

[b] ABC is a right-angled triangle at B , if $2\,AB = \sqrt{3}\,AC$

, **find :** 1 m (∠ C) 2 $\sin^2 A - \cos^2 C$

4 [a] If the two straight lines $L_1 : 3X - 4y - 3 = 0$, $L_2 : ay + 4X - 8 = 0$ are perpendicular , find the value of a

[b] If the points A (3 , 2) , B (4 , 3) , C (- 1 , - 2) , D (- 2 , 3) are the vertices of a rhombus , find the area of the rhombus ABCD

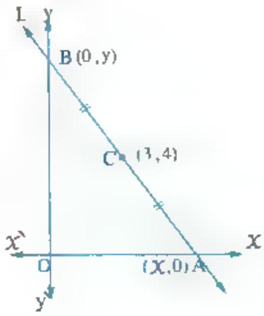
---

5 [a] **Prove that :** $\cos^2 60° = \cos^2 30° \tan^2 30° \tan 45°$

[b] **In the opposite figure :**

The point C is the midpoint of $\overline{AB}$ where C (3 , 4)

Find the perimeter of the triangle AOB



5 El-Sharkia Governorate

Answer the following questions : *(Calculator is allowed)*

1 **Choose the correct answer from those given :**

1 In Δ ABC , if m (∠ B) = 90° , then sin A + cos C = ...

(a) 2 sin C (b) 2 cos A (c) 2 cos C (d) tan A

2 If $\sin 2X = \frac{1}{2}$ where $2X$ is the measure of an acute angle , then X = ... °

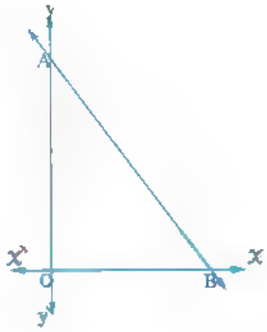
(a) 15 (b) 60 (c) 70 (d) 30

3 **In the opposite figure :**

If AO = 8 length units

, OB = 6 length units

, then the equation of $\overleftrightarrow{AB}$ is ....



(a) $y = \frac{4}{3}X + 8$ (b) $y = -\frac{4}{3}X - 8$

(c) $y = \frac{3}{4}X - 8$ (d) $y = -\frac{4}{3}X + 8$

4 The perpendicular distance between the point (3 , – 4) and X-axis equals .... length units.

(a) 3 (b) – 4 (c) 5 (d) 4

5 In the square XYZL , if the slope of $\overleftrightarrow{XZ} = 1$, then the slope of $\overleftrightarrow{YL} = \cdots\cdots$

(a) 1 (b) -1 (c) ± 1 (d) $45°$

6 ABC is a right-angled triangle at B , where $3\ AC = 5\ BC$, then $\tan A = \cdots\cdots$

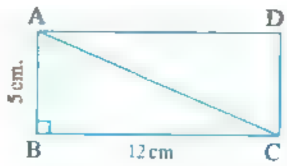
(a) $\frac{3}{5}$ (b) $\frac{5}{3}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

2 [a] If the point C $(4 , y)$ is the midpoint of $\overline{AB}$ where A $(X , 3)$ and B $(6 , 5)$

, find the value of : $X + y$

[b] Prove that the points A $(5 , 3)$, B $(3 , -2)$, C $(-2 , -4)$ are the vertices of a triangle , then prove that the triangle is an obtuse-angled triangle at B

3 [a] **In the opposite figure :**



If ABCD is a rectangle in which AB = 5 cm. , BC = 12 cm.

, find : 1 The length of $\overline{AC}$

2 The value of : $5 \tan (\angle ACD) - 13 \sin (\angle DAC)$

[b] If the two points A $(3 , -1)$, B $(5 , 3)$

, find the equation of the axis of symmetry of $\overline{AB}$

4 [a] **Without using the calculator , find the value of :** $\frac{\cos^2 60° + \cos^2 30°}{\sin 60° \tan 60°}$

[b] If the two equations of the two straight lines L_1 and L_2 are :

$L_1 : 6X + k y - 3 = \text{zero}$ and $L_2 : 3y = 2X + 6$ respectively.

, find the value of k which makes :

1 The two straight lines parallel. 2 The two straight lines perpendicular.

5 [a] Find the equation of the straight line which passes through the point $(1 , 4)$ and is parallel to the straight line : $X + 2y - 4 = \text{zero}$

[b] If ABCD is a square where : A $(2 , 4)$, B $(-3 , \text{zero})$, C $(-7 , 5)$

, find : 1 The coordinates of the point D 2 The area of the square ABCD

6 El-Monofia Governorate

Answer the following questions : *(Using calculator is permitted)*

1 **Choose the correct answer :**

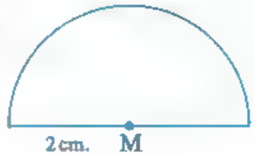
1 The surface area of a square is 25 cm^2 , then the length of its diagonal is $\cdots\cdots$ cm.

(a) 5 (b) 10 (c) $5\sqrt{2}$ (d) $10\sqrt{2}$

2 ABC is a triangle. If $(AC)^2 > (AB)^2 + (BC)^2$, then $\angle$ C is ....

(a) acute. (b) obtuse. (c) right. (d) straight.

3 The opposite figure represents a semicircle with the radius length of its circle is 2 cm. , then the perimeter of this figure = ... cm.



(a) 2π (b) 4π

(c) $2\pi + 4$ (d) $4\pi + 2$

4 If $\cos \frac{X}{2} = \frac{\sqrt{3}}{2}$ where $\frac{X}{2}$ is the measure of an acute angle , then $\tan (X - 15^\circ) = \dots\dots$

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) 1 (d) $\frac{\sqrt{3}}{2}$

5 The equation of a straight line is : $\frac{X}{2} - \frac{y}{3} = 6$, then it intercepts from X-axis a part of length .... length units.

(a) 3 (b) 12 (c) 6 (d) 18

6 If $\frac{-2}{3}$, $\frac{6}{k}$ are the slopes of two perpendicular straight lines , then k = ...

(a) 4 (b) -9 (c) -4 (d) 9

---

2 [a] Determine the type of the triangle ABC where : A (3 , 0) , B (1 , 4) and C (– 1 , 2) with respect to the lengths of its sides.

[b] **Without using calculator , prove that :** $\frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ} = 2 + \sqrt{3}$

---

3 [a] ABCD is a quadrilateral where A (2 , 4) , B (3 , 0) , C (– 7 , 5) and D (– 2 , 9)
Prove that : ABCD is a square.

[b] ABC is a right-angled triangle at C , AC = 6 cm. and BC = 8 cm.
Find the value of : cos A cos B – sin A sin B

---

4 [a] Prove that the straight line which passes through the two points (– 3 , 2) and B (4 , 5) is parallel to the straight line which makes with the positive direction of X-axis an angle its measure is 45°

[b] If $\sqrt{3} \sin X \tan 30^\circ = \tan 45^\circ \cos 2X$, find the value of X (where X is the measure of an acute angle).

---

5 [a] Find the equation of the straight line which is perpendicular to the straight line : $3X - 4y + 7 = 0$ and intercepts from the positive part of y-axis a part of length 4 units.

[b] ABCD is a rectangle in which AB = 3 cm. , AC = 5 cm.

Find : 1 m ($\angle$ ACB) 2 The area of the rectangle ABCD

7 El-Gharbia Governorate

Answer the following questions : *(Calculator is allowed)*

1 Choose the correct answer :

1 The number of the axes of symmetry of the scalene triangle equals

(a) zero (b) 1 (c) 2 (d) 3

2 In the triangle XYZ , if $(YZ)^2 + (XZ)^2 < (XY)^2$, then $\angle$ Z is ....

(a) acute. (b) right. (c) obtuse. (d) straight.

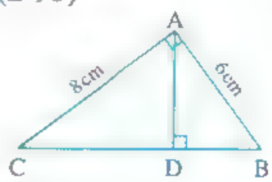
3 If the distance between the two points (a , 0) and (0 , 1) is one length unit , then a = ...

(a) 1 (b) – 1 (c) 0 (d) 2

4 If the origin point is the midpoint of $\overline{AB}$ where A (2 , 3) , then the point B is ....

(a) (– 3 , 2) (b) (– 2 , 3) (c) (– 2 , – 3) (d) (2 , 3)

5 **In the opposite figure :** ABC is a right-angled triangle at A in which $\overrightarrow{AD} \perp \overline{BC}$ cutting it at D , AB = 6 cm. and AC = 8 cm. , then AD = ... cm.



(a) 3.6 (b) 8.4 (c) 4.8 (d) 6.4

6 ABC is a right-angled triangle at B , then sin A + 2 cos C = ...

(a) 2 sin C (b) 3 sin A (c) 2 sin A (d) 3 cos A

2 [a] XYZ is a right-angled triangle at Y in which : XY = 5 cm. and XZ = 13 cm.

Find the value of : cos X cos Z – sin X sin Z

[b] Find the measure of the positive angle that $\overleftrightarrow{AB}$ makes where :

A (3 , – 2) , B (6 , 1) with the negative direction of the X-axis.

3 [a] **Find the value of X if :** $\cos (3X + 6°) = \frac{1}{2}$ where $(3X + 6°)$ is the measure of an acute angle.

[b] Find the equation of the straight line which is parallel to the straight line $\frac{y-1}{X} = \frac{1}{3}$ and intersects from the negative part of y-axis a part equals 3 length units.

4 [a] **Find the value of X which satisfies :** $X - \sin 30° \cos^2 45° = \sin^2 60°$

[b] If the points A (– 3 , 0) , B (3 , 4) and C (1 , 6) are the vertices of an isosceles triangle of vertex A , find the length of the drawn line segment from A perpendicular to $\overline{BC}$

5 [a] If the point M (– 1 , 2) is the centre of the circle passing through the point A (3 , – 1) , find the circumference of the circle $\left(\text{where } \pi = \frac{22}{7}\right)$

[b] Find the equation of the straight line passing through the point (1 , 2) and perpendicular to the straight line passing through the two points A (2 , – 3) and B (5 , – 4)

8 El-Dakahlia Governorate

Answer the following questions : *(Calculator is permitted)*

1 [a] **Choose the correct answer :**

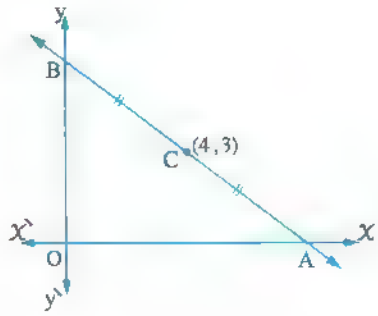
1. If m (∠ A) = 75° , sin A = cos B , ∠ B is acute , then m (∠ B) = ……………
   (a) 45° (b) 75° (c) 15° (d) 105°
2. If ABC is a right-angled triangle at B , AB = BC , then tan A = ……………
   (a) $\frac{1}{3}$ (b) $\sqrt{3}$ (c) 1 (d) $\frac{1}{\sqrt{2}}$
3. If $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = 0$, then the slope of $\overleftrightarrow{CD}$ = ……………
   (a) 1 (b) – 1 (c) zero (d) not defined.

[b] **In the opposite figure :**

The point C is the midpoint of $\overline{AB}$ where C (4 , 3) , O is the origin point in the perpendicular coordinates system.

Find : 1 The coordinates of the two points A , B

2 The area of the triangle AOB



2 [a] **Choose the correct answer :**

1. If cos 3 $X = \frac{1}{2}$, 3 X is the measure of an acute angle , then X = ……………
   (a) 20° (b) 30° (c) 45° (d) 60°
2. The radius length of the circle whose centre is (0 , 0) and passes through (3 , 4) equals …………… length units.
   (a) 7 (b) 1 (c) 12 (d) 5
3. The measure of the exterior angle of the equilateral triangle equals ……………
   (a) 60° (b) 90° (c) 120° (d) 80°

[b] **Without using calculator , find the value of X which satisfies :**

$2 \sin X = \tan^2 60° - 2 \tan 45°$ where X is the measure of an acute angle.

3 [a] Find the equation of the straight line which intercepts from the positive parts of the two axes two parts of lengths 2 units , 3 units from X and y axes respectively.

[b] ABC is a right-angled triangle at C , AC = 5 cm. , BC = 12 cm. **Find the value of :** cos A cos B – sin A sin B

4 [a] ABCD is a parallelogram where A (3 , 2) , B (4 , – 5) , C (0 , – 3)

Find the coordinates of the point at which the two diagonals intersect , then find the coordinates of the point D

[b] **Without using calculator , prove that :** $2 \sin 30° + 4 \cos 60° = \tan^2 60°$

5 [a] Prove that A (5 , 1) , B (3 , – 7) , C (1 , 3) are not collinear points.

[b] Find the equation of the straight line perpendicular to $\overline{AB}$ from its midpoint where A (2 , 1) , B (4 , 5)

Ismailia Governorate

Answer the following questions : *(Calculator is allowed)*

1 **Choose the correct answer from those given :**

1 The parallelogram whose two diagonals are equal in length and perpendicular is the ....

(a) rectangle. (b) rhombus. (c) square. (d) trapezium.

2 If C is the midpoint of $\overline{AB}$ where A (– 3 , 6) , B (3 , – 6) , then C = ...

(a) (6 , – 6) (b) (0 , 0) (c) (3 , 3) (d) (– 3 , 0)

3 The number of diagonals of the triangle equals ....

(a) 3 (b) 2 (c) 1 (d) 0

4 ABC is a triangle in which m (∠ A) = 75° , sin B = cos B , then m (∠ C) = °

(a) 90 (b) 60 (c) 45 (d) 30

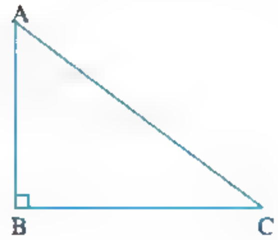
5 If the ratio between the measures of two adjacent supplementary angles is 1 : 2 , then the measure of the greater angle equals ....°

(a) 120 (b) 90 (c) 180 (d) 60

6 The equation of the straight line which passes through the origin point and its slope = 3 is ....

(a) y = X (b) y = 3 (c) X = 3 (d) y = 3 X

2 [a] **In the opposite figure :**



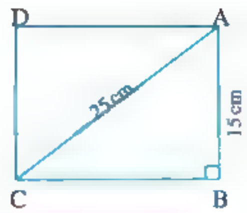
ABC is a right-angled triangle at B

Prove that : $\sin^2 A + \sin^2 C = 1$

[b] Prove that the straight line which passes through the two points $(-1, 3)$, $(2, 4)$ is parallel to the straight line whose equation is $3y - x - 1 = 0$

---

3 [a] **In the opposite figure :**



ABCD is a rectangle , AB = 15 cm. , AC = 25 cm.

Find : m ($\angle$ ACB) in degree measure

, **then find** the area of the rectangle ABCD

[b] The opposite table shows a linear relation.

x	1	2	3
y	1	3	5

Find : 1 The equation of the straight line.

2 The length of the intercepted part from y-axis.

---

4 [a] Prove that the quadrilateral ABCD whose vertices are

A $(-1, 3)$, B $(5, 1)$, C $(7, 4)$ and D $(1, 6)$ is a parallelogram.

[b] Find the slope of the straight line which intersects from the positive parts of two coordinates x-axis and y-axis two parts of lengths 3 units , 4 units respectively , then find the equation of this straight line.

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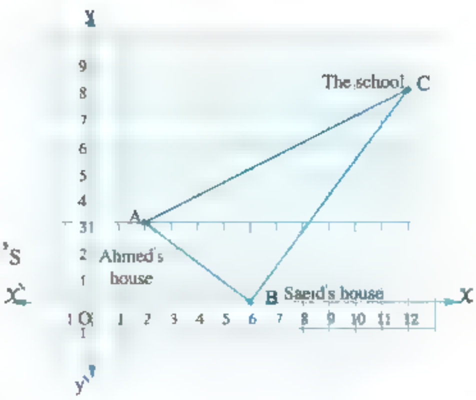
5 [a] **Without using calculator , find the value of :** $\sin 45° \cos 45° + \sin 30° \cos 60° - \cos^2 30°$

[b] **In the opposite figure :**



A represents the location of Ahmed's house

, B represents the location of Saeid's house

, C represents the location of the School.

1 Which is nearer (closer) to the school : Ahmed's house or Saeid's house ? Why ? Without measuring.

2 Are the two roads $\overline{AB}$ and $\overline{BC}$ perpendicular ? giving reason , without measuring.

10 Suez Governorate

Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

1. If $\sin 30° = \cos \theta$ where θ is an acute angle , then $m(\angle \theta) =$ ....°
   (a) 15 (b) 30 (c) 60 (d) 90
2. ABC is a triangle in which : $(AB)^2 > (BC)^2 + (AC)^2$, then $\angle C$ is ....
   (a) acute. (b) obtuse. (c) right. (d) reflex.
3. If A (– 2 , 5) , B (2 , – 5) , then the midpoint of $\overline{AB}$ is ....
   (a) (0 , 0) (b) (2 , 5) (c) (5 , 2) (d) (– 5 , – 2)
4. If $\overleftrightarrow{XY}$ is the axis of symmetry of $\overline{AB}$, then XA .... XB
   (a) > (b) < (c) = (d) ≤
5. If m_1 , m_2 are the slopes of two perpendicular straight lines , then $m_1 \times m_2 =$ ....
   (a) – 1 (b) zero (c) 1 (d) 2
6. The surface area of the rhombus ABCD = ...
   (a) $\frac{1}{2}$ AB × DC (b) $\frac{1}{2}$ AC × BD (c) $\frac{1}{2}$ AB × AD (d) $\frac{1}{2}$ AD × BC

---

2 [a] Find the equation of the straight line whose slope is 2 and intersects from the positive part of the y-axis a part equals 7 units.

[b] **Find the value of X if :** $4X = \cos^2 30° \tan^2 30° \tan^2 45°$

---

3 [a] ABCD is a parallelogram whose diagonals intersect at E
If A (4 , 3) , B (0 , 2) , C (– 2 , – 3) , then find the coordinates of E , D

[b] **Without using calculator , prove that :**

$\tan^2 60° - \tan^2 45° = \sin^2 60° + \cos^2 60° + 2 \sin 30°$

---

4 [a] Prove that the straight line passing through the two points (2 , – 1) , (6 , 3) is parallel to the straight line that makes with the positive direction of the X-axis an angle of measure 45°

[b] ABC is a right-angled triangle at B , if $2 AB = \sqrt{3} AC$
, **find :** sin C , tan A

---

5 [a] Prove that the points A (– 3 , 0) , B (3 , 4) , C (1 , - 6) are the vertices of an isosceles triangle of vertex A

[b] Find the equation of the straight line which passes through the point (3 , 5) and is perpendicular to the straight line whose slope equals $\frac{-1}{2}$

11 Port Said Governorate

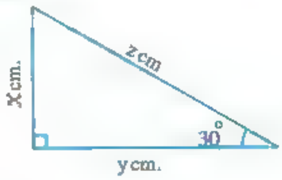
Answer the following questions :

1 Choose the correct answer from those given :

1 The product of multiplying the slopes of two perpendicular straight lines equals ......

(a) 1 (b) -1 (c) ± 1 (d) zero

2 **In the opposite figure :**



(a) $x + y = \frac{1}{2} z$ (b) $z = x^2 + y^2$

(c) $x = \frac{1}{2} z$ (d) $2y = z$

3 $\sin 30° = \cos$ ....

(a) 10° (b) 45° (c) 30° (d) 60°

4 $\tan 45° =$ ....

(a) 1 (b) $2\sqrt{2}$ (c) $\frac{1}{2}$ (d) $\sqrt{2}$

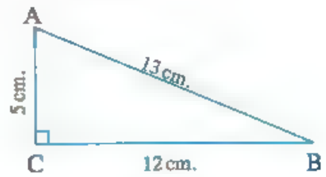
5 If A (5 , 7) , B (1 , – 1) , then the midpoint of $\overline{AB}$ is ....

(a) (2 , 3) (b) (3 , 3) (c) (3 , 2) (d) (3 , 4)

6 If $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = \frac{2}{3}$, then the slope of $\overleftrightarrow{CD} =$ ....

(a) $\frac{3}{2}$ (b) $\frac{-3}{2}$ (c) $-\frac{2}{3}$ (d) $\frac{2}{3}$

2 [a] **In the opposite figure :**



ABC is a right-angled triangle at C

, AB = 13 cm. , BC = 12 cm. , AC = 5 cm.

1 **Prove that :** $\sin A \cos B + \cos A \sin B = 1$

2 **Find :** $1 + \tan^2 A$

[b] **Find the value of the following :** $\sin 45° \cos 45° + \sin 30° \cos 60° - \cos^2 30°$

3 [a] **Find m (∠ E) , where ∠ E is an acute angle :** $\sin E = \sin 60° \cos 30° - \cos 60° \sin 30°$

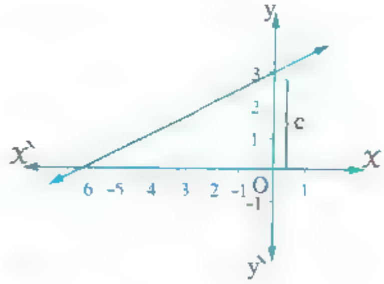
[b] Prove that the straight line passing through the two points (– 3 , – 2) , (4 , 5) is parallel to the straight line that makes with the positive direction of the x-axis an angle of measure 45°

4 [a] Find the equation of the straight line passing through the point (1 , 2) and perpendicular to the straight line passing through the two points A (2 , – 3) , B (5 , – 4)

[b] Prove that the points A (3 , – 1) , B (4 , 6) and C (2 , – 2) are located on the circle whose centre is the point M (– 1 , 2)

5 [a] ABCD is a parallelogram where A (3 , 2) , B (4 , – 5) , C (0 , – 3) , find the coordinates of the point at which the two diagonals intersect , then find the coordinates of the point D

[b] **Using the opposite figure , find the following :**



1 The length of the y-intercept (c)

2 The length of the X-intercept.

3 The slope of the straight line (m)

12 Damietta Governorate

Answer the following questions : (Calculator is allowed)

1 **Choose the correct answer from the given answers :**

1 If the lengths of two sides of an isosceles triangle are 2 cm. and 5 cm. , then the length of the third side is .... cm.

(a) 2 (b) 3 (c) 5 (d) 7

2 If $\sin X = \frac{1}{2}$, X is the measure of an acute angle , then $\sin 2X =$ ....

(a) $\frac{\sqrt{3}}{3}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{\sqrt{2}}{2}$ (d) 1

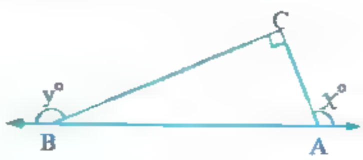
3 The surface area of the square is equal to the square of the length of the diagonal divided by ....

(a) 1 (b) 2 (c) 3 (d) 4

4 The equation of the straight line which passes through the point (– 2 , 5) and is parallel to X-axis is ....

(a) $X = -2$ (b) $X = 5$ (c) $y = -2$ (d) $y = 5$

5 **In the opposite figure :**



$A \in \overleftrightarrow{AB}$, $B \in \overleftrightarrow{AB}$, $m(\angle C) = 90°$

, then $X + y =$ ....

(a) 90° (b) 180° (c) 270° (d) 360°

6 If $\overleftrightarrow{AB}$, $\overleftrightarrow{DC}$ are parallel , their slopes are m_1 , m_2 , then ....

(a) $m_1 = -m_2$ (b) $m_1 - m_2 = 0$ (c) $m_1 m_2 = -1$ (d) $m_1 m_2 = 1$

2 [a] ABC is a right-angled triangle at C , AC = 6 cm. , BC = 8 cm.

Find : cos A cos B – sin A sin B

[b] Find the equation of the straight line which intercepts from the positive parts of the two axes two parts of lengths 3 units and 2 units for X and y axes respectively and find its slope.

3 [a] If the distance of the point $(X , 5)$ from the point $(6 , 1)$ equals $2\sqrt{5}$ length units , then find the value of X

[b] Find the equation of the straight line which passes through the points $(2 , -1)$, $(1 , 1)$ and if the point $(0 , k) \in$ the straight line , find the value of k

4 [a] **Find the value of X if :** $4X = \cos^2 30° \tan^2 30° \tan^2 45°$ (Indicating the steps of the solution)

[b] If the straight line passing through the two points $(a , 0)$, $(0 , 3)$ is perpendicular to the straight line that makes an angle of measure 30° with the positive direction of the X-axis find a.

5 [a] **Prove that :** $\sin 45° \cos 45° + \sin 30° \cos 60° - \cos^2 30° = 0$ (Indicating the steps of the solution)

[b] Find the equation of the straight line perpendicular to $\overline{AB}$ from its midpoint C where A $(1 , 3)$ and B $(3 , 5)$

13 Kafr El-Sheikh Governorate

Answer the following questions : (Calculators are permitted)

1 **Choose the correct answer from those given :**

1. In Δ ABC , if $m(\angle A) = 60°$, $\sin B = \cos B$, then $m(\angle C) =$ ....
   (a) 30° (b) 75° (c) 90° (d) 105°
2. The area of the triangle bounded by the straight lines : $X = 0$, $y = 0$, $5X + 2y = 10$ is .... square units.
   (a) 10 (b) 8 (c) 7 (d) 5
3. If the straight line passing through the two points $\left(\sqrt{3} , 1\right)$, $\left(2\sqrt{3} , y\right)$ its slope equals $\tan 60°$, then y = ...
   (a) 2 (b) 3 (c) 4 (d) 5
4. If the straight line $aX + (2 - a)y = 5$ is parallel to the straight line passing through the two points $(1 , 4)$, $(3 , 5)$, then a = ...
   (a) 3 (b) -2 (c) 1 (d) zero
5. If the point $(\ell - 3 , 2)$ is in the first quadrant , then ℓ can be equal to ....
   (a) -3 (b) 2 (c) 7 (d) zero
6. The complement of the angle whose measure is 65° is of measure ....
   (a) 35° (b) 25° (c) 115° (d) 45°

2 [a] ABC is a right-angled triangle at B , AC = 13 cm. , BC = 12 cm.

Prove that : $\sin^2 C + \sin^2 A = 1$

[b] If the point A (5 , 2) lies on the circle of centre M (1 , - 1) , **then find :**

1. The surface area of the circle in terms of π
2. The equation of the straight line which passes through A and M

3 [a] If A (- 3 , 5) , B (1 , 7) , find the equation of the axis of symmetry of $\overline{AB}$

[b] **Without using the calculator , prove that :**

$\tan^2 60° - \tan^2 45° = \sin^2 60° + \cos^2 60° + 2 \sin 30°$

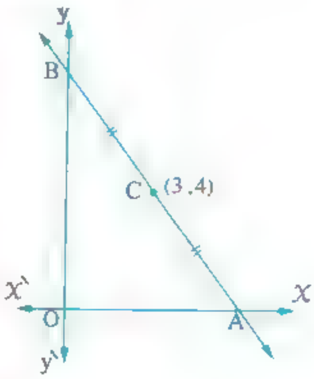
4 [a] Prove that the points A (1 , 3) , B (5 , 1) , C (7 , 4) , D (1 , 6) are the vertices of the parallelogram ABCD

[b] ABCD is an isosceles trapezoid in which $\overline{AD} // \overline{BC}$, AD = 4 cm. , AB = 5 cm. , BC = 12 cm. , **then calculate :** $\frac{\tan B \cos C}{\cos^2 C + \sin^2 C}$

5 [a] If the straight line L_1 passes through the two points (3 , 1) , (2 , k) and the straight line L_2 makes with the positive direction of X-axis an angle of measure 45°

, find the value of k if : 1 $L_1 // L_2$ 2 $L_1 \perp L_2$

[b] **In the opposite figure :**



The point C is the midpoint of $\overline{AB}$ where C (3 , 4) , O is the origin point of the perpendicular coordinates system.

Find : 1 The coordinates of the two points A and B

2 The equation of $\overleftrightarrow{AB}$

14 El-Beheira Governorate

Answer the following questions : (Calculator is permitted)

1 **Choose the correct answer from the given ones :**

1 If A (5 , 7) and B (1 , – 1) , then the midpoint of $\overline{AB}$ is ....

(a) (2 , 3) (b) (3 , 3) (c) (3 , 2) (d) (3 , 4)

2 If m (∠ B) = 80° , then m (reflex ∠ B) = ...

(a) 10° (b) 100° (c) 80° (d) 280°

3 The slope of the straight line which is parallel to the straight line passing through the two points (2 , 3) , (– 2 , 4) equals ....

(a) – 1 (b) $\frac{-1}{4}$ (c) $\frac{1}{4}$ (d) 1

4 If $\tan (X + 10°) = \sqrt{3}$ where X is the measure of an acute angle , then $X =$ ....

(a) 30° (b) 45° (c) 50° (d) 60°

5 In a parallelogram , the two diagonals are ....

(a) perpendicular. (b) equal in length.

(c) equal in length and perpendicular. (d) bisecting each other.

6 The triangle whose sides lengths are 2 cm. , $(X + 2)$ cm. and 5 cm. becomes an isosceles triangle when $X =$ ....

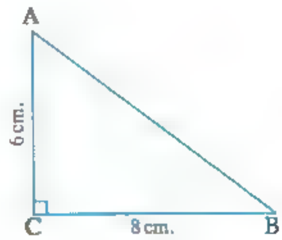
(a) zero (b) 2 (c) 3 (d) 5

2 [a] **In the opposite figure :**

ABC is a right-angled triangle at C , AC = 6 cm. , BC = 8 cm.

Find : 1 cos A cos B – sin A sin B

2 m (∠ B)



[b] State the kind of the triangle whose vertices are the points A (– 2 , 4) , B (3 , – 1) , C (4 , 5) with respect to its sides.

3 [a] **Without using the calculator , prove that :**

$\tan^2 60° - \tan^2 45° = \cos^2 30° + \cos^2 60° + 2 \sin 30°$

[b] Find the equation of the straight line whose slope equals 2 and intersects from the negative part of the y-axis a part equals 3 units and draw it.

4 [a] **Find the value of X which satisfies :** $X \sin 30° \cos^2 45° = \sin^2 60°$

[b] If the straight line L_1 passes through the two points (3 , 1) , (2 , k) and the straight line L_2 makes with the positive direction of the X-axis an angle of measure 45° , find the value of k , if $L_1 // L_2$

5 [a] If the point (3 , 1) is the midpoint of $\overline{AB}$ where A (1 , y) and B (X , 3) , find the point (X , y)

[b] Find the equation of the straight line passing through the point (3 , – 5) and perpendicular to the straight line : $X + 2y - 7 = 0$

15 El-Fayoum Governorate

Answer the following questions : (Using calculators is allowed)

1 Choose the correct answer :

1 If $\tan 3X = \sqrt{3}$ where X is the measure of an acute angle , then $X = \cdots$ °

(a) 10 (b) 15 (c) 20 (d) 30

2 If the perimeter of a square is 16 cm. , then its area is .... cm^2

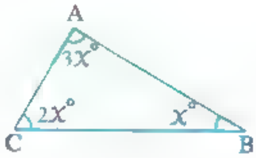
(a) 4 (b) 16 (c) 60 (d) 90

3 The perpendicular distance between the two straight lines : $X - 2 = 0$, $X + 3 = 0$ equals .... length units.

(a) 1 (b) 5 (c) 2 (d) 3

4 **In the opposite figure :**

Δ ABC is .... triangle.



(a) an isosceles. (b) an equilateral.

(c) an obtuse-angled. (d) a right-angled.

5 The area of the triangle identified by the straight lines :

$3X - 4y = 12$, $X = 0$, $y = 0$ equals .... square units.

(a) 6 (b) 7 (c) 12 (d) 5

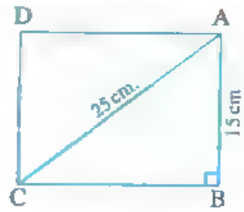
6 The measure of the angle of the regular hexagon is ....

(a) 108° (b) 90° (c) 120° (d) 60°

2 [a] In the opposite figure :

ABCD is a rectangle in which AB = 15 cm.

, AC = 25 cm.



Find : 1 m (∠ ACB)

2 The surface area of the rectangle ABCD

[b] If the distance between the two points (a , 7) , (– 2 , 3) equals 5 length units , find the values of a

3 [a] Without using the calculator , find the value of X (where X is the measure of an acute angle) which satisfies :

$2 \sin X = \sin 30° \cos 60° + \cos 30° \sin 60°$

[b] Prove that the straight line passing through the two points (– 1 , 3) , (2 , 4) is parallel to the straight line $3y - X - 1 = 0$

4 [a] ABCD is a quadrilateral , where A (5 , 3) , B (6 , – 2) , C (1 , – 1) , D (0 , 4).

Prove that : ABCD is a rhombus.

[b] If A (5 , – 6) , B (3 , 7) and C (1 , – 3) , find the equation of the straight line passing through the point A and the midpoint of $\overline{BC}$

5 [a] **Without using the calculator , prove that :**

$$\frac{\cos^2 60° + \cos^2 30° + \tan^2 45°}{\sin 60° \tan 60° - \sin 30°} = 2$$

[b] If the straight line L_1 passes through the two points A (3 , 1) , B (2 , y) and the straight line L_2 makes an angle whose measure is 45° with the positive direction of X-axis , then find the value of y if $L_1 \perp L_2$

10 Beni Suef Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer from those given :**

1 The product of multiplying the slopes of two perpendicular straight lines equals ……………

(a) zero (b) 1 (c) – 1 (d) $\frac{1}{2}$

2 If $\overline{AB}$ is a diameter in a circle of centre M , where A (2 , 4) and B (– 2 , 0) , then M = …………

(a) (0 , 2) (b) (2 , 0) (c) (0 , 0) (d) (2 , 2)

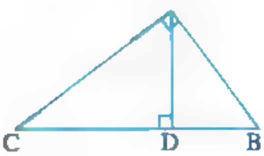
3 The quadrilateral whose diagonals are equal in length and perpendicular is the ……… .

(a) parallelogram. (b) rhombus. (c) rectangle. (d) square.

4 If the lengths of two sides of a triangle are 2 cm. and 5 cm. , then the length of the third side ∈ ……………

(a)]2 , 5[(b)]3 , 7[(c)]2 , 7[(d)]3 , 5[

5 **In the opposite figure :**



If m (∠ BAC) = 90° , $\overline{AD} \perp \overline{BC}$, then $(AD)^2$ = ……………

(a) AB × AC (b) DB × DC (c) BD × BC (d) $(AB)^2 + (BD)^2$

6 If tan (X + 15°) = 1 , where X is the measure of an acute angle , then X = ………… .

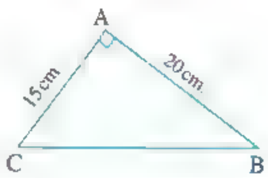
(a) 60° (b) 45° (c) 30° (d) 15°

2 [a] Find the area of the rectangle ABCD where A (– 1 , 3) , B (5 , 1) , C (6 , 4) and D (0 , 6)

[b] **Find the value of X if :** $X \cos 60° = \sin 30° + \tan 45°$

3 [a] Prove that the straight line passing through the two points (– 1 , 0) and (3 , 4) is parallel to the straight line that makes a positive angle of measure 45° with the positive direction of the X-axis.

[b] **In the opposite figure :**



ABC is a right-angled triangle at A

, AB = 20 cm. and AC = 15 cm.

Prove that : $\cos C \cos B - \sin C \sin B = \text{zero}$

4 [a] If C (X , – 3) is the midpoint of $\overline{AB}$ where A (– 3 , y) , B (9 , 11)

, **find the value of :** $X + y$

[b] **Without using the calculator , find the value of the expression :**

$\sin 45° \cos 45° + 3 \sin 30° \cos 60° - \cos^2 30°$

5 [a] Find the equation of the straight line passing through the point (2 , 5) and perpendicular to the straight line whose equation is $y - 2X + 7 = \text{zero}$

[b] Prove that the points A (2 , 3) , B (6 , 2) , C (0 , – 1) and D (– 2 , 1) are the vertices of a trapezoid.

17 El-Menia Governorate

Answer the following questions : (Calculator is allowed)

1 Choose the correct answer :

1 The measure of the exterior angle of the equilateral triangle equals ....

(a) 60° (b) 90° (c) 120° (d) 180°

2 If L_1 , L_2 are two lines parallel and their slopes are $\frac{-2}{3}$, $\frac{k}{6}$, then k = ...

(a) – 12 (b) – 9 (c) 4 (d) – 4

3 The lengths of two sides of an isosceles triangle equal 2 cm. , 5 cm. , then the length of the third side equals .... cm.

(a) 5 (b) 2 (c) 3 (d) 7

4 The distance between the point (5 , 12) and the point of origin equals .... units.

(a) 5 (b) 13 (c) 12 (d) $\sqrt{17}$

5 The area of the square whose perimeter is 16 cm. equals ······ ······ cm^2

(a) 4 (b) 8 (c) 16 (d) 256

6 XYZ is an isosceles triangle right-angled at Z , then tan X = ···············

(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$ (c) 1 (d) $\frac{1}{3}$

---

2 [a] Prove that the triangle whose vertices are A (6 , 0) , B (2 , – 4) , C (– 4 , 2) is right-angled at B

[b] XYZ is a right-angled triangle at Z where X Z = 7 cm. **Find the value of :** tan X × tan Y

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3 [a] **Find X where :** $4X = \cos^2 30° \tan^2 30° \tan^2 45°$

[b] Find the equation of the straight line passing through the point (3 , – 5) and parallel to the straight line $X + 2y - 7 = 0$

---

4 [a] ABCD is a parallelogram , A (– 2 , 5) , B (3 , 3) , C (– 4 , 2) Find the two coordinates of the point at which the two diagonals intersect , then find the coordinates of the point D.

[b] **Without using the calculator , prove that :** $\sin^2 30° = 5\cos^2 60° - \tan^2 45°$

---

5 [a] If the straight line L_1 passes through the two points (3 , 1) , (2 , k) and the straight line L_2 makes with the positive direction of the X-axis an angle whose measure is 45° , then find k , if the two straight lines L_1 , L_2 are perpendicular.

[b] Find the equation of the straight line which intersects from the positive parts of X and y axes two parts of lengths 2 units , 3 units respectively.

18 Assiut Governorate

Answer the following questions : (Calculator is permitted)

1 **Choose the correct answer :**

1 The sum of the measures of the interior angles of a triangle equals ······ ··

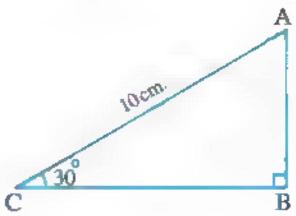
(a) 90° (b) 180° (c) 360° (d) 540°

2 **In the opposite figure :**

AB = ··············· cm.

(a) 5 (b) 15

(c) 20 (d) 40



3 The measure of the interior angle of a regular hexagon equals ....

(a) 108° (b) 120° (c) 90° (d) 180°

4 If $2 \sin X = 1$ (where X is the measure of an acute angle) , then $X =$ ....

(a) 45° (b) 90° (c) 30° (d) 60°

5 The equation of the straight line which passes through the point (2 , – 3) and is parallel to X-axis is ....

(a) $X = 2$ (b) $y = -3$ (c) $X = -2$ (d) $y = 3$

6 If the origin point is the midpoint of $\overline{AB}$, A (5 , – 2) , then B = ...

(a) (5 , 2) (b) (– 5 , – 2) (c) (– 5 , 2) (d) (0 , 0)

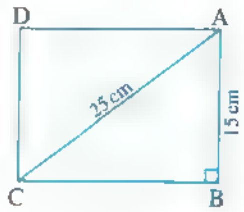
2 [a] Prove that the points A (– 3 , – 1) , B (6 , 5) and C (3 , 3) are collinear.

[b] **Find the value of X that satisfies :** $X \sin 30° \cos^2 45° = \sin^2 60°$

3 [a] If the triangle whose vertices are Y (4 , 2) , X (3 , 5) and Z (– 5 , a) is right-angled at Y , then find the value of a

[b] Find the equation of the straight line whose slope is 2 and intersects from the positive part of the y-axis a part that equals 7 units.

4 [a] **In the opposite figure :**



ABCD is a rectangle in which AB = 15 cm.

and AC = 25 cm.

Find : 1 m (∠ ACB)

2 The surface area of the rectangle ABCD

[b] Prove that the straight line which passes through the points (2 , 3) , (0 , 0) is parallel to the straight line which passes through (– 1 , 4) , (1 , 7)

5 [a] ABCD is a quadrilateral , where A (5 , 3) , B (6 , – 2) , C (1 , – 1) and D (0 , 4)

Prove that : ABCD is a rhombus.

[b] Find the slope and the intercepted part of y-axis by the straight line :

$2X - 3y - 6 = 0$

19 Souhag Governorate

Answer the following questions : *(Calculator is permitted)*

1 Choose the correct answer :

1 If $\sin \frac{X}{2} = \frac{1}{2}$, X is the measure of an acute angle , then $X =$ .... °

(a) 30 (b) 60 (c) 10 (d) 90

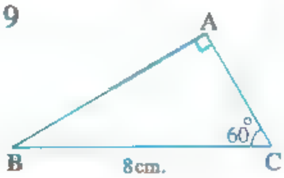
2 The perimeter of the square whose surface area is 100 cm^2 equals .... cm.

(a) 10 (b) 20 (c) 40 (d) 50

3 If $\frac{-2}{3}$, $\frac{6}{k}$ are the slopes of two perpendicular straight lines , then k = ...

(a) 4 (b) -9 (c) -4 (d) 9

4 **In the opposite figure :**



The length of $\overline{AC}$ = ... cm.

(a) 2 (b) 6

(c) 4 (d) 8

5 The equation of the straight line passing through the origin point and its slope = 1 is ....

(a) $y = X$ (b) $y = -X$ (c) $y = 2X$ (d) $y = 0$

6 If the numbers 3 , 7 , ℓ are lengths of sides of a triangle , then ℓ can be equal to ....

(a) 3 (b) 7 (c) 4 (d) 10

2 [a] If the midpoint of $\overline{BC}$ is A (2 , 3) and C (– 1 , 3) , find the point B

[b] If $\cos X = \sin 30° \cos 60°$, **find :**

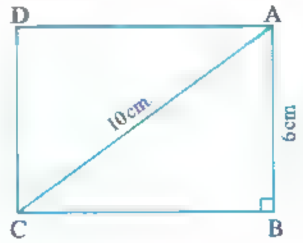
1 The measure of $\angle X$ (where X is an acute angle)

2 $\tan X$

3 [a] If the straight line whose equation is : $a X + 2y - 7 = 0$ is parallel to the straight line which makes an angle of measure 45° with the positive direction of X-axis , find the value of a

[b] **Without using calculator , prove that :** $\tan^2 60° - \tan^2 45° = 4 \sin 30°$

4 [a] **In the opposite figure :**



ABCD is a rectangle where AB = 6 cm. , AC = 10 cm.

Find : 1 m ($\angle$ ACB)

2 The surface area of the rectangle ABCD

[b] Find the equation of the straight line passing through the point $(1 , 2)$ and perpendicular to the straight line $X + 3y + 7 = 0$

5 [a] Prove that the points $A(3 , -1)$, $B(\ 4 , 6)$, $C(2 ,\ 2)$ which belong to a perpendicular coordinates plane lie on the circle whose centre is the point $M(-1 , 2)$, then find the area of the circle.

[b] Find the slope and the intercepted part of y-axis by the straight line where its equation is $4X + 5y - 10 = 0$

20 Qena Governorate

Answer the following questions :

1 **Choose the correct answer from those given :**

1 $\sin 30° = \cdots\cdots$

(a) 1 (b) $\frac{\sqrt{3}}{2}$ (c) $\cos 60°$ (d) $\frac{1}{\sqrt{2}}$

2 The number of diagonals of the hexagon equals ....

(a) 5 (b) 6 (c) 2 (d) 9

3 If O the origin point is the midpoint of $\overline{AB}$ as $A = (-2 , 5)$, then $B = \cdots\cdots$

(a) $(2 , 5)$ (b) $(2 , -5)$ (c) $(-2 , 5)$ (d) $(-2 , -5)$

4 If the measure of two angles of a triangle are $70°$, $40°$, then the number of its axes equals ....

(a) 1 (b) 2 (c) 3 (d) zero

5 If L_1 , L_2 are two parallel straight lines of slopes m_1 , m_2 respectively , then $\cdots\cdots$

(a) $m_1 - m_2 = \text{zero}$ (b) $m_1 = -m_2$ (c) $m_1 \times m_2 = 1$ (d) $m_1 \times m_2 = -1$

6 If the lengths of two sides of a triangle are 2 cm. , 5 cm. , then the length of the third side can be $\cdots\cdots$

(a) 2 cm. (b) 3 cm. (c) 4 cm. (d) 1 cm.

2 [a] **Without using calculator , find the value of :** $\cos 60° \sin 30° - \sin 60° \cos 30°$

[b] Find the equation of the straight line which makes with the positive direction of X-axis a positive angle of measure $135°$ and intercepts from the positive part of y-axis a part of length 5 length units.

3 [a] Prove that the points $A(1 , 4)$, $B(-1 , -2)$, $C(2 , -3)$ are the vertices of a right-angled triangle , find its area.

[b] **In the opposite figure :**

Δ ABC is a right-angled triangle at C

, AB = 6 cm. , m ($\angle$ B) = 60°

Find : The length of $\overline{AC}$

---

4 [a] Find the slope of the straight line whose equation is :

$2X - 6y = 12$, then find the points of intersection with the coordinates axes.

[b] **Without using calculator , find the value of X (where X is the measure of an acute angle) that satisfies :** $\tan X = 4 \cos 60° \sin 30°$

---

5 [a] Prove that the straight line which passes through the two points (1 , 3) , (2 , 4) is parallel to the straight line whose equation is : $y - X = 5$

[b] Prove that the figure ABCD is a rectangle where A (1 , 0) , B (1 , 4) , C (7 , 8) , D (9 , 4)

21 Luxor Governorate

Answer the following questions :

1 **Choose the correct answer :**

1 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.

(a) quarter. (b) twice. (c) half. (d) third.

2 If $\tan(2X - 5) = 1$ where X is the measure of an acute angle , then X = ...

(a) 15° (b) 75° (c) 50° (d) 25°

3 If the diagonal length of a square is 10 cm. , then its area = ... cm^2

(a) 100 (b) 75 (c) 50 (d) 25

4 The straight line passing by the two points (0 , 0) , (2 , 3) is parallel to the straight line whose slope is ....

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{-3}{2}$ (d) $-\frac{2}{3}$

5 The image of the point (3 , – 2) by reflection in the X-axis is ....

(a) (– 2 , 3) (b) (3 , 2) (c) (2 , – 3) (d) (– 3 , – 2)

6 The slope of the straight line $X - 5 = 0$ is ....

(a) 5 (b) $\frac{1}{5}$ (c) zero (d) undefined.

2 [a] **Find in degrees the value of x if :** $\tan 2x = 4 \sin 30° \cos 30°$ where $0° < x < 90°$

[b] Find the equation of the straight line passing by the point $(3 , 5)$ and is parallel to the straight line $2x - 3y + 6 = 0$

3 [a] Prove that the straight line passing by the two points $(7 , -3)$, $(5 , -1)$ is perpendicular to the straight line which makes an angle of measure $45°$ with the positive direction of x-axis.

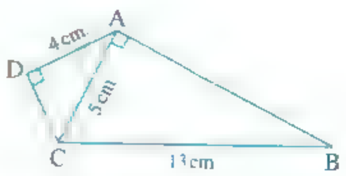
[b] **Without using the calculator , prove that :** $2 \sin 30° + 4 \cos 60° = \tan^2 60°$

4 [a] If the distance between the points $(a , 0)$, $(0 , 1)$ equals $\sqrt{2}$ length unit , find a

[b] If $\overline{AB}$ is a diameter in the circle M where $A (4 , -1)$, $B (-2 , 7)$, find the coordinates of the point M and the radius length of the circle.

5 [a] Prove that the points $A (-1 , -4)$, $B (1 , 0)$, $C (2 , 2)$ are collinear.

[b] **In the opposite figure :**



$m (\angle ADC) = m (\angle BAC) = 90°$

, $AD = 4$ cm. , $AC = 5$ cm. , $BC = 13$ cm.

Find the value of :

$\tan (\angle DAC) \sin (\angle ACB) - \sin (\angle B) \cos (\angle CAD)$

22 Aswan Governorate

Answer the following questions : (Calculator is allowed)

1 **Choose the correct answer from those given :**

1 The measure of the exterior angle of the equilateral triangle is ....°

(a) 60 (b) 90 (c) 120 (d) 180

2 $4 \sin 30° \cos 60° =$ ....

(a) 1 (b) 2 (c) 3 (d) 4

3 The length of the opposite side of the angle with measure $30°$ in the right-angled triangle equals .... the length of the hypotenuse.

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

4 The equation of the straight line passing through the point (2 , – 3) and parallel to X-axis is ....

(a) $y = -2$ (b) $y = -3$ (c) $X = -2$ (d) $X = -3$

5 Δ ABC is an isosceles triangle in which AB = 3 cm. , BC = 7 cm. , then AC = ... cm.

(a) 3 (b) 4 (c) 7 (d) 10

6 The distance between the two straight lines $X - 2 = 0$, $X + 3 = 0$ equals .... length units.

(a) 1 (b) 2 (c) 3 (d) 5

---

2 [a] Find the equation of the straight line which passes through the two points (1 , 3) , (– 1 , - 3)

[b] Prove that the points A (3 , – 1) , B (– 4 , 6) , C (2 , – 2) lie on the circle whose centre is M (– 1 , 2) , then find the circumference of the circle.

---

3 [a] **Without using calculator , find the measure of $\angle$ E (Such that E is an acute angle)**

if : $2 \sin E = \sin 30° \cos 60° + \cos 30° \sin 60°$

[b] If C is the midpoint of $\overline{AB}$, then find X , y where A (X , 3) , B (6 , y) , C (4 , 6)

---

4 [a] Δ ABC is right-angled at C in which AC = 6 cm. , BC = 8 cm.

Find : 1 $\cos A \cos B - \sin A \sin B$ 2 m($\angle$ B)

[b] If the straight line L_1 passes through the two points (3 , 1) , (2 , k) and the straight line L_2 makes with the positive direction of the X-axis an angle of measure 45°

, find the value of k if the two straight lines are : 1 Parallel. 2 Perpendicular.

---

5 [a] Find the equation of the straight line which passes through the point (3 , – 5) and is parallel to the straight line $X + 2y - 7 = 0$

[b] **Find the value of X if :** $X \sin 30° \cos^2 45° = \sin^2 60°$

24 North Sinai Governorate

Answer the following questions :

1 **Choose the correct answer from those given :**

1 If a = b , a , b are the measures of two complementary angles , then a = ...°

(a) 30 (b) 45 (c) 60 (d) 90

2 If $\tan 3X = \sqrt{3}$, where X is the measure of an acute angle , then $X =$ .... °

(a) 10 (b) 20 (c) 30 (d) 60

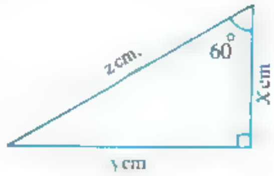
3 The sum of measures of the interior angles of the quadrilateral equals .... °

(a) 360 (b) 180 (c) 90 (d) 540

4 If A (1 , – 6) , B (9 , 2) , then the midpoint of $\overline{AB}$ is ....

(a) (– 2 , 5) (b) (2 , – 5) (c) (5 , – 2) (d) (– 5 , 2)

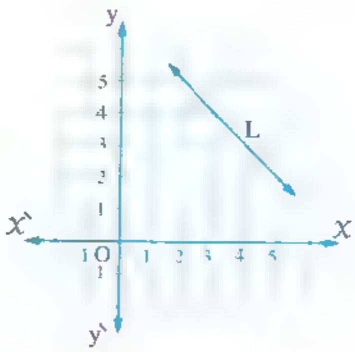
5 **In the opposite figure :**



(a) $X + y = z$ (b) $z = X^2 + y^2$

(c) $2X = z$ (d) $y = \frac{1}{2} z$

6 **In the opposite figure :**



L is a straight line passing through the two points (2 , 5) , (5 , 2) , then the point .... $\in$ L

(a) (1 , 6) (b) (2 , 3)

(c) (0 , 0) (d) (3 , – 4)

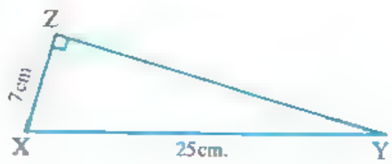
2 [a] **Without using the calculator , prove that :** $\sin 60° = 2 \sin 30° \cos 30°$

[b] ABCD is a quadrilateral , where A (2 , 4) , B (– 3 , 0) , C (– 7 , 5) , D (– 2 , 9)

Prove that : ABCD is a square.

3 [a] Find the equation of the straight line whose slope is 3 and passes through the point (5 , 0)

[b] **In the opposite figure :** XYZ is a right-angled triangle at Z , XZ = 7 cm. , XY = 25 cm.



1 **Find the value of :** $\tan X \times \tan Y$

2 **Prove that :** $\sin^2 X + \sin^2 Y = 1$

4 [a] **Without using the calculator , find the value of X if :** $2 \sin X = \tan^2 60° - 2 \tan 45°$ where X is the measure of an acute angle.

[b] Prove that the points A (– 1 , – 4) , B (1 , 0) , C (2 , 2) are collinear.

5 [a] Prove that the straight line passing through the two points $(-3, -2)$, $(4, 5)$ is parallel to the straight line which makes with the positive direction of the X-axis an angle of measure $45°$

[b] If the straight line passing through the two points $(-2, 3)$, $(1, k)$ is perpendicular to the straight line whose slope equals -3, then find the value of k

24 Red Sea Governorate

Answer the following questions :

1 **Choose the correct answer from those given :**

1. $2 \sin 30° = \dots\dots$
   (a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) 1 (d) 2

2. The measure of the exterior angle of the equilateral triangle equals $\dots\dots$
   (a) $30°$ (b) $60°$ (c) $90°$ (d) $120°$

3. The distance between the point $(3, 4)$ and the point of origin equals $\dots\dots$ length units.
   (a) 3 (b) 4 (c) 5 (d) 7

4. If 3 cm., 7 cm., ℓ are the lengths of the sides of a triangle, then ℓ can be equal to $\dots\dots$ cm.
   (a) 3 (b) 7 (c) 4 (d) 10

5. If $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = \frac{2}{3}$, then the slope of $\overleftrightarrow{CD} = \dots\dots$
   (a) $\frac{2}{3}$ (b) $-\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $-\frac{3}{2}$

6. The image of the point $(3, -2)$ by reflection in the origin point is $\dots\dots$
   (a) $(-3, 2)$ (b) $(-3, -2)$ (c) $(3, 2)$ (d) $(-2, 3)$

---

2 [a] **Find the value of :** $\cos 60° \sin 30° - \sin 60° \tan 60° + \cos^2 30°$

[b] Prove that the straight line which passes through the two points $(3, -2)$, $(4, 5)$ is parallel to the straight line which makes an angle of measure $45°$ with the positive direction of the X-axis.

---

3 [a] Find the slope of the straight line $3X + 4y - 5 = 0$, then find the length of the intercepted part from y-axis.

[b] **Find the value of X where :** $X \sin 30° \cos^2 45° = \sin^2 60°$

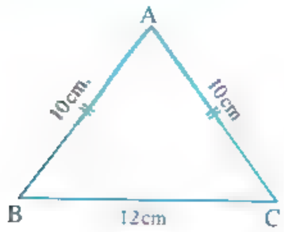
4 [a] **In the opposite figure :**

ABC is a triangle in which AB = AC = 10 cm.

, BC = 12 cm.

1 **Find :** m ($\angle$ B)

2 **Prove that :** $\sin^2 B + \cos^2 B = 1$



[b] Prove that the triangle whose vertices are A (1 , 4) , B (– 1 , – 2) , C (2 , – 3) is right-angled , then find its area.

5 [a] Find the equation of the straight line which passes through the point A (4 , 6) and the midpoint of $\overline{BC}$ where B (3 , 7) , C (1 , – 3)

[b] ABCD is a parallelogram where A (3 , 3) , B (2 , – 2) , C (5 , – 1) , M is the intersection point of its diagonals. **Find :**

1 The coordinates of M

2 The coordinates of D

25 Matrouh Governorate

Answer the following questions : (Calculator is allowed)

1 **Choose the correct answer from those given :**

1 The area of the square whose perimeter is 16 cm. equals .... cm^2.

(a) 4 (b) 8 (c) 16 (d) 256

2 The equation of the straight line whose slope is 1 and passes through the origin point is ....

(a) $x = 1$ (b) $y = 1$ (c) $y = x$ (d) $y = -x$

3 If $\cos 2x = \frac{1}{2}$, then $x =$ ....

(a) 15° (b) 30° (c) 45° (d) 60°

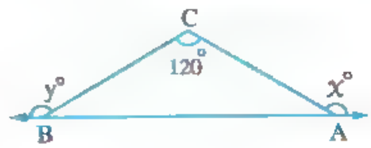
4 A right circular cylinder , if its height equals the length of its base radius = r cm. , then its volume = ... cm^3.

(a) πr^3 (b) $2 \pi r^2$ (c) $2 \pi r^3$ (d) $\frac{4}{3} \pi r^3$

5 The slope of the straight line which is parallel to the x-axis is ....

(a) – 1 (b) zero (c) 1 (d) undefined.

6 **In the opposite figure :**



If m (∠ C) = 120°

, then $X° + y° =$ ....

(a) 90° (b) 180° (c) 300° (d) 360°

---

2 [a] **Without using calculator , find the value of X if :** $4 X = \cos^2 30° \tan^2 30° \tan^2 45°$

[b] $\overline{AB}$ is a diameter of the circle M , if B (8 , 11) , M (5 , 7)

, find : 1 The coordinates of A

2 The length of the radius of the circle.

---

3 [a] Prove that the points A (– 2 , 5) , B (3 , 3) , C (4 , 2) are not collinear and if D (– 9 , 4) , prove that the figure ABCD is a parallelogram.

[b] **Explaining the steps and without using calculator , find :**

$$\frac{\cos^2 60° + \cos^2 30° - \tan^2 45°}{\sin 60° \tan 60° - \sin 30°}$$

---

4 [a] Find the equation of the straight line which passes through the point (3 , 4) and is perpendicular to the straight line $5 X - 2 y + 7 = 0$

[b] ABCD is an isosceles trapezoid , $\overline{AD} // \overline{BC}$, AD = 4 cm. , AB = 5 cm. where BC = 12 cm.

Prove that : $\frac{5 \tan B \cos C}{\sin^2 C + \cos^2 C} = 3$

---

5 [a] If the straight line L_1 passes through the two points (3 , 1) , (2 , k) and the straight line L_2 makes with the positive direction of the X-axis an angle whose measure is 45°

, then find k if the two straight lines L_1 , L_2 are :

1 Parallel. 2 Perpendicular.

[b] **Find the slope and the intercepted part of y-axis by the straight line :** $2 X = 3 y + 6$

Multidisciplinary Exams

Selected math exams from the multidisciplinary exams of the previous year

1 Cairo Governorate — Administration of Education

Choose the correct answer :

1. If $(X , 2) = (5 , y)$, then $X + y =$ ....
   (a) 2 (b) 5 (c) 7 (d) 10
2. If 5 , 7 , X and 14 are proportional quantities , then $X =$ ....
   (a) 5 (b) 10 (c) 12 (d) 14
3. If $f(X) = 4X^3 + X^2 - 1$, then f is called a polynomial function of .... degree.
   (a) the first (b) the second (c) the third (d) the fourth
4. The range of the set of the values 8 , 1 , 7 , 9 and 6 is ....
   (a) 3 (b) 5 (c) 6 (d) 8
5. If $\sin 65° = \cos X$ (where X is an acute angle) , then $m(\angle X) =$ ....°
   (a) 25 (b) 30 (c) 45 (d) 60
6. If $A = (7 , 2)$, $B = (1 , 4)$, then the midpoint of $\overline{AB}$ is ....
   (a) (6 , 2) (b) (4 , 3) (c) (8 , 6) (d) (3 , 1)
7. $2 \sin 30° - \tan 45° =$ ....
   (a) zero (b) 1 (c) 2 (d) 3
8. The straight line whose equation is $y = 2X + 5$ intercepts from y-axis a part of .... length unit.
   (a) 1 (b) 2 (c) 3 (d) 5

2 Giza Governorate — El-Dokki Directorate

Choose the correct answer :

1. The point $(3 , -4)$ lies in the .... quadrant.
   (a) first (b) second (c) third (d) fourth
2. If 2 , 3 , 6 and X are proportional quantities , then $X =$ ....
   (a) 3 (b) 9 (c) 11 (d) 18

3 The range of the set of values 7 , 3 , 6 , 9 and 5 is ......

(a) 3 (b) 4 (c) 6 (d) 12

4 If $y \propto X$, y = 2 at $X = 8$, then y = 3 at $X =$ ....

(a) 6 (b) 12 (c) 16 (d) 24

5 If $\cos X = \frac{1}{2}$, then $2\, m(\angle X) =$ ....°

(a) 30 (b) 60 (c) 90 (d) 120

6 The equation of the straight line passing through the point (– 5 , – 3) parallel to X-axis is ....

(a) $X = -5$ (b) $X = -3$ (c) $y = -5$ (d) $y = -3$

7 If the two straight lines whose slopes are $-\frac{1}{2}$ and $\frac{6}{X}$ are parallel , then $X =$ ......

(a) 6 (b) – 6 (c) – 12 (d) 12

8 If A = (5 , 7) , B = (1 , – 1) , then the midpoint of $\overline{AB}$ is ....

(a) (2 , 3) (b) (3 , 3) (c) (3 , 2) (d) (3 , 4)

8 Alexandria Governorate

El-Montaza Directorate – A

Choose the correct answer :

1 If the point (5 , b – 7) lies on the X-axis , then b = ...

(a) 2 (b) 5 (c) 7 (d) 12

2 The range of the values 10 , 15 , 20 , 24 equals ....

(a) 14 (b) 17 (c) 10 (d) 24

3 If $f(X) = X^2 + 3$, then $f(-1) =$ ....

(a) 2 (b) 3 (c) 4 (d) – 4

4 The relation which represents a direct variation between the two variables X and y is ....

(a) $X y = 5$ (b) $X + y = 5$ (c) $\frac{X}{3} = \frac{7}{y}$ (d) $5\, X = 2\, y$

5 If m_1 and m_2 are the slopes of two perpendicular straight lines , then $m_1 \times m_2 =$ ......

(a) – 1 (b) $\frac{1}{2}$ (c) $\frac{-1}{2}$ (d) 1

6 If A (5 , 7) , B (1 , – 1) , then the midpoint of $\overline{AB}$ is ....

(a) (2 , 3) (b) (3 , 3) (c) (3 , 2) (d) (3 , 4)

7 If $\tan (X + 30°) = \sqrt{3}$, where X is the measure of an acute angle , then $X =$ .... °

(a) 30 (b) 45 (c) 50 (d) 60

8 The equation of the straight line passing through the origin point and its slope = 3 is ....

(a) $y = X + 3$ (b) $y = 3X$ (c) $X = 3y$ (d) $y = 3X + 1$

4 El-Kalyoubia Governorate — Administration of Education

Choose the correct answer :

1 If $(2 , 3) \in \{2 , 5\} \times \{6 , X\}$, then $X =$ ....

(a) 6 (b) 5 (c) 3 (d) 2

2 The fourth proportional of the quantities 2 , 3 , 6 is ....

(a) 9 (b) 3 (c) 12 (d) 18

3 If y varies inversely as X and $X = \sqrt{3}$ when $y = \frac{2}{\sqrt{3}}$, then the constant of variation = ...

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) 2 (d) 6

4 If $\Sigma (X - \overline{X})^2 = 48$ for a set of 12 values , then $\sigma =$ ....

(a) – 2 (b) 2 (c) 4 (d) 6

5 $2 \sin 30° \tan 60° =$ ....

(a) $\sqrt{3}$ (b) 3 (c) $\frac{\sqrt{3}}{3}$ (d) $\frac{1}{2}$

6 The distance between the two points (3 , 0) and (0 , – 4) equals .. .. length unit.

(a) 4 (b) 5 (c) 6 (d) 7

7 If A (5 , 7) and B (1 , – 1) , then the midpoint of $\overline{AB}$ is . ...

(a) (2 , 3) (b) (3 , 3) (c) (3 , 2) (d) (3 , 4)

8 The equation of the straight line which passes through the point (– 2 , 5) and is parallel to X-axis is ....

(a) $X = -2$ (b) $X = 5$ (c) $y = -2$ (d) $y = 5$

5 El-Sharkia Governorate

El-Sharkia Directorate

Choose the correct answer :

1 If $X = \{3, 4, 5\}$, $Y = \{3, 6\}$, then $n(X \times Y) = \ldots\ldots$

(a) 3 (b) 5 (c) 6 (d) 9

2 If a, 2, b, 6 are proportional quantities, then $\frac{a}{b} = \ldots\ldots$

(a) 6 (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 4

3 If $x y = 12$, then $y \propto \ldots\ldots$

(a) $12x$ (b) $x - 12$ (c) x (d) $\frac{1}{x}$

4 If the range of the values 2, 7, 9 and x is 11 where $x > 0$, then $x = \ldots\ldots$

(a) 13 (b) 7 (c) 9 (d) 3

5 If $\tan(x + 15°) = 1$ where x is the measure of an acute angle, then $x = \ldots\ldots °$

(a) 20 (b) 30 (c) 40 (d) 50

6 The straight line whose equation is $y = 2x + 4$ intercepts from the positive part of y-axis a part of length .... length unit.

(a) 10 (b) 8 (c) 6 (d) 4

7 If A (6, 1) and B (2, 3), then the midpoint of $\overline{AB}$ is ....

(a) (4, 2) (b) (3, 4) (c) (8, 4) (d) (3, 2)

8 The equation of the straight line whose slope is 3 and passes through the origin point is ....

(a) $y = 2x$ (b) $y = -x$ (c) $y = 3x$ (d) $y = 3$

6 El-Gharbia Governorate

Administration of Education

Choose the correct answer :

1 If $X = \{2\}$, $Y = \{3\}$, then $X \times Y = \ldots\ldots$

(a) 6 (b) $\{6\}$ (c) (2, 3) (d) $\{(2, 3)\}$

2 If $f(x) = 2$, then $f(3) - f(1) = \ldots\ldots$

(a) $f(2)$ (b) 2 (c) zero (d) 10

3 If $X \propto y$ and $X = 3$ when $y = 6$, then $X = 5$ when $y =$ ....

(a) 2 (b) 6 (c) 8 (d) 10

4 If $\sum \left(X - \overline{X}\right)^2 = 48$ for a set of the values whose number equals 12 , then $\sigma =$ ....

(a) -4 (b) -2 (c) 2 (d) 4

5 If $2 \sin X = \tan 60°$ (where X is an acute angle) , then $m(\angle X) =$ .... °

(a) 30 (b) 45 (c) 60 (d) 40

6 If X , y are the measures of two complementary angles and $\sin X = \frac{3}{5}$, then $\cos y =$ ....

(a) $\frac{4}{5}$ (b) $\frac{3}{5}$ (c) $\frac{3}{4}$ (d) $\frac{5}{3}$

7 The radius length of the circle of center (7 , 4) and passing through the point (3 , 1) equals .... length unit.

(a) 5 (b) 10 (c) 2.5 (d) 25

8 If the two straight lines : $3X - 4y - 3 = 0$, $ky + 4X - 8 = 0$ are perpendicular , then $k =$ ....

(a) -4 (b) -3 (c) 3 (d) 4

7 Ismailia Governorate — El-Tall Directorate

Choose the correct answer :

1 The function $f : f(X) = X(X + 8)$ is polynomial of the .... degree.

(a) 1^{st} (b) 2^{nd} (c) 3^{rd} (d) 4^{th}

2 The range of the set of values 5 , 14 , 23 and 15 is ....

(a) 12 (b) 14 (c) 18 (d) 23

3 The third proportional of the two numbers 3 and 6 is ....

(a) $\frac{1}{2}$ (b) 9 (c) 2 (d) 12

4 If $Xy = 7$, then y varies directly with ....

(a) $\frac{1}{X}$ (b) $X - 7$ (c) X (d) $X + 7$

5 $4 \cos 30° \tan 60° =$ ....

(a) 3 (b) $2\sqrt{3}$ (c) 6 (d) 12

6 The midpoint of $\overline{AB}$ where A (6 , 1) , B (– 2 , 3) is the point ....

(a) (4 , 2) (b) (2 , 2) (c) (4 , 4) (d) (4 , 8)

7 If $\overleftrightarrow{AB}$ // $\overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = \frac{2}{3}$, then the slope of $\overleftrightarrow{CD}$ = ...

(a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) $\frac{-2}{3}$ (d) $\frac{-3}{2}$

8 The equation of the straight line which passes through the point (3 , – 2) and is parallel to y-axis is ....

(a) $x = -2$ (b) $y = -2$ (c) $x = 3$ (d) $y = 3$

8 Kafr El-Sheikh Governorate

East Kafr El-Sheikh Directorate

Choose the correct answer :

1 If $(3 , 5) \in \{3 , 4\} \times \{x , 8\}$, then x = ...

(a) 3 (b) 5 (c) 6 (d) 8

2 The relation which represents a direct variation between x and y is ....

(a) $x y = 5$ (b) $y = x + 1$ (c) $\frac{x}{2} = \frac{3}{y}$ (d) $\frac{x}{3} = \frac{y}{2}$

3 The difference between the greatest and the smallest value is the ....

(a) arithmetic mean (b) mode (c) median (d) range

4 If 4 , x and 9 are in proportion , then x = ...

(a) – 12 (b) – 6 (c) ± 12 (d) ± 6

5 If $\cos 2x = \frac{1}{2}$ where $2x$ is the measure of an acute angle , then x = ...

(a) 15° (b) 30° (c) 45° (d) 60°

6 If (0 , 4) is the midpoint of $\overline{AB}$, A (– 1 , – 1) , then B is ....

(a) (1 , 9) (b) (– 1 , 9) (c) (2 , 3) (d) (1 , 3)

7 The slope of the straight line whose equation is $3y = 6x + 1$ is ....

(a) 0 (b) 1 (c) 2 (d) 3

8 If A (5 , 4) , B (8 , 8) , then AB = ... length unit.

(a) 5 (b) 6 (c) 8 (d) 12

9 Assiut Governorate

Administration of Education

Choose the correct answer :

1 If $n\left(X^2\right) = 4$, $n(X \times Y) = 6$, then $n(Y) =$ ……………

(a) 2 (b) 3 (c) 6 (d) 10

2 If the all set of values are equal , then ……

(a) $\sigma = 0$ (b) $\sigma = -4$ (c) $X - \overline{X} > 0$ (d) $X - \overline{X} < 0$

3 If $y = 8X$, then ……………

(a) $y \propto X^2$ (b) $y \propto \frac{1}{X}$ (c) $y \propto X$ (d) $y \propto \frac{1}{X^2}$

4 The third proportional of the two numbers 3 and 6 is ……………

(a) 4 (b) 12 (c) 18 (d) 36

5 If $\sin (X + 5)^\circ = \frac{1}{2}$ where $(X + 5)^\circ$ is the measure of an acute angle , then $X =$ ……………°

(a) 30 (b) 60 (c) 25 (d) 55

6 The slope of the straight line which is perpendicular to the straight line passing through the two points $(-1 , 4)$, $(2 , 5)$ is ……………

(a) -3 (b) 3 (c) $\frac{-1}{3}$ (d) $\frac{1}{3}$

7 If $(0 , 4)$ is the midpoint of the line segment whose terminals are $(1 , -1)$, (X , y) , then the point (X , y) is ……………

(a) $(-1 , 9)$ (b) $(-1 , -9)$ (c) $(-1 , 3)$ (d) $(1 , 9)$

8 The intercepted part of the y-axis by the straight line of the equation : $3y = 2X + 5$ equals ……

(a) $\frac{2}{3}$ (b) $\frac{5}{3}$ (c) $\frac{3}{2}$ (d) $\frac{3}{5}$

10 Qena Governorate

Nagaa Hamady Directorate

Choose the correct answer :

1 The range of the values 5 , 11 , 6 , 8 and 3 is ……………

(a) 5 (b) 11 (c) 6 (d) 8

2 The length of y intercept by the straight line : $y = -3X + 5$, equals …………… length unit.

(a) -3 (b) 5 (c) 3 (d) -5

3 If $5xy = 13$, then $y \propto$ ....

(a) $5x$ (b) x (c) $\frac{1}{x}$ (d) $\frac{1}{x^2}$

4 If $(3 , k) \in \{3 , 2\} \times \{9\}$, then k = ...

(a) 3 (b) 2 (c) 27 (d) 9

5 The middle proportional between 2 and 8 is ....

(a) ± 4 (b) 4 (c) – 4 (d) 16

6 If 2 sin X = 1 where X is an acute angle , then m (∠ X) = ...°

(a) 60 (b) 30 (c) 45 (d) 90

7 If A (3 , 5) , B (1 , – 1) , then the midpoint of $\overline{AB}$ is ....

(a) (4 , 4) (b) (4 , 2) (c) (2 , – 2) (d) (2 , 2)

8 4 sin 30° tan 45° = ...

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 3

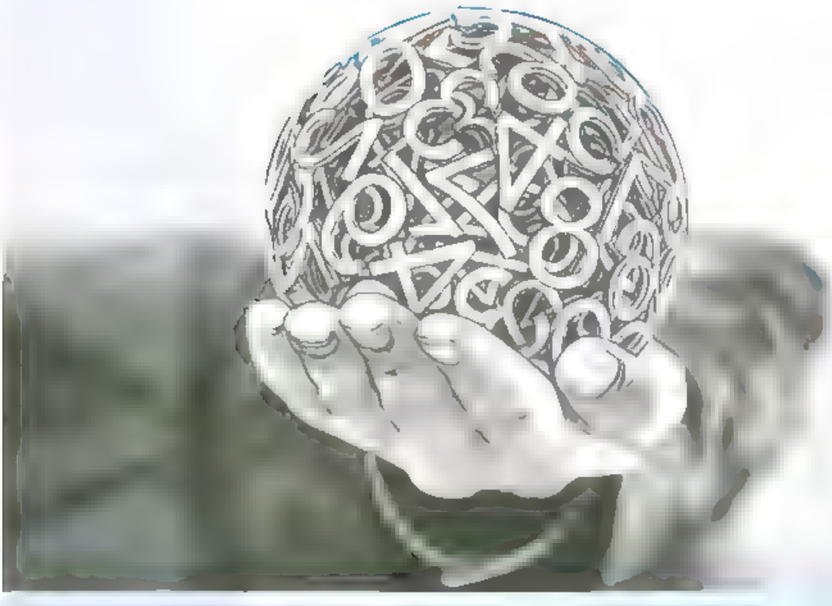
GUIDE ANSWERS

3rd PREP 2022

FIRST TERM

Maths

of Algebra and Statistics Exercises



Answers of unit one

Answers of Exercise 1

1

1) $a = 5$, $b = 9$ 2) $a = \sqrt{25} = 5$, $b = \sqrt[3]{27} = 3$

3) $a - 2 = 2$ $\therefore a = 4$, $b + 1 = 3$ $\therefore b = 2$

4) $2 - a = 6$ $\therefore a = -4$, $b - 3 = -1$ $\therefore b = 2$

5) $a - 7 = -2$ $\therefore a = 5$, $b^3 - 1 = 26$

$b^3 = 27$ $\therefore b = \sqrt[3]{27} = 3$

6) $a = 2 - a$ $\therefore 2a = 2$ $\therefore a = 1$

, $b = 2b - 3$ $\therefore b = 3$

7) $a^5 = 32$ $\therefore a^5 = 2^5$ $\therefore a = 2$

, $b^2 - 1 = \sqrt[3]{27}$ $\therefore b^2 - 1 = 3$

$b^2 = 4$ $\therefore b = \pm 2$

8) $b = 7$, $a = b^2 = 49$

9) $a = 7$, $2a = 2b + 1$ $\therefore 2b + 1 = 14$

$\therefore 2b = 13$ $\therefore b = 6.5$

10) $5a - 1 = 3$ $\therefore 5a = 4$ $\therefore a = \frac{4}{5}$

, $b = 4a$ $\therefore b = 4 \times \frac{4}{5}$ $\therefore b = \frac{16}{5}$

2

1) a 2) d 3) c 4) c 5) d

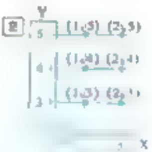
3

$X \times Y = \{(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5)\}$

1)

The arrow diagram

2)



The Cartesian diagram

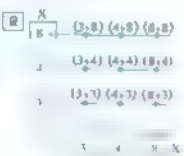
4

$X^2 = \{(3, 3), (3, 4), (3, 8), (4, 3), (4, 4), (4, 8), (8, 3), (8, 4), (8, 8)\}$

1)

The arrow diagram

2)



The Cartesian diagram

5

1) $X \times Y = \{(1, 4), (2, 4), (3, 4)\}$

2) $Y \times X = \{(4, 1), (4, 2), (4, 3)\}$

3) $Y^2 = \{(4, 4)\}$

4) $n(X^2) = 3^2 = 9$

6

1) $X \times Y = \{(2, 4), (2, 0), (-1, 4), (-1, 0)\}$

2) $Y \times Z = \{(4, 4), (4, 5), (4, -2), (0, 4), (0, 5), (0, -2)\}$

3) $X^2 = \{(2, 2), (2, -1), (-1, 2), (-1, -1)\}$

4) $n(X \times Z) = 2 \times 3 = 6$

5) $n(Y^2) = 2 \times 2 = 4$

6) $n(Z^2) = 3 \times 3 = 9$

7

1) b 2) b 3) d 4) d 5) a

6) d 7) a 8) a 9) b 10) b

11) a 12) c 13) c 14) c

8

$X = \{2, 3, 5\}$, $Y = \{6, 9\}$

9

1) $X = \{1\}$, $Y = \{1, 3, 5\}$

2) $Y \times X = \{(1, 1), (3, 1), (5, 1)\}$

3) $Y^2 = \{(1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5)\}$

10

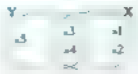
$X = \{1, 2\}$

11

$X = \{3\}$

$X^2 = \{3\} \times \{3\} = \{(3, 3)\}$

12



1 $(X \cap Y) \times Y =$

$\{3, 4\} \times \{3, 4, 5\}$

$= \{(3, 3), (3, 4), (3, 5), (4, 3), (4, 4), (4, 5)\}$

2 $(X - Y) \times Y = \{1, 2\} \times \{3, 4, 5\}$

$= \{(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5)\}$

3 $(Y - X) \times X = \{5\} \times \{1, 2, 3, 4\}$

$= \{(5, 1), (5, 2), (5, 3), (5, 4)\}$

1 $X \times (Y \cap Z) = \{3, 4\} \times \{5\} = \{(3, 5), (4, 5)\}$

2 $(X - Y) \times Z = \{3\} \times \{6, 5\} = \{(3, 6), (3, 5)\}$

3 $(X - Y) \times (Y - Z) = \{3\} \times \{4\} = \{(3, 4)\}$

14

First :

1 $X \times Y = \{(1, 2), (1, 3)\}$

2 $Y \times Z = \{(2, 2), (2, 5), (2, 6), (3, 2), (3, 5), (3, 6)\}$

3 $X \times Z = \{(1, 2), (1, 5), (1, 6)\}$

4 $Y^2 = \{(2, 2), (2, 3), (3, 2), (3, 3)\}$

Second :

$(X \times Y) \cup (Y \times Z) = \{(1, 2), (1, 3), (2, 2), (2, 5), (2, 6), (3, 2), (3, 5), (3, 6)\}$

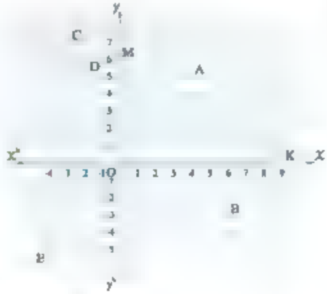
Third : $X \times (Y \cap Z) = \{1\} \times \{2\} = \{(1, 2)\}$

Fourth : $(X \times Y) \cap (X \times Z) = \{(1, 2)\}$

Fifth : $(Z - Y) \times (X \cup Y) = \{5, 6\} \times \{1, 2, 3\}$

$= \{(5, 1), (5, 2), (5, 3), (6, 1), (6, 2), (6, 3)\}$

15



A Lies on the first quadrant
B Lies on the fourth quadrant
C Lies on the second quadrant
D Lies on the second quadrant
E Lies on the third quadrant
M Lies on y-axis K Lies on X-axis

16

1 b	2 a	3 c	4 b	5 b	6 d	7 d
8 c	9 a	10 c	11 a	12 c	13 b	14 c

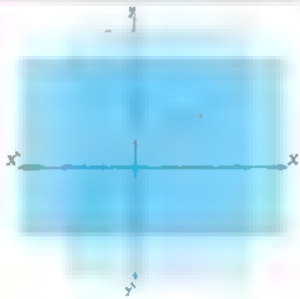
17

$\because$ AB = 3 length unit , BC = 4 length unit

The area of $\Delta ABC = \frac{1}{2} \times AB \times BC$

$= \frac{1}{2} \times 3 \times 4 = 6$ square unit

18

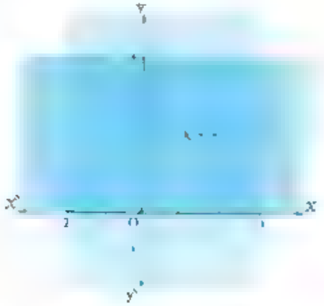


$A \in X \times X$ $B \in X \times X$

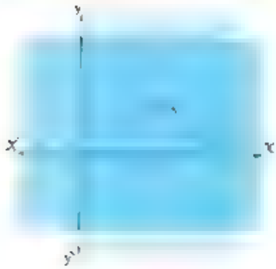
$C \notin X \times X$ $D \in X \times X$

19

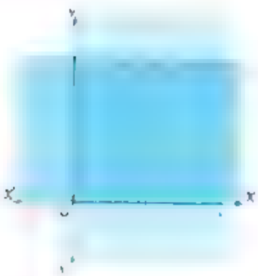
1 $X \times Y$



2 $Y \times X$



3 Y^2



20

1 d 2 a 3 a

21

$\because X = \{a , 2\} , Y = \{1 , 2 , 3\}$

$, \because X \subset Y \qquad \therefore a = 1$ or $a = 3$

22

$X = \{4 , 1\} \quad , \quad Y = \{4 , 1 , 7\}$

$, X \times Y = \{(4 , 4) , (4 , 1) , (4 , 7) , (1 , 4)$
$, (1 , 1) , (1 , 7)\}$

Answers of Exercise 2

1

1 b 2 b 3 c 4 a

2

R_1 is a function and its range $= \{1 , 9\}$

R_2 is a function and its range $= \{1 , 4 , 9\}$

R_3 is not a function.

R_4 is not a function

3

f_1 is a function

$, f_1 = \{(1 , 4) , (2 , 3) , (3 , 8) , (4 , 6) , (5 , 9)\}$

and its range $= \{4 , 3 , 8 , 6 , 9\} = Y$

f_2 is not a function because $2 \in X$ has two images and $3 \in X$ has no image

f_3 is a function $, f_3 = \{(a , 3) , (b , 3) , (c , 3)$
$, (d , 3) , (e , 3)\}$, its range $= \{3\}$

4

1 R_1 is not a function because $c \in X$ has no image

2 R_2 is not a function because $b \in X$ has two images

3 R_3 is a function because each element of X appears only once as a first projection in an ordered pair of the relation

and the range $= \{2 , 8 , 10\}$

5

1. $R = \{(-3, 1), (-3, 8), (1, 2), (4, 4)\}$
2. R is not a function because $-3 \in X$ has two images
3. $\because (X, 2) \in R$ $\therefore X = 1$

6

$R = \{(1, 3), (2, 6), (3, 9)\}$

, R is a function from X to Y because each element of X has one image in Y

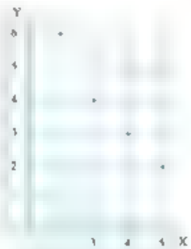
and the range $= \{3, 6, 9\}$

7

$R = \{(4, 2), (6, 3), (8, 4), (10, 5)\}$

8

$R = \{(1, 6), (3, 4), (4, 3), (5, 2)\}$



9

$R = \{(0, 1), (0, 3), (0, 5), (0, 6), (1, 1), (1, 3), (1, 5), (1, 6), (4, 1), (4, 3)\}$

R is not a function because $0 \in X$, $1 \in X$, $4 \in X$ each of them has more than one image in Y

also $7 \in X$ has no image in Y

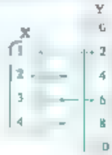
10

$R = \{(2, 4), (2, 5), (2, 6), (2, 7), (2, 9), (4, 4), (4, 5), (4, 6), (4, 7), (4, 9), (5, 5), (5, 6), (5, 7), (5, 9), (7, 7), (7, 9)\}$

Represent by yourself

11

1. $R = \{(1, 2), (2, 4), (3, 6), (4, 8)\}$



2. R is a function from X to Y because each element of X has one image in Y

, its range $= \{2, 4, 6, 8\}$

12

1. $R = \{(1, 2), (2, 3), (3, 2)\}$

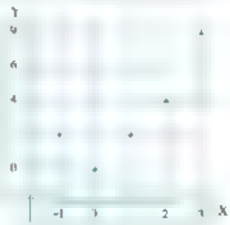
Yes, R is a function

2. $\because (2, 3) \in R$, $\therefore 2a\ R\ 3$

$\therefore 2a = 2$ $\therefore a = 1$

13

1. $R = \{(-1, 1), (0, 0), (1, 1), (2, 4), (3, 9)\}$
2. R is a function from X to Y because each element of X has one image in Y



14

$R = \{(1, 1), (2, 8)\}$ Represent by yourself

15

$R = \left\{\left(-2, \frac{1}{4}\right), \left(-1, \frac{1}{2}\right), (0, 1), (1, 2), (2, 4)\right\}$

Represent by yourself

, R is a function from X to Y because each element of X has one image in Y

The range $= \left\{\frac{1}{4}, \frac{1}{2}, 1, 2, 4\right\}$

$R = \{(2, 10), (2, 16), (2, 24), (2, 30), (5, 10), (5, 30), (8, 16), (8, 24)\}$

R is not a function because $2 \in X$ has more than one image in Y

also $5 \in X$, $8 \in X$ each of them has two images in Y

Represent by yourself

$R = \{(2, 6), (2, 8), (2, 10), (3, 6), (3, 15), (4, 8)\}$

18

1 a   2 c

19

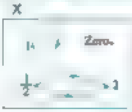
The arrow diagram number (2)

$R = \{(6, -6), (4, -4), (2, -2), (0, 0), (-2, 2), (-4, 4), (-6, 6)\}$

∴ R is a function on X because each element of X has a unique image in X

its range = X

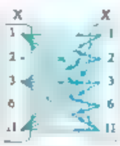
$R = \{(1, 1), (2, \frac{1}{2}), (\frac{1}{2}, 2)\}$



∴ R is not a function because $0 \in X$ has no image in X

22

$R = \{(1, 1), (1, 2), (1, 3), (1, 6), (1, 11), (3, 1), (3, 2), (3, 3), (3, 6), (3, 11), (11, 1), (11, 2), (11, 3), (11, 6), (11, 11)\}$



R is not a function because each of $1 \in X$, $3 \in X$, $11 \in X$ has more than one image in X

also each of $2 \in X$, $6 \in X$ has no image in X

23

$X = \{1, 2, 3\}$

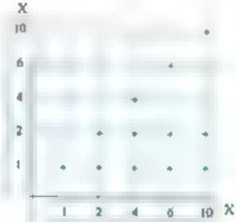
$R = \{(1, 2), (2, 1), (3, 3)\}$

∴ R is a function

, its range $= \{1, 2, 3\}$

24

$R = \{(1, 1), (2, 1), (2, 2), (4, 1), (4, 2), (4, 4), (6, 1), (6, 2), (6, 6), (10, 1), (10, 2), (10, 10)\}$



R is not a function because each of $2 \in X$, $4 \in X$, $6 \in X$ and $10 \in X$ has more than one image in X

25

$R = \{(-2, 2), (-1, 1), (0, 0), (1, 1), (2, 2)\}$

R is a function on X

26

1 ∵ R is a function from X to Y

∴ each element in X has only one image in Y

, ∵ the image of $-2 = (-2)^2 - 1 = 3 \in Y$

, the image of $2 = (2)^2 - 1 = 3 \in Y$

, the image of $5 = (5)^2 - 1 = 24 \in Y$

∴ $\ell = 24$

2 Represent by yourself

27

1. $R_1 = \{(0, 0), (4, 2), (16, 4)\}$

   $\because R_1$ is a function from X to Y

2. $R_2 = \{(0, 0)\}$

   $\therefore R_2$ is not a function because each of $4 \in X$, $16 \in X$ has no image in Y

3. $R_3 = \{(0, 0), (4, 2)\}$

   $\therefore R_3$ is not a function because $16 \in X$ has no image in Y

28

a R b , a × b = 2

1. X R 4 $X \times 4 =$

2. y R 3 y $y \times 3y = 12$

   $3y^2 = 12$ $y^2 = 4$

   $\therefore y = 2$ or $y = -2$ (refused because $y \in \mathbb{N}$)

29

$\because (a, 2) \in R$, $y^2 = 2X$

$2^2 = 2a$ $a = 2$

$\because (\frac{2}{9}, b) \in R$ $b^2 = 2 \times \frac{2}{9} = \frac{4}{9}$

$b = -\frac{2}{3}$ (refused) or $b = \frac{2}{3}$

$\because (c, 3) \in R$ $3^2 = 2c$ $c = 4\frac{1}{2}$

$\because (\frac{9}{32}, d) \in R$ $d^2 = 2 \times \frac{9}{32} = \frac{9}{16}$

$d = -\frac{3}{4}$ (refused) or $d = \frac{3}{4}$

30

$R_1 = \{(1, -1), (0, 0), (-1, 1)\}$

$, R_2 = \{(1, 1), (-1, -1)\}$, $R = R_1 \cap R_2$

$\therefore R = \varnothing$, R is not a function

31

1. $R = \{(1, 13), (1, 31), (2, 23), (3, 13),$

   $(3, 31), (3, 23)\}$ (Represent by yourself)

2. 2 R 65 false (say why by yourself)

   1 R 31 true (say why by yourself)

   3 R 13 true (say why by yourself)

3. $M = \{(2, 23), (3, 23)\}$

32

$R = \{(-1, 1), (1, 5), (2, 7)\}$

33

1. L does not represent a relation because $L \not\subset X \times Y$
2. M represents a relation because $M \subset X \times Y$

34

Each element in X has to appear only once as a first projection in R

a = 3 , b = 5 or a = 5 , b = 3

a + b = 3 + 5 = 8

35

$R = \{(-1, 1), (0, 0), (1, 1)\}$

R is not a function because $-2 \in X$, $2 \in X$ did not appear as a first projection in the ordered pairs of R

36

$\because$ a divides b

, $X \cup Y = \{2, 3, 5, 11, 14, 9, 35\}$

, f is a function from X to Y

$\because$ 2 divides 14 $\therefore 2 \in X$, $14 \in Y$

, $\because$ 3 divides 9 $\therefore 3 \in X$, $9 \in Y$

, $\because$ 5 divides 35 $\therefore 5 \in X$, $35 \in Y$

$\because n(X) = 3$ $X = \{2, 3, 5\}$

, $\because n(X \times Y) = 12$ $\therefore n(Y) = 4$

$\therefore Y = \{14, 9, 35, 11\}$

, $R = \{(2, 14), (3, 9), (5, 35)\}$

, its range = $\{14, 9, 35\}$

37

$\because X \cup Y = \{4, 8, 9, 27\}$, $n(X) = 4$

$\therefore X = \{4, 8, 9, 27\}$

$\because$ a is a multiple of b , f is a function from X to Y

, $n(Y) = 2$ $\therefore Y = \{4, 9\}$

$\therefore R = \{(4, 4), (8, 4), (9, 9), (27, 9)\}$

, the range of the function = $\{4, 9\}$

Answers of Exercise 3

1

1 c	2 d	3 b	4 c	5 d
6 c	7 a	8 d	9 c	10 d
11 d	12 c	13 d	14 c	15 a
16 c	17 a	18 c	19 b	20 d
21 d				

2

	Degree	$f(-2)$	$f(0)$	$f\left(\frac{1}{2}\right)$
1	First	7	3	2
2	Second	zero	4	$-3\frac{3}{4}$

3

$f(2) = 2 \times (2)^2 - 5 \times 2 + 2 = \text{zero}$

$, f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^2 - 5 \times \frac{1}{2} + 2 = \text{zero}$

$\therefore f(2) = f\left(\frac{1}{2}\right)$

4

$\because f(2) = 2 \times 2 - 1 = 3 , f(1) = 2 \times 1 - 1 = 1$

$\therefore f(2) - 3 f(1) = 3 - 3 \times 1 = \text{zero}$

5

1. $f(\sqrt{2}) + 3 g(\sqrt{2}) = (\sqrt{2})^2 - 3(\sqrt{2}) + 3(\sqrt{2} - 3)$

$= 2 - 3\sqrt{2} + 3\sqrt{2} - 9 = -7$

2. $\because f(3) = (3)^2 - 3 \times 3 = 9 - 9 = \text{zero}$

$, g(3) = 3 - 3 = \text{zero}$

$\therefore f(3) = g(3) = \text{zero}$

6

$f(1 + \sqrt{6}) = (1 + \sqrt{6})^2 - 2(1 + \sqrt{6}) - 5$

$= 1 + 2\sqrt{6} + 6 - 2 - 2\sqrt{6} - 5 = \text{zero}$

$, f(1 - \sqrt{6}) = (1 - \sqrt{6})^2 - 2(1 - \sqrt{6}) - 5$

$= 1 - 2\sqrt{6} + 6 - 2 + 2\sqrt{6} - 5 = \text{zero}$

$\therefore f(1 + \sqrt{6}) = f(1 - \sqrt{6}) = \text{zero}$

7

1. $a = \text{zero}$ $\quad \therefore f(X) = b X + 5$

$\therefore f$ is of the first degree

2. $f(3) = 11$ $\quad b \times 3 + 5 = 11$

$\therefore 3 b = 6$ $\quad b = \frac{6}{3} = 2$

8

$f(1) = 5 \times 1 - b = 5 - b , h(3) = 3 - 2 b$

$, f(1) + h(3) = -7$ $\quad 5 - b + 3 - 2 b = -7$

$8 - 3 b = -7$ $\quad 8 + 7 = 3 b$

$15 = 3 b$ $\quad b = \frac{15}{3} = 5$

$f(X) = 5 X - 5$

$f(3) = 5 \times 3 - 5 = 15 - 5 = 10$

$h(X) = X - 10$ $\quad h(1) = 1 - 10 = -9$

$f(3) + h(1) = 10 - 9 = 1$

9

$\because f(X) = r(X)$ $\quad \therefore (X - 3)^2 = X - 3$

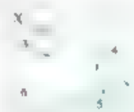
$\therefore X^2 - 6 X + 9 - X + 3 = 0$

$\therefore X^2 - 7 X + 12 = 0$ $\quad \therefore (X - 3)(X - 4) = 0$

$\therefore X = 3$ or $X = 4$

10

1.



2. $f = \{(3 , 3) , (4 , 5) , (5 , 5) , (6 , 5)\}$

$,$ its range $= \{3 , 5\}$

11

1. $\because f(0) = 5 - 0 = 5$

also $, f(1) = 4 , f(3) = 2$

$\therefore$ The range of $f = \{5 , 4 , 2\}$

2.

12

1. $t(0) = 2 \times 0 + 3 = 3$
   also $t(1) = 5$
   , $t(2) = 7$, $t(3) = 9$
   , $t(4) = 11$
   , $t(5) = 13$

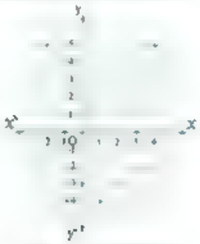
2.

3. The range of t
   $= \{3 , 5 , 7 , 9 , 11 , 13 , \ldots\}$
   = The set of odd natural numbers except $\{1\}$

13

1. $f(4) = (4)^2 - 2 \times 4 - 3 = 16 - 8 - 3 = 5$
   , $f(3) = 0$, $f(2) = -3$, $f(1) = -4$
   , $f(0) = -3$, $f(-1) = 0$, $f(-2) = 5$

2.



3. From 1 : $\because f(4) = 5$, $f(-2) = 5$
   $\therefore X = 4$ or -2

14

$\because f(a) = b$ $\quad \therefore b = a^2 + b$ $\quad \therefore a^2 = 0$

$\therefore a = 0$ $\quad \therefore a\,b^2 + 5 = 0 \times b^2 + 5 = 5$

15

1. The domain $= \{1 , 2 , 3 , 4 , 5\}$
2. The range $= \{3 , 5 , 7 , 9 , 11\}$
3. The rule of the function f is : $f(X) = 2X + 1$

16

$f(0) = 0$ $\quad \therefore 2 \times (0)^2 + b \times 0 + c = 0$

$\therefore c = 0$ $\quad \therefore f(X) = 2X^2 + bX$

$\because f(3) = 0$ $\quad \therefore 0 = 2(3)^2 + 3b$

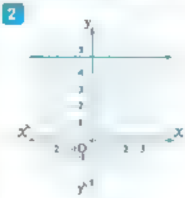
$\therefore 0 = 18 + 3b$ $\quad \therefore b = -6$

Answers of Exercise 4

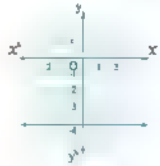
1

1 a	2 b	3 c	4 c	5 b
6 a	7 a	8 c	9 b	10 b
11 b	12 c	13 c	14 a	15 a

2

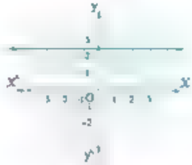


1



2

3

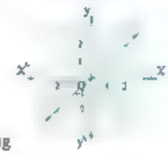


4

3

1. $f(X) = X$

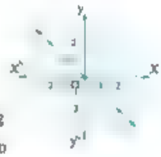
X	-2	zero	2
$f(X)$	-2	zero	2



The straight line representing the function intersects the two coordinate axes at the origin point O (0 , 0)

2. $f(X) = -X$

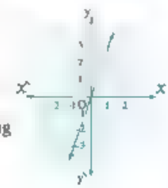
X	-2	zero	2
$f(X)$	2	zero	-2



The straight line representing the function intersects the two coordinate axes at the origin point O (0 , 0)

3. $f(X) = 3X$

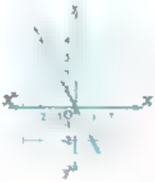
X	-1	zero	1
$f(X)$	-3	zero	3



The straight line representing the function intersects the two coordinate axes at the origin point O (0 , 0)

4 $f(X) = -2X$

X	-2	zero	1
$f(X)$	4	zero	-2



The straight line representing the function intersects the two coordinate axes at the origin point O $(0, 0)$

5 $f(X) = X + 2$

X	-2	zero	2
$f(X)$	zero	2	4

The straight line representing the function intersects the X-axis at the point $(-2, 0)$ and the y-axis at the point $(0, 2)$

6 $f(X) = 2 - X$

X	-2	zero	2
$f(X)$	4	2	zero

The straight line representing the function intersects the X-axis at the point $(2, 0)$ and the y-axis at the point $(0, 2)$

7 $f(X) = 3X - 1$

X	zero	1	2
$f(X)$	-1	2	5

Represent by yourself.

From the graph we find that : the straight line representing the function intersects the X-axis at the point $\left(\frac{1}{3}, 0\right)$ and the y-axis at the point $(0, -1)$

8 $f(X) = -2X + 3$

X	-1	zero	1
$f(X)$	5	3	1

Represent by yourself

From the graph we find that :

The straight line representing the function intersects the X-axis at the point $(1.5, 0)$ and the y-axis at $(0, 3)$

9 $f(X) = \frac{1}{2}X$

X	-2	zero	2
$f(X)$	-1	zero	1

Represent by yourself

From the graph we find that :

The straight line representing the function intersects the two coordinate axes at the origin point O $(0, 0)$

10 $f(X) = 5 - \frac{1}{2}X$

X	zero	2	4
$f(X)$	5	4	3

Represent by yourself.

From the graph we find that :

The straight line representing the function intersects the X-axis at the point $(10, 0)$ and the y-axis at the point $(0, 5)$

4

The straight line intersects the y-axis at $(b, 2)$

$\therefore b = 0$ $\quad \because (0, 2)$ satisfies the function

$\therefore 6 \times 0 - a = 2$ $\quad \therefore a = -2$

5

$\because (a, 2a)$ satisfies the function

$\therefore 2a = 3 \times a - 6$ $\quad \therefore 2a = 3a - 6$

$\therefore 3a - 2a = 6$ $\quad \therefore a = 6$

, at $X = 0$ $\quad \therefore f(0) = 3 \times 0 - 6 = -6$

$\therefore$ The straight line intersects the y-axis at $(0, -6)$

6

1 $\because f(3) = 9$ $\quad \therefore 2 \times 3 + a = 9$

$\therefore 6 + a = 9$ $\quad \therefore a = 3$

2 At $y = 0$: $\therefore 2X + 3 = 0$ $\quad \therefore X = -\frac{3}{2}$

$\therefore$ The straight line intersects the X-axis at $\left(-\frac{3}{2}, 0\right)$

∵ The straight line cuts a positive part of the y-axis of length 3 units

∴ The straight line passes through (0 , 3)

∴ (0 , 3) satisfies the relation

∴ $3 = a \times 0 + b$ ∴ $b = 3$

∴ $f(X) = a X + 3$

, (1 , 5) satisfies the relation

∴ $5 = a \times 1 + 3$ ∴ $a = 2$

8

∵ The point (0 , −3) satisfies the relation $f(X) = a X + b$

∴ $-3 = a \times 0 + b$ ∴ $b = -3$

∴ $f(X) = a X - 3$

, ∵ the point (3 , 0) satisfies the relation $f(X) = a X - 3$

∴ $0 = 3 \times a - 3$ ∴ $3a = 3$ ∴ $a = 1$

∴ $f(X) = X - 3$ ∴ $f(1) = 1 - 3 = -2$

9

1 ∵ $r(2) = 9 - 2 = 7$, also $r(3) = 6$, $r(6) = 3$

∴ The set of images of elements of the set X with the function r = {7 , 6 , 3}

2 r is not a linear function because each of the domain and the codomain is not the set of real numbers

10

1 Let A (X , 0)

, ∵ A (X , 0) belongs to the straight line representing the function f

$4 - 2X = 0$ ∴ $-2X = -4$

∴ $X = \frac{-4}{-2} = 2$ ∴ A (2 , 0)

Let B (0 , y)

, ∵ B (0 , y) belongs to the straight line representing the function f

∴ $4 - 2 \times 0 = y$ ∴ $y = 4$

∴ B (0 , 4)

2 The area of Δ AOB $= \frac{1}{2} \times 2 \times 4 = 4$ square unit

1 ∵ f is a constant function , passes through the point A (2 , 3) and is represented graphically by a straight line parallel to X-axis

∴ The rule of the function f is $f(X) = 3$

, ∵ g is a linear function and passes through A (2 , 3) , O (0 , 0)

The rule of the function g is $g(X) = bX + c$

, $(0 , 0) \in \overleftrightarrow{OA}$

$0 = b \times 0 + c$ ∴ $c = 0$

$g(X) = bX$

, $(2 , 3) \in \overleftrightarrow{OA}$ $3 = 2 \times b$

∴ $b = \frac{3}{2}$ $g(X) = \frac{3}{2}X$

2 $f(-10) + g(6) = 3 + \frac{3}{2} \times 6 = 12$

12

$\overline{AB}$ represents the function $f : f(X) = 4$

, the point B ∈ y-axis

B = (0 , 4) ∴ OB = 4 length unit

, ∵ the area of Δ ABO = 4 square unit

$\frac{1}{2}$ AB × OB = 4 ∴ $\frac{1}{2}$ AB × 4 = 4

∴ $\frac{1}{2}$ AB = 1 ∴ AB = 2 length unit

∴ A = (2 , 4)

, ∵ the point O (0 , 0) belongs to the straight line representing the function g : $g(X) = nX + k$

∴ $0 = n \times 0 + k$ ∴ $k = 0$

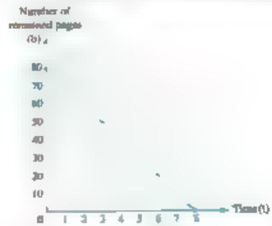
∴ $g(X) = nX$

, ∵ the point A (2 , 4) belongs to the straight line representing the function g : $g(X) = nX$

∴ $4 = 2n$ ∴ $n = 2$

13

1



You can find the algebraic relation easily after studying the equation of the straight line (the last lesson in geometry) as follows .

Taking the two points (3 , 50) and (6 , 20)

∴ The slope $= \frac{50 - 20}{3 - 6} = -10$

∴ $b = -10t + 80$

2 8 hours 3 80 pages

14

1 a	2 b	3 b	4 b	5 c
6 d	7 d	8 c	9 d	10 c

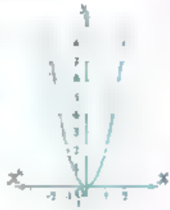
15

1 $f(X) = 2X^2$

X	- 2	1	0	1	2
$f(X)$	8	2	0	2	8

From the graph :

- The vertex of the curve is (0 , 0)
- The equation of the line of symmetry is $X = 0$
- The minimum value = zero

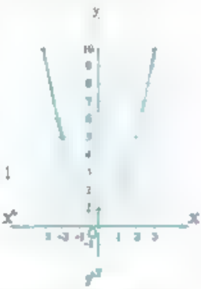


2 $f(X) = X^2 + 1$

X	3	2	- 1	0	1	2	3
$f(X)$	10	5	2	1	2	5	10

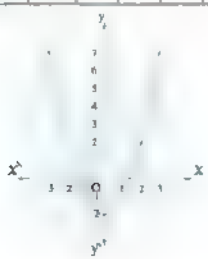
From the graph :

- The vertex of the curve is (0 , 1)
- The equation of the line of symmetry is $X = 0$
- The minimum value = 1



3 $f(X) = X^2 - 2$

X	−3	−2	−1	0	1	2	3
$f(X)$	7	2	−1	−2	−1	2	7



From the graph :

- The vertex of the curve is (0 , −2)
- The equation of the line of symmetry is $X = 0$
- The minimum value = 2

4 $f(X) = 2 \quad X^2$

X	3	- 2	· 1	0	1	2	3
$f(X)$	7	2	1	2	1	2	7

Represent by yourself

From the graph , we find that :

- The vertex of the curve is (0 , 2)
- The equation of the line of symmetry is $X = 0$
- The maximum value = 2

5 $f(X) = X^2 - 2X$

X	−2	−1	0	1	2	3	4
$f(X)$	8	3	0	−1	0	3	8

Represent by yourself

From the graph , we find that :

- The vertex of the curve is (1 , − 1)
- The equation of the line of symmetry is $X = 1$
- The minimum value = − 1

6 $f(X) = X^2 + 2X + 1$

X	4	3	2	1	0	1	2
$f(X)$	9	4	1	0	1	4	9

Represent by yourself

From the graph , we find that :

- The vertex of the curve is (− 1 , 0)
- The equation of the line of symmetry is $X = \quad 1$
- The minimum value = 0

7 $f(X) = (X - 2)^2 = X^2 - 4X + 4$

X	−1	0	1	2	3	4	5
$f(X)$	9	4	1	0	1	4	9

Represent by yourself

From the graph , we find that :

- The vertex of the curve is (2 , 0)
- The equation of the line of symmetry is $X = 2$
- The minimum value = zero

8 $f(X) = X^2 - 2X - 3$

X	-2	-1	0	1	2	3	4
$f(X)$	5	0	-3	-4	-3	0	5

Represent by yourself.

From the graph , we find that :

- The vertex of the curve is $(1 , -4)$
- The equation of the line of symmetry is $X = 1$
- The minimum value = -4

9 $f(X) = 3 - 2X - X^2$

X	-4	-3	-2	-1	0	1	2
$f(X)$	-5	0	3	4	3	0	-5

Represent by yourself

From the graph , we find that :

- The vertex of the curve is $(-1 , 4)$
- The equation of the line of symmetry is $X = -1$
- The maximum value = 4

10 $f(X) = 4X + 3 - 2X^2$

X	-2	-1	0	1	2	3
$f(X)$	-13	-3	3	5	3	-3

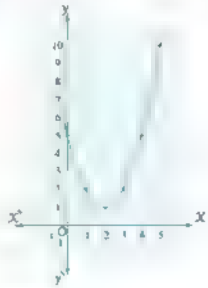
From the graph :

- The vertex of the curve is $(1 , 5)$
- The equation of the axis of symmetry is $X = 1$

  «notice that : the domain of f is $\mathbb{R}$ and the given interval is for facilitating representation only»
- The maximum value = 5

11 $f(X) = X^2 - 4X + 5$

X	0	1	2	3	4	5
$f(X)$	5	2	1	2	5	10



From the graph :

- The vertex of the curve is $(2 , 1)$
- The equation of the axis of symmetry is $X = 2$

  «notice that : the domain of f is $\mathbb{R}$ and the given interval is for facilitating representation only»
- The minimum value = 1

12 $f(X) = 1 - 3X + X^2$

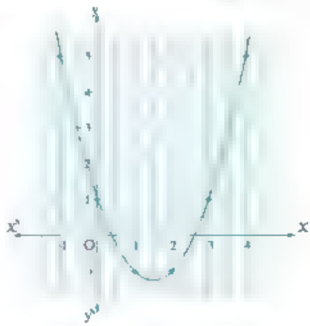
X	-1	0	1	2	3	4
$f(X)$	5	1	-1	-1	1	5

The X-coordinate of the vertex of the curve

$= -\frac{b}{2a} = \frac{-(-3)}{2} = \frac{3}{2} = 1\frac{1}{2}$

$, f\left(\frac{3}{2}\right) = 1 - 3\left(\frac{3}{2}\right) + \frac{9}{4} = -\frac{5}{4} = -1\frac{1}{4}$

The vertex of the curve is $\left(1\frac{1}{2} , -1\frac{1}{4}\right)$



- The equation of the axis of symmetry is $x = 1\frac{1}{2}$
- The minimum value $= -1\frac{1}{4}$

16

∵ The curve of the function f intersects the x-axis at the point $(-2, b)$

∴ $b = 0$

$(-2, 0)$ satisfies the relation $f(x) = m - x^2$

∴ $m - (-2)^2 = 0$ ∴ $m - 4 = 0$

∴ $m = 4$ $m^b + 2m = 4^0 + 2 \times 4 = 9$

17

∵ $3f(2) + 3\ell(x) = 6$ ∴ $f(2) + \ell(x) = 2$

∴ $a + 2^2 + c = 2$ ∴ $a + 4 + c = 2$

∴ $a + c = -2$

∴ $2f(0) + 2\ell(7) = 2[f(0) + \ell(7)] = 2[a + (0)^2 + c]$
$= 2[a + c] = 2 \times (-2) = -4$

18

[1] Let $A = (x, 0)$ and $C = (-x, 0)$

∵ The curve of the function intersects the x-axis at the two points A and C

∴ $0 = 9 - x^2$ ∴ $x^2 = 9$

∴ $x = 3$ or $x = -3$

∴ $A = (3, 0)$, $C = (-3, 0)$

[2] Let $B = (0, y)$

, ∵ the point $B = (0, y)$ belongs to the curve of the function f

∴ $y = 9 - (0)^2$ $y = 9$

∴ $b = (0, 9)$ ∴ $OB = 9$ length units.

, ∵ $A = (3, 0)$ and $C = (-3, 0)$

∴ $AC = 6$ length units.

∴ The area of $\Delta ABC = \frac{1}{2} \times 6 \times 9$
$= 27$ square units.

19

∵ $AO = 4$ length units ∴ $A(0, 4)$

$A(0, 4)$ belongs to the curve of the function f

∴ A satisfies the equation of the curve

∴ $4 = m - (0)^2$ ∴ $m = 4$ (The first req.)

, The curve of the function intersects x-axis at the two points B and C

∴ $0 = 4 - x^2$ ∴ $x^2 = 4$

∴ $x = 2$ or -2

∴ $B = (2, 0)$, $C = (-2, 0)$ (The second req.)

∴ $BC = 4$ length units

∴ The area of $\Delta ABC = \frac{1}{2} \times 4 \times 4 = 8$ square units (The third req.)

20

$A(0, -7)$ ∴ $OA = 7$ length units

, the area of $\Delta ABC = \frac{1}{2} \times BC \times AO$

$21 = \frac{1}{2} \times BC \times 7$ $BC = \frac{21 \times 2}{7} = 6$ length units

$OB = OC = \frac{6}{2} = 3$ length units

$B = (3, 0)$

, $B(3, 0) \in$ the curve of the function f

∴ $\ell \times 3^2 - 7 = 0$ ∴ $9\ell - 7 = 0$

$9\ell = 7$ $\ell = \frac{7}{9}$

21

[1] The domain of the function $f = \mathbb{R}$

[2] ∵ The range of the function f = the set of images of the elements of the set $\mathbb{R}$ by the function f

∴ The range of the function $f =]-\infty, 4\frac{1}{2}]$

[3] The equation of the line of symmetry of the curve of the function f is : $x = 2$

[4] The maximum value of $f = 4\frac{1}{2}$

[5] $f(1) = 4$

[6] ∵ $(2, 4\frac{1}{2}) \in$ the curve of the function f

∴ $a(2-2)^2 + k = 4\frac{1}{2}$ ∴ $k = 4\frac{1}{2}$

, ∵ $(5, 0) \in$ the curve of the function f

∴ $a(5-2)^2 + 4\frac{1}{2} = 0$

∴ $9a = -4\frac{1}{2}$ $a = \frac{-4\frac{1}{2}}{9} = -\frac{1}{2}$

∴ $a + k = -\frac{1}{2} + 4\frac{1}{2} = 4$

22

The curve of the function intersects the x-axis at the two points $A(1, 0)$ and $B(4, 0)$

∴ $f(1) = 0$, $f(4) = 0$ ∴ $f(1) = f(4)$

, ∴ the function is symmetric

∴ The quation of the axis of symmetry is $x = \frac{4+1}{2} = \frac{5}{2}$

$\because \frac{2+7}{2} = \frac{5}{2}$ $\qquad f(-2) = f(7)$

$f(-2) + f(-2) = 8$ $\qquad f(-2) = 4$

23

Let $C = (0 , \ell)$

$\because$ the curve of the function f passes through the point C

$\therefore \ell = 0^2 - (k - 2) \times 0 - k + 4$

$\ell = 4 - k$

$\because$ the X-coordinate of the vertex of the curve $= \frac{k-2}{2}$

$\therefore AO = 2 \times \frac{k-2}{2} = k - 2$

$\because \ell = AO$ $\qquad 4 - k = k - 2$

$2k = 6$ $\qquad k = 3$

24

$OB = 5\,OA$ $\qquad \therefore \frac{OB}{OA} = \frac{5}{1}$

$\therefore OB = 5\,m , OA = m$

$\therefore B(5\,m , 0) , A(-m , 0)$

$\because f(5\,m) = f(-m)$

$-25\,m^2 + 20\,m + k - 1 = -m^2 - 4\,m + k - 1$

$24\,m^2 - 24\,m = 0$ $\qquad 24\,m\,(m - 1) = 0$

$\therefore m = 0$ (refused) or $m = 1$

$\therefore B(5 , 0)$

By substituting in the rule of the function f

$\therefore 0 = -25 + 20 + k - 1$ $\qquad \therefore k = 6$

Answers of exams on unit one

Model (1)

1

1 a	2 d	3 b
4 b	5 c	6 a

2

[a] $R = \{(1 , 2) , (2 , 3) , (3 , 4)\}$

, R is a function because every element of X has only one image in Y

, its range $= \{2 , 3 , 4\}$

[b] 1 $X \times (Y \cap Z) = \{(2 , 5) , (3 , 5)\}$

2 $(X - Y) \times Z = \{(2 , 7) , (2 , 5)\}$

3 $(X - Y) \times (Y - Z) = \{(2 , 3)\}$

3

[a] 1 The domain of the function $f = \{3 , 5 , 7\}$

2 The range of the function $f = \{9 , 15 , 21\}$

3 The rule of the function f is : $f(X) = 3\,X$

[b] Represent by yourself

The straight line representing the function intersects X-axis at the point $(1.5 , 0)$ and y-axis at the point $(0 , 3)$

4

[a] 1 $R = \{(-1 , 1) , (1 , 1) , (0 , 0)\}$

, represent by yourself

2 R is not a function because $2 \in X$ has no image in X

[b] Represent by yourself

1 The vertex of the curve is $(2 , 1)$

2 The equation of the line of symmetry is $X = 2$

3 The minimum value $= 1$

5

[a] 1 The function is of the first degree

2 $f(0) + g(0) = -4$

[b] 1 The equation of the axis of symmetry is $X = 0$

, the maximum value of the function $= 4$

2 $B(-2 , 0)$ $\qquad$ 3 $k = 1$

Model 2

1

1 b　　2 b　　3 c

4 a　　5 c　　6 c

2 [a] 1 $R = \{(1 , 7) , (4 , 4) , (7 , 1) , (7 , -1)\}$, represent by yourself

2 R is not a function because $7 \in X$ has two images in Y

[b] 1 $Y \times X = \{(1 , 1) , (5 , 1) , (3 , 1) , (1 , 4) , (5 , 4) , (3 , 4)\}$

2 $X = \{1 , 4\}$, $X^2 = \{(1 , 1) , (1 , 4) , (4 , 1) , (4 , 4)\}$

3 [a] Represent by yourself.

1 The vertex of the curve is $(-1 , -5)$

2 The equation of the line of symmetry is $X = -1$

3 The minimum value $= -5$

[b] $m = 2$, $k = 4$

4 [a] $\sqrt{3X + 2y} = 5$

[b] 1 $(X - Y) \times Z = \{(1 , 4) , (1 , 5)\}$

2 $n(X \times Y) + n(Z^2) = 8$

5 [a] 1 36　　2 $\{5 , 7\}$

[b] $f(2) - f\left(\frac{1}{2}\right) =$ zero

Answers of unit two

Answers of Exercise 5

1

1 a	2 d	3 c	4 c	5 c
6 d	7 d	8 d	9 a	10 b
11 b	12 b	13 c	14 a	15 b
16 a	17 a	18 b	19 b	20 d
21 a	22 c	23 a	24 c	25 b
26 a	27 c			

2

1 Let the first proportional be x $\quad \frac{x}{\sqrt{8}} = \frac{7}{14\sqrt{2}}$

$\therefore x = \frac{7 \times \sqrt{8}}{14\sqrt{2}} = \frac{7 \times 2\sqrt{2}}{14\sqrt{2}} = 1$

2 Let the third proportional be x

$\frac{a}{(a+b)} = \frac{x}{(a^2 - b^2)}$

$\therefore x = \frac{a(a^2 - b^2)}{(a+b)} = \frac{a(a+b)(a-b)}{(a+b)} = a(a - b)$

3 Let the fourth proportional be x

$\therefore \frac{(a+b)}{(a-b)} = \frac{(a-b)}{x} \qquad \therefore x = \frac{(a-b)^2}{(a+b)}$

3

1 $\frac{2x - 3}{x - 5} = \frac{1}{4} \qquad \therefore x - 5 = 4(2x - 3)$

$\therefore x - 5 = 8x - 12 \qquad \therefore 7x = 7 \qquad \therefore x = 1$

2 $\frac{x-5}{5x+3} = \frac{2}{3} \qquad \therefore 3(x-5) = 2(5x+3)$

$\therefore 3x - 15 = 10x + 6 \qquad \therefore -7x = 21$

$\therefore x = -3$

3 $\frac{x^2 - 8}{2x^2 + 1} = \frac{1}{3} \qquad 3x^2 - 24 = 2x^2 + 1$

$\therefore x^2 = 25 \qquad \therefore x = \pm 5$

4 $\frac{x^2 + 10x}{2x^2 - 3} = \frac{24}{5} \qquad \therefore 5x^2 + 50x = 48x^2 - 72$

$\therefore 43x^2 - 50x - 72 = 0$

$\therefore (x - 2)(43x + 36) = 0$

$\therefore x = 2$ or $x = \frac{-36}{43}$ (refused)

4

$3x - 6y = x + 3y \qquad \therefore 2x = 9y$

$\frac{y}{x} = \frac{2}{9}$

5

$\because \frac{2x+3}{2x-3} = \frac{2y+5}{2y-5}$

$\therefore 4xy + 6y - 10x - 15 = 4xy - 6y + 10x - 15$

$\therefore 12y = 20x \qquad \therefore \frac{x}{y} = \frac{12}{20} = \frac{3}{5}$

6

$\because x^2 - 3xy - 4y^2 = 0 \qquad \therefore (x+y)(x-4y) = 0$

$\therefore x + y = 0 \qquad \therefore x = -y \quad \therefore x : y = -1 : 1$

or $x - 4y = 0 \qquad \therefore x = 4y \quad \therefore x : y = 4 : 1$

7

$\because 3x^2 - 10xy + 7y^2 = 0 \qquad \therefore (x-y)(3x-7y) = 0$

$x = y$ (refused) or $3x - 7y = 0 \quad \therefore 3x = 7y$

$x : y = 7 : 3$

8

$\frac{x}{y} = \frac{2}{3} \qquad \therefore x = 2m, y = 3m$

$\frac{3x + 2y}{6y - x} = \frac{6m + 6m}{18m - 2m} = \frac{12m}{16m} = \frac{3}{4}$

9

$\frac{a}{b} = \frac{3}{5} \qquad a = 3m, b = 5m$

$\frac{7a + 9b}{4a + 2b} = \frac{21m + 45m}{12m + 10m} = \frac{66m}{22m} = 3$

10

$\frac{a}{b} = \frac{3}{4} \qquad a = 3m, b = 4m$

1 $\frac{4a + b}{2a - b} = \frac{12m + 4m}{6m - 4m} = \frac{16m}{2m} = 8$

2 $\frac{b^2 - a^2}{a^2 - b^2} = \frac{16m^2 - 9m^2}{9m^2 - 16m^2} = \frac{7m^2}{-7m^2} = -1$

11

$\frac{a}{b} = \frac{1}{3} \qquad a = m, b = 3m$

$\frac{c}{d} = \frac{7}{2} \qquad c = 7k, d = 2k$

$\therefore \frac{2ac + bd}{bc - 3ad} = \frac{2 \times m \times 7k + 3m \times 2k}{3m \times 7k - 3 \times m \times 2k} = \frac{14mk + 6mk}{21mk - 6mk}$

$= \frac{20mk}{15mk} = \frac{4}{3}$

12

$\because \frac{7x - 3y}{x + y} = \frac{3}{1}$ $\quad\therefore 7x - 3y = 3x + 3y$

$\therefore 4x = 6y$ $\quad\therefore \frac{x}{y} = \frac{6}{4} = \frac{3}{2}$

$x = 3m , y = 2m$

$\therefore \frac{12x + 9y}{11x - 3y} = \frac{36m + 18m}{33m - 6m} = \frac{54m}{27m} = 2 : 1$

13

$\frac{21x + a}{7x + b} = \frac{a}{b}$ $\quad\therefore 21bx + ab = 7ax + ab$

$\therefore 21bx = 7ax$ $\quad\therefore 3b = a$

$\frac{a + 2b}{2a} = \frac{3b + 2b}{2 \times 3b} = \frac{5b}{6b} = \frac{5}{6}$

14

Let the number be x

$\therefore 3 + x , 5 + x , 8 + x , 12 + x$ are proportional

$\frac{3 + x}{5 + x} = \frac{8 + x}{12 + x}$

$\therefore 40 + 13x + x^2 = 36 + 15x + x^2$

$40 - 36 = 15x - 13x$ $\quad 4 = 2x$ $\quad\therefore x = 2$

$\therefore$ The required number $= 2$

15

Let the number be x

$16 - x , 21 - x , 14 - x , 18 - x$ are proportional

$\therefore \frac{16 - x}{21 - x} = \frac{14 - x}{18 - x}$

$\therefore (21 - x)(14 - x) = (16 - x)(18 - x)$

$\therefore 294 - 35x + x^2 = 288 - 34x + x^2$

$\therefore x = 6$ $\quad\therefore$ The required number $= 6$

16

[1] $\because \frac{a + b}{b} = \frac{c + d}{d}$ $\quad\therefore d(a + b) = b(c + d)$

$ad + bd = bc + bd$ $\quad\therefore ad = bc$

$\therefore \frac{a}{b} = \frac{c}{d}$

$\therefore a , b , c , d$ are proportional.

Another solution :

$\frac{a + b}{b} = \frac{c + d}{d}$ $\quad \frac{a}{b} + \frac{b}{b} = \frac{c}{d} + \frac{d}{d}$

$\frac{a}{b} + 1 = \frac{c}{d} + 1$ $\quad \frac{a}{b} = \frac{c}{d}$

a , b , c , d are proportional

[2] $\frac{a}{b - a} = \frac{c}{d - c}$ $\quad\therefore a(d - c) = c(b - a)$

$\therefore ad - ac = cb - ca$ $\quad ad = cb$

$\therefore \frac{a}{b} = \frac{c}{d}$ $\quad\therefore a , b , c , d$ are proportional.

Another solution :

$\frac{a}{b - a} = \frac{c}{d - c}$ $\quad\therefore \frac{b - a}{a} = \frac{d - c}{c}$

$\frac{b}{a} - 1 = \frac{d}{c} - 1$ $\quad\therefore \frac{b}{a} = \frac{d}{c}$

$\therefore \frac{a}{b} = \frac{c}{d}$ $\quad\therefore a , b , c , d$ are proportional

[3] $\frac{a - b}{a + b} = \frac{c - d}{c + d}$ $\quad\therefore (a - b)(c + d) = (a + b)(c - d)$

$\therefore ac + ad - bc - bd = ac - ad + bc - bd$

$\therefore 2ad = 2bc$ $\quad\therefore ad = bc$ $\quad\therefore \frac{a}{b} = \frac{c}{d}$

a , b , c , d are proportional

[4] $\frac{a^2 - 2c^2}{b^2 - 2d^2} = \frac{a^2}{b^2}$

$\therefore a^2(b^2 - 2d^2) = b^2(a^2 - 2c^2)$

$a^2b^2 - 2a^2d^2 = a^2b^2 - 2b^2c^2$

$a^2d^2 = b^2c^2$ $\quad\therefore ad = bc$

$\frac{a}{b} = \frac{c}{d}$ $\quad\therefore a , b , c , d$ are proportional

17

$a : b : c = 5 : 7 : 3$ $\quad\therefore a = 5m , b = 7m , c = 3m$

$a + b = 27.6$ $\quad\therefore 5m + 7m = 27.6$

$\therefore 12m = 27.6$ $\quad\therefore m = 2.3$

$a = 5 \times 2.3 = 11.5 , b = 7 \times 2.3 = 16.1 , c = 3 \times 2.3 = 6.9$

18

$\because a : b : c = 3 : 4 : 5$ $\quad\therefore a = 3m , b = 4m , c = 5m$

$\therefore \frac{a^2 + b^2 + c^2}{a(b + c)} = \frac{9m^2 + 16m^2 + 25m^2}{3m(4m + 5m)} = \frac{50m^2}{27m^2} = \frac{50}{27}$

19

$\because 2a = 3b = 4c$ $\quad\therefore 2a = 3b$ $\quad\therefore a = \frac{3}{2}b$

$, 3b = 4c$ $\quad\therefore c = \frac{3}{4}b$

$\therefore a : b : c = \frac{3}{2}b : b : \frac{3}{4}b$

multiplying by 4 $\quad\therefore a : b : c = 6b : 4b : 3b$

dividing by b $\quad\therefore a : b : c := 6 : 4 : 3$

Another solution :

$\because 2a = 3b = 4c$ $\quad$ (dividing by 12)

$\therefore \frac{2a}{12} = \frac{3b}{12} = \frac{4c}{12}$ $\quad\therefore \frac{a}{6} = \frac{b}{4} = \frac{c}{3}$

$\therefore a : b : c = 6 : 4 : 3$

[illegible]

[1] Let the number be x $\quad\therefore \frac{7 + x}{11 + x} = \frac{2}{3}$

$\therefore 21 + 3x = 22 + 2x$ $\quad\therefore x = 1$

$\therefore$ The required number is 1

2 Let the number be x $\therefore \frac{49-3x}{69-3x}=\frac{2}{3}$

$\therefore 147-9x=138-6x$ $\therefore 3x=9$

$\therefore x=3$

$\therefore$ The required number = 3

3 Let the number be x $\therefore \frac{7+x^2}{11+x^2}=\frac{4}{5}$

$\therefore 35+5x^2=44+4x^2$ $\therefore x^2=9$

$\therefore x=\pm 3$

$\therefore$ The required number is 3 or -3

4 Let the number be x $\therefore \frac{5+x^2}{11+x^2}=\frac{3}{5}$

$\therefore 25+5x^2=33+3x^2$ $\therefore 2x^2=8$

$\therefore x^2=4$

$\therefore x=2$ or $x=-2$ (refused)

$\therefore$ The required number = 2

5 Let the number be x $\therefore \frac{15-x}{13+x}=\frac{3}{4}$

$60-4x=39+3x$ $\therefore 7x=21$

$\therefore x=3$

$\therefore$ The required number = 3

6 Let the two numbers be a and b

$\therefore \frac{a}{b}=\frac{3}{7}$ $\therefore a=3m$, $b=7m$

$\therefore \frac{3m-5}{7m-5}=\frac{1}{3}$ $\therefore 9m-15=7m-5$

$\therefore 2m=10$ $\therefore m=5$

The two numbers are 15 and 35

7 Let the two numbers be a and b

$\therefore \frac{a}{b}=\frac{2}{3}$ $\therefore a=2m$, $b=3m$

$\frac{2m+7}{3m-12}=\frac{5}{3}$ $\therefore 6m+21=15m-60$

$\therefore 81=9m$ $\therefore m=9$

$\therefore$ The two numbers are 18 and 27

8 Let the two numbers be a and b

$\therefore \frac{a}{b}=\frac{4}{7}$ $a=4m$, $b=7m$

$\therefore (4m)^2-5(7m)=39$

$\therefore 16m^2-35m-39=0$

$\therefore (m-3)(16m+13)=0$

$m=3$ or $m=\frac{-13}{16}$ (refused)

The two numbers are 12 and 21

21

Let the two dimensions of the rectangle be a and b in centimetres

$\frac{a}{b}=\frac{4}{7}$ $\therefore a=4m$, $b=7m$

$\therefore 2(4m+7m)=88$ $\therefore 22m=88$ $\therefore m=4$

$\therefore$ The two dimensions of the rectangle are

$4\times 4=16$ cm. , $7\times 4=28$ cm

$\therefore$ The area $=16\times 28=448$ cm^2.

22

Let the base length = a cm and the height = b cm.

$\therefore \frac{a}{b}=\frac{3}{2}$ $\therefore a=3m$, $b=2m$

$\therefore \frac{1}{2}\times 3m\times 2m=48$ $\therefore 3m^2=48$

$\therefore m^2=16$ $m=4$

$\therefore$ The base length $=3\times 4=12$ cm

The height $=2\times 4=8$ cm.

23

The area of the unshaded part from the circle

$=1-\frac{5}{6}=\frac{1}{6}$ of the area of the circle

, the area of the unshaded part from the triangle

$=1-\frac{2}{3}=\frac{1}{3}$ of the area of the triangle.

$\therefore \frac{1}{6}$ of the area of the circle

$=\frac{1}{3}$ of the area of the triangle

$\therefore$ The area of the circle : the area of the triangle

$=\frac{1}{3}:\frac{1}{6}$ (multiply by 6) $=2:1$

24

Let the share of the second be x pounds

$\therefore \frac{2}{3}=\frac{30}{x}$ $\therefore x=\frac{30\times 3}{2}=45$ pounds.

25

$\therefore \frac{\text{The length of the shadow of the tree}}{\text{The length of the shadow of Islam}}$

$=\frac{\text{The height of the tree}}{\text{The height of Islam}}$

$\frac{300}{120}=\frac{\text{The height of the tree}}{180}$

$\therefore$ The height of the tree

$=\frac{300\times 180}{120}=450$ cm. $=4\frac{1}{2}$ m.

26

Let the costs of building the school be x

, the costs of building the medical unit = y

and the costs of building the youth centre = z

$x = \frac{3}{2}y$, $y = \frac{5}{6}z$, $z = \frac{6}{5}y$

$\because x + y + z = 1.85 \times 10^6$

$\therefore \frac{3}{2}y + y + \frac{6}{5}y = 1.85 \times 10^6$ $\therefore \frac{37}{10}y = 1.85 \times 10^6$

$\therefore 37y = 1.85 \times 10^7$ $\therefore y = 5 \times 10^5$

$x = \frac{3}{2} \times 5 \times 10^5 = 7.5 \times 10^5$, $z = \frac{6}{5} \times 5 \times 10^5 = 6 \times 10^5$

27

Let the number of boys = x and the number of girls = y

$\because$ The total number of pupils = $x + y$

The number of succeeded boys = $x \times \frac{79}{100} = 0.79\,x$

The number of succeeded girls = $y \times \frac{89}{100} = 0.89\,y$

The total number of succeeded pupils = $0.79\,x + 0.89\,y$

$\because$ The ratio of success in 3rd grade preparatory

$\frac{0.79\,x + 0.89\,y}{x + y} = 0.83$

$(0.79)\,x + (0.89)\,y = (0.83)\,x + (0.83)\,y$

$\therefore (0.89)\,y - (0.83)\,y = (0.83)\,x - (0.79)\,x$

$\therefore (0.06)\,y = (0.04)\,x$ $\therefore x : y = 6 : 4 = 3 : 2$

$\therefore$ The number of boys : the number of girls = 3 : 2

28

Let the circumference of the circle be a cm

and the perimeter of the square be b cm

$\therefore \frac{a}{b} = \frac{11}{8}$ $\therefore a = 11\,m$, $b = 8\,m$

$\therefore 11\,m + 8\,m = 152$

$\therefore 19\,m = 152$ $\therefore m = 8$

$\therefore$ The circumference of the circle = $11 \times 8 = 88$ cm

$\therefore 2 \times \frac{22}{7} \times r = 88$ $\therefore r = 14$ cm.

The area of the circle = $\pi r^2 = \frac{22}{7} \times 14 \times 14 = 616\ cm^2$

, the perimeter of the square = $8 \times 8 = 64$ cm

$\therefore$ The side length of the square = $\frac{64}{4} = 16$ cm

$\therefore$ The area of the square = $16 \times 16 = 256\ cm^2$

$\therefore$ The area of the square : The area of the circle

$= \frac{256}{616} = \frac{32}{77}$

29

Let the second proportional be x

$\therefore$ The numbers are : $x - 2$, x , 8 and x^2

$\because \frac{x - 2}{x} = \frac{8}{x^2}$ $\therefore x^3 - 2x^2 = 8x$

$\therefore x^3 - 2x^2 - 8x = 0$ $\therefore x(x^2 - 2x - 8) = 0$

$\therefore x(x - 4)(x + 2) = 0$ $\therefore x = 0$ (refused)

or $x = 4$ thus , the numbers are : 2 , 4 , 8 and 16

or $x = -2$ thus , the numbers are : -4 , -2 , 8 and 4

30

$x + y = 8$ $\therefore y = 8 - x$ (1)

$y + z = 14$ $z = 14 - y = 14 - (8 - x) = 6 + x$ (2)

$\because z + \ell = 24$ $\ell = 24 - z = 24 - (6 + x) = 18 - x$ (3)

$\because x$, y , z , ℓ are proportional $\therefore \frac{x}{y} = \frac{z}{\ell}$

From (1) , (2) and (3) $\therefore \frac{x}{8 - x} = \frac{6 + x}{18 - x}$

$\therefore 18x - x^2 = 48 + 2x - x^2$ $\therefore 16x = 48$

$\therefore x = 3$

$y = 8 - 3 = 5$, $z = 6 + 3 = 9$, $\ell = 18 - 3 = 15$

31

Let the number be x Its multiplicative inverse = $\frac{1}{x}$

$\frac{2}{3 + \frac{1}{x}} = \frac{3}{5}$

Multiplying the two terms of the ratio in the left side by x

$\therefore \frac{2x}{3x + 1} = \frac{3}{5}$ $\therefore 10x = 9x + 3$

$x = 3$ $\therefore$ The number = 3

1

1 c	2 d	3 c	4 b	5 b	6 d
7 b	8 c	9 a	10 c	11 d	12 b
13 d	14 d	15 d	16 c		

2

Let $\frac{a}{b} = \frac{c}{d} = m$ where $m > 0$

$a = bm$, $c = dm$

1 L.H.S $= \frac{3a + c}{5a - 2c} = \frac{3bm + dm}{5bm - 2dm}$

$= \frac{m(3b + d)}{m(5b - 2d)} = \frac{3b + d}{5b - 2d}$ = R.H.S

2 L.H.S. $= \frac{3a - 2c}{5a + 3c} = \frac{3bm - 2dm}{5bm + 3dm}$

$= \frac{m(3b - 2d)}{m(5b + 3d)} = \frac{3b - 2d}{5b + 3d}$

= R.H.S

3 L.H.S. $= \frac{b^2 m^2 + d^2 m^2}{b^2 m + d^2 m} = \frac{m^2 (b^2 + d^2)}{m (b^2 + d^2)} = m$ (1)

, R.H.S $= \frac{b\,m}{b} = m$ (2)

From (1) and (2) $\therefore$ The two sides are equal

4 L.H.S. $= \frac{a^2 + c^2}{b^2 + d^2} = \frac{b^2 m^2 + d^2 m^2}{b^2 + d^2} = \frac{m^2 (b^2 + d^2)}{b^2 + d^2} = m^2$ (1)

R.H.S. $= \frac{a\,c}{b\,d} = \frac{b\,m \times d\,m}{b\,d} = m^2$ (2)

From (1) and (2) $\therefore$ The two sides are equal.

5 L.H.S. $= \frac{a\,c}{b\,d} = \frac{b\,m \times d\,m}{b\,d} = m^2$ (1)

R.H.S. $= \left(\frac{a - c}{b - d}\right)^2 = \left(\frac{b\,m - d\,m}{b - d}\right)^2 = \left(\frac{m (b - d)}{b - d}\right)^2 = m^2$ (2)

From (1) and (2) $\therefore$ The two sides are equal

6 L.H.S $= \left(\frac{a + b}{c + d}\right)^2 = \left(\frac{b\,m + b}{d\,m + d}\right)^2 = \left(\frac{b (m + 1)}{d (m + 1)}\right)^2 = \frac{b^2}{d^2}$ (1)

R.H.S $= \frac{2 a^2 - 3 b^2}{2 c^2 - 3 d^2} = \frac{2 b^2 m^2 - 3 b^2}{2 d^2 m^2 - 3 d^2} = \frac{b^2 (2 m^2 - 3)}{d^2 (2 m^2 - 3)} = \frac{b^2}{d^2}$ (2)

From (1) and (2) $\therefore$ The two sides are equal

7 L.H.S $= \sqrt{\frac{3 a^2 - 5 c^2}{3 b^2 - 5 d^2}} = \sqrt{\frac{3 b^2 m^2 - 5 d^2 m^2}{3 b^2 - 5 d^2}} = \sqrt{\frac{m^2 (3 b^2 - 5 d^2)}{(3 b^2 - 5 d^2)}} = m$ (1)

R.H.S. $= \frac{a}{b} = \frac{b\,m}{b} = m$ (2)

From (1) and (2) $\therefore$ The two sides are equal

8 L.H.S $= \sqrt[3]{\frac{5 a^3 - 3 c^3}{5 b^3 - 3 d^3}} = \sqrt[3]{\frac{5 b^3 m^3 - 3 d^3 m^3}{5 b^3 - 3 d^3}} = \sqrt[3]{\frac{m^3 (5 b^3 - 3 d^3)}{(5 b^3 - 3 d^3)}} = \sqrt[3]{m^3} = m$ (1)

R.H.S. $= \frac{b\,m + d\,m}{b + d} = \frac{m (b + d)}{(b + d)} = m$ (2)

From (1) and (2) : $\therefore$ The two sides are equal

9 L.H.S $= \frac{a^2 - 2 a c + c^2}{a\,c} = \frac{(a - c)^2}{a\,c} = \frac{(b\,m - d\,m)^2}{b\,m \times d\,m} = \frac{\left(m (b - d)\right)^2}{b\,d\,m^2} = \frac{m^2 (b - d)^2}{b\,d\,m^2} = \frac{(b - d)^2}{b\,d}$ (1)

R.H.S. $= \frac{b^2 - 2 b d + d^2}{b\,d} = \frac{(b - d)^2}{b\,d}$ (2)

From (1) and (2) : $\therefore$ The two sides are equal.

3

Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = m$ where $m > 0$

$a = b\,m$, $c = d\,m$, $e = f\,m$

1 L.H.S. $= \frac{a + 5 c}{b + 5 d} = \frac{b\,m + 5 d\,m}{b + 5 d} = \frac{m (b + 5 d)}{(b + 5 d)} = m$ (1)

R.H.S $= \frac{c - 3 e}{d - 3 f} = \frac{d\,m - 3 f\,m}{d - 3 f} = \frac{m (d - 3 f)}{d - 3 f} = m$ (2)

From (1) and (2) : $\therefore$ The two sides are equal.

2 L.H.S. $= \frac{2 a + 7 c - 4 e}{2 b + 7 d - 4 f} = \frac{2 b\,m + 7 d\,m - 4 f\,m}{2 b + 7 d - 4 f} = \frac{m (2 b + 7 d - 4 f)}{(2 b + 7 d - 4 f)} = m$ (1)

R.H.S. $= \frac{a - 8 e}{b - 8 f} = \frac{b\,m - 8 f\,m}{b - 8 f} = \frac{m (b - 8 f)}{b - 8 f} = m$ (2)

From (1) and (2) : $\therefore$ The two sides are equal

3 L.H.S. $= \frac{2 a^4 b^2 + 3 a^2 e^2 - 5 e^4 f}{2 b^6 + 3 b^2 f^2 - 5 f^5} = \frac{(2 b^6 m^4 + 3 b^2 f^2 m^4 - 5 f^5 m^4)}{2 b^6 + 3 b^2 f^2 - 5 f^5} = \frac{m^4 (2 b^6 + 3 b^2 f^2 - 5 f^5)}{2 b^6 + 3 b^2 f^2 - 5 f^5} = m^4$ (1)

R.H.S $= \frac{a^4}{b^4} = \frac{b^4 m^4}{b^4} = m^4$ (2)

From (1) and (2) : $\therefore$ The two sides are equal

4 L.H.S. $= \sqrt{\frac{5 a^2 - 7 c e}{5 b^2 - 7 d f}} = \sqrt{\frac{5 b^2 m^2 - 7 d f m^2}{5 b^2 - 7 d f}} = \sqrt{\frac{m^2 (5 b^2 - 7 d f)}{5 b^2 - 7 d f}} = \sqrt{m^2} = m$ (1)

R.H.S. $= \frac{2 a + c}{2 b + d} = \frac{2 b\,m + d\,m}{2 b + d} = \frac{m (2 b + d)}{2 b + d} = m$ (2)

From (1) and (2) : $\therefore$ The two sides are equal.

4

Let $\frac{x}{3} = \frac{y}{4} = \frac{z}{5} = m$ where $m > 0$

$\therefore x = 3 m$, $y = 4 m$, $z = 5 m$

1 L.H.S $= \frac{2y-z}{3x-2y+z} = \frac{8m-5m}{9m-8m+5m}$

$= \frac{3m}{6m} = \frac{1}{2} = $ R.H.S

2 $\sqrt{3x^2+3y^2+z^2} = \sqrt{27m^2+48m^2+25m^2}$

$= \sqrt{100m^2} = 10m \quad (1)$

, $2x+y = 6m+4m = 10m \quad (2)$

From (1) and (2) : $\therefore \sqrt{3x^2+3y^2+z^2} = 2x+y$

5

Let $\frac{x}{1} = \frac{y}{2} = \frac{z}{3} = m$

$x = m$, $y = 2m$, $z = 3m$

$\therefore$ L.H.S $= \frac{x+y-2z}{x-3z} = \frac{m+2m-6m}{m-9m} = \frac{-3m}{-8m}$

$= \frac{3}{8} = $ R.H.S

Another solution :

$x = \frac{y}{2} = \frac{z}{3} \qquad \therefore y = 2x$, $z = 3x$

$\therefore$ L.H.S $= \frac{x+y-2z}{x-3z} = \frac{x+2x-6x}{x-9x} = \frac{-3x}{-8x}$

$= \frac{3}{8} = $ R.H.S

6

$\because \frac{a}{2} = \frac{b}{3} = \frac{c}{4}$

, multiplying the two terms of the 1st ratio by 2 and the 2nd by − 5 and the 3rd by 3 and adding the antecedents and consequents of the three ratios.

$\therefore \frac{2a-5b+3c}{4-15+12} = $ one of the given ratios

$\therefore 2a-5b+3c = $ one of the given ratios

7

$\because \frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a-b+5c}{3x}$

, multiplying the two terms of the 1st ratio by 2 and the 2nd by − 1 and the 3rd by 5 and adding the antecedents and consequents of the three ratios

$\therefore \frac{2a-b+5c}{4-3+20} = $ one of the given ratios.

$\therefore \frac{2a-b+5c}{21} = \frac{2a-b+5c}{3x}$

$\therefore 3x = 21 \qquad \therefore x = 7$

8

$\because \frac{a}{4x+y} = \frac{b}{x-4y}$

, adding the antecedents and consequents of the two ratios

$\frac{a+b}{4x+y+x-4y} = \frac{a+b}{5x-3y}$

$= $ one of the given ratios. (1)

subtracting the antecedents and consequents of the 2nd ratio from the 1st ratio

$\frac{a-b}{4x+y-x+4y} = \frac{a-b}{3x+5y}$

$= $ one of the given ratios. (2)

From (1) and (2) $\therefore \frac{a+b}{5x-3y} = \frac{a-b}{3x+5y}$

9

$\because \frac{x+y}{19} = \frac{y+z}{7}$

, adding the antecedents and consequents of the two ratios.

$\therefore \frac{x+y+y+z}{19+7} = \frac{x+2y+z}{26}$

$= $ one of the given ratios. (1)

, subtracting the antecedent and consequent of the 2nd ratio from the antecedent and consequent of the 1st ratio.

$\therefore \frac{x+y-y-z}{19-7} = \frac{x-z}{12} = $ one of the given ratios. (2)

From (1) and (2) : $\therefore \frac{x+2y+z}{26} = \frac{x-z}{12}$

$\therefore \frac{x+2y+z}{13} = \frac{x-z}{6}$

10

$\because \frac{y}{x-z} = \frac{x}{y} = \frac{x+y}{z}$

, adding the antecedents and consequents of the three ratios , $\frac{y+x+x+y}{x-z+y+z} = \frac{2(x+y)}{(x+y)} = 2$

$= $ one of the given ratios

$\therefore$ Each ratio $= 2$ unless $x+y \neq 0$

$\because \frac{x}{y} = 2 \qquad \therefore x = 2y$

, $\frac{x+y}{z} = 2 \qquad \therefore x+y = 2z \qquad \therefore 2y+y = 2z$

$\therefore 3y = 2z \qquad \therefore z = \frac{3}{2}y$

$\therefore x : y : z = 2y : y : \frac{3}{2}y = 4 : 2 : 3$

11

$\because \frac{x}{a-b+c} = \frac{y}{b-c+a} = \frac{z}{c-a+b}$

, adding the antecedents and consequents of the 1st and 2nd ratios.

$\therefore \frac{x+y}{a-b+c+b-c+a} = \frac{x+y}{2a}$

= one of the given ratios. (1)

, adding the antecedents and consequents of the 2nd and 3rd ratios.

$\therefore \frac{y+z}{b-c+a+c-a+b} = \frac{y+z}{2b}$

= one of the given ratios (2)

From (1) and (2) $\frac{x+y}{2a} = \frac{y+z}{2b}$

$\frac{x+y}{a} = \frac{y+z}{b}$

12

$\frac{x}{2a+b} = \frac{y}{2b-c} = \frac{z}{2c-a}$

, multiplying the two terms of the 1st ratio by 2 and adding the antecedents and consequents of the 1st and the 2nd ratios.

$\therefore \frac{2x+y}{4a+2b+2b-c} = \frac{2x+y}{4a+4b-c}$

= one of the given ratios (1)

, multiplying the terms of the 1st ratio by 2 and the 2nd by 2 and adding the antecedents and consequents of the three ratios

$\therefore \frac{2x+2y+z}{4a+2b+4b-2c+2c-a} = \frac{2x+2y+z}{3a+6b}$

= one of the given ratios (2)

From (1) and (2)

$\therefore \frac{2x+y}{4a+4b-c} = \frac{2x+2y+z}{3a+6b}$

13

$\because \frac{a}{2x-y} = \frac{b}{2y-x}$

, multiplying the terms of the 1st ratio by 2 and adding the antecedents and consequents of the two ratios

$\therefore \frac{2a+b}{4x-2y+2y-x} = \frac{2a+b}{3x}$

= one of the given ratios. (1)

, multiplying the terms of the 2nd ratio by 2 and adding the antecedents and consequents of the two ratios.

$\therefore \frac{a+2b}{2x-y+4y-2x} = \frac{a+2b}{3y}$

= one of the given ratios. (2)

From (1) and (2) $\therefore \frac{2a+b}{3x} = \frac{a+2b}{3y}$

$\frac{2a+b}{a+2b} = \frac{3x}{3y} = \frac{x}{y}$

14

$\because \frac{a}{2x+y} = \frac{b}{3y-x} = \frac{c}{4x+5y}$

, multiplying the terms of the 2nd ratio by 2 and adding the antecedents and consequents of the 1st and 2nd ratios

$\therefore \frac{a+2b}{2x+y+6y-2x} = \frac{a+2b}{7y}$

= one of the given ratios. (1)

, multiplying the terms of the 2nd ratio by 4 and adding the antecedents and consequents of the 2nd and 3rd ratios.

$\therefore \frac{4b+c}{12y-4x+4x+5y} = \frac{4b+c}{17y}$

= one of the given ratios. (2)

From (1) and (2) : $\therefore \frac{a+2b}{7y} = \frac{4b+c}{17y}$

$\frac{a+2b}{4b+c} = \frac{7y}{17y} = \frac{7}{17}$

15

$\frac{a}{2} = \frac{b}{7} = \frac{c}{3}$

, multiplying the terms of the 2nd ratio by 2 and adding the antecedents and consequents of the 1st and 2nd ratios.

$\frac{a+2b}{2+14} = \frac{a+2b}{16}$ = one of the given ratios. (1)

Subtracting the antecedents and consequents of the 3rd ratio from the antecedents and consequents of the 2nd ratio

$\therefore \frac{b-c}{7-3} = \frac{b-c}{4}$ = one of the given ratios. (2)

From (1) and (2) : $\therefore \frac{a+2b}{16} = \frac{b-c}{4}$

$\therefore \frac{a+2b}{b-c} = \frac{16}{4} = 4$

16

$\because \frac{x+y}{7} = \frac{y+z}{5} = \frac{z+x}{8}$

, adding the antecedents and consequents of the three ratios.

$\therefore \frac{x+y+y+z+z+x}{7+5+8} = \frac{2(x+y+z)}{20} = \frac{x+y+z}{10}$

= one of the given ratios. (1)

, multiplying the terms of the 2nd ratio by (– 1) and adding the antecedents and consequents of the 1st and 2nd ratios.

$\therefore \frac{x+y \quad y \quad z}{7 \quad 5} = \frac{x-z}{2}$ = one of the given ratios (2)

From (1) and (2) $\therefore \frac{x+y+z}{10} = \frac{x-z}{2}$

$\therefore \frac{x+y+z}{x-z} \quad \frac{10}{2} = 5$

17

$\frac{a+b}{4} = \frac{b+c}{5} = \frac{c+a}{7}$

, adding the antecedents and consequents of the three ratios.

$\frac{a+b+b+c+c+a}{4+5+7} = \frac{2(a+b+c)}{16} = \frac{a+b+c}{8}$

= one of the given ratios. (1)

, multiplying the terms of the 2^{nd} ratio by (–1) and adding the antecedents and consequents of the three ratios.

$\frac{a+b-b-c+c+a}{4-5+7} = \frac{2a}{6} = \frac{a}{3}$

= one of the given ratios. (2)

From (1) and (2) :

$\frac{a+b+c}{8} = \frac{a}{3}$

18

$\frac{x+y}{3} = \frac{y+z}{8} = \frac{z+x}{6}$

, adding the antecedents and consequents of the three ratios.

$\therefore \frac{x+y+y+z+z+x}{3+8+6} = \frac{2x+2y+2z}{17} = \frac{2(x+y+z)}{17}$

= one of the given ratios. (1)

, multiplying the terms of the 2^{nd} ratio by 2 and adding the antecedents and consequents of the three ratios.

$\therefore \frac{x+y+2y+2z+z+x}{3+16+6} = \frac{2x+3y+3z}{25}$

= one of the given ratios.(2)

From (1) and (2) : $\therefore \frac{2(x+y+z)}{17} = \frac{2x+3y+3z}{25}$

$\therefore \frac{x+y+z}{2x+3y+3z} = \frac{17}{50}$

19

Multiplying the terms of the 2^{nd} ratio by (– 1) and adding the antecedents and consequents of the three ratios

$\therefore \frac{x+y \quad y \quad z+z+x}{5-8+7}$

$= \frac{2x}{4} = \frac{x}{2}$ = one of the given ratios (1)

, multiplying the terms of the 3^{rd} ratio by (– 1) and adding the antecedents and consequents of the three ratios

$\therefore \frac{x+y+y+z-z-x}{5+8-7} = \frac{2y}{6} = \frac{y}{3}$

= one of the given ratios (2)

, multiplying the terms of the 1^{st} ratio by (– 1) and adding the antecedents and consequents of the three ratios

$\therefore \frac{x \quad y+y+z+z+x}{-5+8+7} = \frac{2z}{10} = \frac{z}{5}$

= one of the given ratios (3)

From (1) , (2) and (3) : $\therefore \frac{x}{2} = \frac{y}{3} = \frac{z}{5}$

20

$\frac{x+y}{25} = \frac{x-y}{11} = \frac{x+y-z}{8}$

, adding the antecedents and consequents of the 1^{st} and 2^{nd} ratios.

$\therefore \frac{2x}{36} = \frac{x}{18}$ = one of the given ratios. (1)

, subtracting the antecedent and consequent of the 3^{rd} ratio from the antecedent and consequent of the 1^{st} ratio.

$\frac{z}{17}$ = one of the given ratios. (2)

, subtracting the antecedent and consequent of the 2^{nd} ratio from the antecedent and consequent of the 1^{st} ratio.

$\therefore \frac{2y}{14} = \frac{y}{7}$ = one of the given ratios. (3)

From (1) , (2) , (3)

$\therefore \frac{x}{18} = \frac{y}{7} = \frac{z}{17}$ $\qquad x \quad y \quad z = 18 \quad 7 \quad 17$

21

Multiplying the terms of the 2^{nd} ratio by (– 1) and adding the antecedents and consequents of the three ratios.

$\frac{a+3b \quad 3b \quad 5c+5c+a}{x+6y-6y-10z+10z+x}$

$= \frac{2a}{2x} = \frac{a}{x}$ = one of the given ratios (1)

, multiplying the terms of the 3^{rd} ratio by (– 1) and adding the antecedents and consequents of the three ratios

$\therefore \frac{a+3b+3b+5c-5c-a}{x+6y+6y+10z-10z-x}$

$= \frac{6b}{12y} = \frac{b}{2y}$ = one of the given ratios (2)

From (1) and (2) : $\therefore \frac{a}{x} = \frac{b}{2y}$

$\frac{a}{b} = \frac{x}{2y}$

, multiplying the terms of the 1st ratio by (– 1) and adding the antecedents and consequents of the three ratios

$\therefore \frac{-a-3b+3b+5c+5c+a}{-x-6y+6y+10z+10z+x} = \frac{10c}{20z} = \frac{c}{2z}$

= one of the given ratios. (3)

From (1) , (2) and (3)

$\frac{a}{x} = \frac{b}{2y} = \frac{c}{2z}$

$a : b : c = x : 2y : 2z$

22

Multiplying the terms of the 1st ratio by (– 2) and the 3rd ratio by 3 and adding the antecedents and consequents of the 1st and 3rd ratios

$\frac{3c-2a}{3y+6x-6x-8y} = \frac{3c-2a}{-5y}$

= one of the given ratios (1)

, multiplying the terms of the 2nd ratio by 2 and adding the antecedents and consequents of the 1st and 2nd ratios

$\therefore \frac{a+2b}{3x+4y+10x-4y} = \frac{a+2b}{13x}$

= one of the given ratios (2)

From (1) and (2) $\therefore \frac{3c-2a}{-5y} = \frac{a+2b}{13x}$

$\therefore 13x(3c-2a) = -5y(a+2b)$

$13x(3c-2a) + 5y(a+2b) = 0$

23

$\frac{x}{7} = \frac{y}{3} = m$ $\therefore x = 7m , y = 3m$

$\because \frac{2x-3y}{x+2y} = \frac{2(7m)-3(3m)}{7m+2(3m)} = \frac{14m-9m}{7m+6m}$

$= \frac{5m}{13m} = \frac{5}{13}$ (1)

$\frac{10}{26} = \frac{5}{13}$ (2)

From (1) and (2) $\frac{2x-3y}{x+2y} = \frac{10}{26}$

$\therefore (2x-3y) , (x+2y) , 10 , 26$ are proportional

24

$\frac{a}{b} = \frac{3}{5}$ $\therefore b = \frac{5a}{3}$

, $\frac{a}{c} = \frac{3}{7}$ $\therefore c = \frac{7a}{3}$

$a+b+c = a + \frac{5a}{3} + \frac{7a}{3} = 5a$

25

$\frac{a}{b} = \frac{2}{3}$ $\therefore b = \frac{3}{2}a$

, $\frac{a}{c} = \frac{3}{5}$ $\therefore c = \frac{5}{3}a$

$\because a+b+c = 75$ $\therefore a + \frac{3}{2}a + \frac{5}{3}a = 75$

$\frac{25}{6}a = 75$ $\therefore a = 18$

$, b = \frac{3}{2} \times 18 = 27$ $, c = \frac{5}{3} \times 18 = 30$

26

$\because \Delta ABC \sim \Delta DEF$ $\therefore \frac{DE}{AB} = \frac{EF}{BC} = \frac{DF}{AC} = \frac{2}{3}$

Adding the antecedents and the consequents of the three ratios

$\therefore \frac{DE+EF+DF}{AB+BC+AC}$ = one of the given ratios

$\therefore \frac{22}{\text{perimeter of } \Delta ABC} = \frac{2}{3}$

The perimeter of ΔABC = 33 cm

27

$\frac{a}{x-y+z} = \frac{b}{x+y-z} = \frac{c}{y+z-x}$

, multiplying the terms of the 1st ratio by x and the 2nd by y and the 3rd by z and adding the antecedents and consequents of the three ratios each ratio

$= \frac{ax+by+cz}{x^2-xy+xz+xy+y^2-zy+zy+z^2-xz}$

$= \frac{ax+by+cz}{x^2+y^2+z^2}$

28

$\because \frac{2x+y}{x} = \frac{4y+z}{y} = \frac{4z+3x}{z}$

, adding the antecedents and consequents of the three ratios

$\therefore \frac{5x+5y+5z}{x+y+z} = \frac{5(x+y+z)}{x+y+z} = 5$

= one of the given ratios

$\because$ Each ratio = 5 $\qquad \therefore \frac{2x+y}{x} = 5$

$\therefore 2x + y = 5x \qquad \therefore y = 3x \qquad (1)$

$, \frac{4z+3x}{z} = 5 \qquad \therefore 4z + 3x = 5z$

$\therefore z = 3x \qquad (2)$

From (1) and (2)

$x : y : z = x : 3x : 3x = 1 : 3 : 3$

$, \frac{2x+y+z}{3x-y+2z} = \frac{2x+3x+3x}{3x-3x+6x} = \frac{8x}{6x} = \frac{4}{3}$

29

$\frac{a+2b}{5} = \frac{3b-c}{3} = \frac{c-a}{2} = m$

$\therefore a + 2b = 5m \ (1) , 3b - c = 3m \ (2) , c - a = 2m \ (3)$

Adding (1) , (2) and (3)

$\therefore 5b = 10m \qquad \therefore b = 2m$

From (1) $\therefore a + 4m = 5m \qquad \therefore a = m$

From (3) $\therefore c - m = 2m \qquad \therefore c = 3m$

[1] $a + b - c = m + 2m - 3m =$ zero

[2] $\frac{3b-a}{2b+c} = \frac{6m-m}{4m+3m} = \frac{5m}{7m} = \frac{5}{7}$

Answers of Exercise 7

1

[1] The middle proportional $= \pm\sqrt{3 \times 27}$

$= \pm\sqrt{81} = \pm 9$

[2] The middle proportional $= \pm\sqrt{9 \times 25} = \pm\sqrt{225} = \pm 15$

[3] The middle proportional $= \pm\sqrt{-2 \times -8}$

$= \pm\sqrt{16} = \pm 4$

[4] The middle proportional $= \pm\sqrt{\frac{1}{5} \times 125}$

$= \pm\sqrt{25} = \pm 5$

[5] The middle proportional $= \pm\sqrt{2ab \times 8ab^2}$

$= \pm\sqrt{16a^2b^2} = \pm 4ab$

[6] The middle proportional $= \pm\sqrt{(\ell^2 - m^2)^2}$

$= \pm(\ell^2 - m^2)$

2

[1] Let the third proportional be c

$\frac{6}{12} = \frac{12}{c} \qquad \therefore c = \frac{12 \times 12}{6} = 24$

[2] Let the third proportional be c

$\therefore \frac{x^2}{-5x} = \frac{-5x}{c} \qquad \therefore c = \frac{-5x \times -5x}{x^2} = 25$

[3] Let the third proportional be c

$\frac{x^2}{3x^2} = \frac{3x^2}{c} \qquad \therefore c = \frac{3x^2 \times 3x^2}{x^2} = 9x^2$

3

1 Let $\frac{a}{b} = \frac{b}{c} = m \qquad \therefore b = cm \ , a = cm^2$

$\therefore \frac{a}{c} = \frac{cm^2}{c} = m^2 \quad (1) \quad , \frac{b^2}{c^2} = \frac{c^2m^2}{c^2} = m^2 \quad (2)$

From (1) and (2) : $\therefore \frac{a}{c} = \frac{b^2}{c^2}$

Another solution :

$\because b^2 = ac \qquad \therefore \frac{b^2}{c^2} = \frac{ac}{c^2} = \frac{a}{c} =$ L.H.S

2 Let $\frac{a}{b} = \frac{b}{c} = m \qquad \therefore b = cm \ , a = cm^2$

$\therefore \frac{2a+3b}{2b+3c} = \frac{2cm^2+3cm}{2cm+3c} = \frac{cm(2m+3)}{c(2m+3)} = m \qquad (1)$

$, \frac{a}{b} = \frac{cm^2}{cm} = m \qquad (2)$

From (1) and (2) : $\therefore \frac{2a+3b}{2b+3c} = \frac{a}{b}$

[3] Let $\frac{a}{b} = \frac{b}{c} = m \qquad \therefore b = cm \ , a = cm^2$

$\therefore \frac{a-b}{b-c} = \frac{cm^2-cm}{cm-c} = \frac{cm(m-1)}{c(m-1)} = m \qquad (1)$

$, \frac{a+3b}{3c+b} = \frac{cm^2+3cm}{3c+cm} = \frac{cm(m+3)}{c(3+m)} = m \qquad (2)$

From (1) and (2) $\therefore \frac{a-b}{b-c} = \frac{a+3b}{3c+b}$

4 Let $\frac{a}{b} = \frac{b}{c} = m \qquad \therefore b = cm \ , a = cm^2$

$\therefore \frac{a^2+b^2}{b^2+c^2} = \frac{c^2m^4+c^2m^2}{c^2m^2+c^2} = \frac{c^2m^2(m^2+1)}{c^2(m^2+1)} = m^2 \ (1)$

$, \frac{a}{c} = \frac{cm^2}{c} = m^2 \qquad (2)$

From (1) and (2) $\therefore \frac{a^2+b^2}{b^2+c^2} = \frac{a}{c}$

Another solution :

$\because b^2 = ac$

$\therefore \frac{a^2+b^2}{b^2+c^2} = \frac{a^2+ac}{ac+c^2} = \frac{a(a+c)}{c(a+c)} = \frac{a}{c} =$ R.H.S

5 Let $\frac{a}{b} = \frac{b}{c} = m$ $\therefore b = c\,m \;,\; a = c\,m^2$

$$\therefore \left(\frac{b-c}{a-b}\right)^2 = \left(\frac{c\,m - c}{c\,m^2 - c\,m}\right)^2 = \left(\frac{c\,(m-1)}{c\,m\,(m-1)}\right)^2 = \frac{1}{m^2} \quad (1)$$

$$, \frac{c}{a} = \frac{c}{c\,m^2} = \frac{1}{m^2} \quad (2)$$

From (1) and (2) $\left(\frac{b-c}{a-b}\right)^2 = \frac{c}{a}$

6 Let $\frac{a}{b} = \frac{b}{c} = m$ $\therefore b = c\,m \;,\; a = c\,m^2$

$$\therefore \frac{a^3+b^3}{b^3+c^3} = \frac{c^3m^6+c^3m^3}{c^3m^3+c^3} = \frac{c^3m^3(m^3+1)}{c^3(m^3+1)} = m^3 \quad (1)$$

$$, \frac{a^2}{c\,b} = \frac{c^2m^4}{c \times c\,m} = m^3 \quad (2)$$

From (1) and (2) $\therefore \frac{a^3+b^3}{b^3+c^3} = \frac{a^2}{c\,b}$

7 Let $\frac{a}{b} = \frac{b}{c} = m$ $\therefore b = c\,m \;,\; a = c\,m^2$

$$\therefore \frac{a^3-4b^3}{b^3-4c^3} = \frac{c^3m^6-4c^3m^3}{c^3m^3-4c^3} = \frac{c^3m^3(m^3-4)}{c^3(m^3-4)} = m^3 \quad (1)$$

$$, \frac{b^3}{c^3} = \frac{c^3m^3}{c^3} = m^3 \quad (2)$$

From (1) and (2) $\frac{a^3-4b^3}{b^3-4c^3} = \frac{b^3}{c^3}$

8 Let $\frac{a}{b} = \frac{b}{c} = m$ $\therefore b = c\,m \;,\; a = c\,m^2$

$$\therefore \frac{2c^2-3b^2}{2b^2-3a^2} = \frac{2c^2-3c^2m^2}{2c^2m^2-3c^2m^4} = \frac{c^2(2-3m^2)}{c^2m^2(2-3m^2)} = \frac{1}{m^2} \quad (1)$$

$$, \frac{c}{a} = \frac{c}{c\,m^2} = \frac{1}{m^2} \quad (2) \qquad , \frac{c^2}{b^2} = \frac{c^2}{c^2m^2} = \frac{1}{m^2} \quad (3)$$

From (1) , (2) and (3) $\therefore \frac{2c^2-3b^2}{2b^2-3a^2} = \frac{c}{a} = \frac{c^2}{b^2}$

Another solution : $\because b^2 = a\,c$

$$\therefore \frac{2c^2-3b^2}{2b^2-3a^2} = \frac{2c^2-3ac}{2ac-3a^2} = \frac{c\,(2c-3a)}{a\,(2c-3a)} = \frac{c}{a}$$

$$, \frac{c^2}{b^2} = \frac{c^2}{a\,c} = \frac{c}{a}$$

$$\therefore \frac{2c^2-3b^2}{2b^2-3a^2} = \frac{c}{a} = \frac{c^2}{b^2}$$

9 Let $\frac{a}{b} = \frac{b}{c} = m$ $\therefore b = c\,m \;,\; a = c\,m^2$

$$\therefore \frac{a^2+a\,b+b^2}{b^2+b\,c+c^2} = \frac{c^2m^4+c^2m^3+c^2m^2}{c^2m^2+c^2m+c^2} = \frac{c^2m^2(m^2+m+1)}{c^2(m^2+m+1)} = m^2 \quad (1)$$

$$, \frac{a^2-b^2}{b^2-c^2} = \frac{c^2m^4-c^2m^2}{c^2m^2-c^2} = \frac{c^2m^2(m^2-1)}{c^2(m^2-1)} = m^2 \quad (2)$$

From (1) and (2) $\therefore \frac{a^2+a\,b+b^2}{b^2+b\,c+c^2} = \frac{a^2-b^2}{b^2-c^2}$

10 Let $\frac{a}{b} = \frac{b}{c} = m$ $\therefore b = c\,m \;,\; a = c\,m^2$

$$\therefore \frac{2a}{c} = \frac{2c\,m^2}{c} = 2m^2 \quad (1)$$

$$, \frac{a^2}{b^2} + \frac{b^2}{c^2} = m^2 + m^2 = 2m^2 \quad (2)$$

From (1) and (2) $\therefore \frac{2a}{c} = \frac{a^2}{b^2} + \frac{b^2}{c^2}$

Another solution : $b^2 = a\,c$

$$\therefore \frac{a^2}{b^2} + \frac{b^2}{c^2} = \frac{a^2}{ac} + \frac{ac}{c^2} = \frac{a}{c} + \frac{a}{c} = \frac{2a}{c} = \text{R.H.S}$$

11 Let $\frac{a}{b} = \frac{b}{c} = m$ $\therefore b = c\,m \;,\; a = c\,m^2$

$$\therefore \frac{a+b+c}{a^{-1}+b^{-1}+c^{-1}} = \frac{c\,m^2+c\,m+c}{c^{-1}m^{-2}+c^{-1}m^{-1}+c^{-1}}$$

$$= \frac{c\,(m^2+m+1)}{c^{-1}m^{-2}(1+m+m^2)}$$

$$= c \times c\,m^2 = c^2m^2 = b^2$$

12 Let $\frac{a}{b} = \frac{b}{c} = m$ $\therefore b = c\,m \;,\; a = c\,m^2$

$$\therefore \frac{a\,c}{b\,(b+c)} = \frac{c\,m^2 \times c}{c\,m\,(c\,m+c)} = \frac{c^2m^2}{c^2m\,(m+1)} = \frac{m}{m+1} \quad (1)$$

$$, \frac{a}{a+b} = \frac{c\,m^2}{c\,m^2+c\,m} = \frac{c\,m^2}{c\,m\,(m+1)} = \frac{m}{m+1} \quad (2)$$

From (1) and (2) $\therefore \frac{a\,c}{b\,(b+c)} = \frac{a}{a+b}$

Another solution : $\because b^2 = a\,c$

$$\therefore \frac{a\,c}{b\,(b+c)} = \frac{a\,c}{b^2+b\,c} = \frac{a\,c}{a\,c+b\,c} = \frac{a\,c}{c\,(a+b)}$$

$$= \frac{a}{a+b} = \text{R.H.S}$$

4

1 Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = d\,m \;,\; b = d\,m^2 \;,\; a = d\,m^3$

$$\therefore \frac{a-2b}{b-2c} = \frac{d\,m^3-2d\,m^2}{d\,m^2-2d\,m} = \frac{d\,m^2(m-2)}{d\,m\,(m-2)} = m \quad (1)$$

$$, \frac{3b+4c}{3c+4d} = \frac{3d\,m^2+4d\,m}{3d\,m+4d} = \frac{d\,m\,(3m+4)}{d\,(3m+4)} = m \quad (2)$$

From (1) and (2) : $\therefore \frac{a-2b}{b-2c} = \frac{3b+4c}{3c+4d}$

2 Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = dm \; , \; b = dm^2 \; , \; a = dm^3$

$\therefore \frac{3a+5c}{3b+5d} = \frac{3dm^3+5dm}{3dm^2+5d} = \frac{dm(3m^2+5)}{d(3m^2+5)} = m$ (1)

$, \frac{a-4c}{b-4d} = \frac{dm^3-4dm}{dm^2-4d} = \frac{dm(m^2-4)}{d(m^2-4)} = m$ (2)

From (1) and (2) : $\therefore \frac{3a+5c}{3b+5d} = \frac{a-4c}{b-4d}$

3 Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = dm \; , \; b = dm^2 \; , \; a = dm^3$

$\frac{3a-5c}{a-b+c} = \frac{3dm^3-5dm}{dm^3-dm^2+dm}$

$= \frac{dm(3m^2-5)}{dm(m^2-m+1)} = \frac{3m^2-5}{m^2-m+1}$ (1)

$, \frac{3b-5d}{b-c+d} = \frac{3dm^2-5d}{dm^2-dm+d}$

$= \frac{d(3m^2-5)}{d(m^2-m+1)} = \frac{3m^2-5}{m^2-m+1}$ (2)

From (1) and (2) : $\therefore \frac{3a-5c}{a-b+c} = \frac{3b-5d}{b-c+d}$

4 Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = dm \; , \; b = dm^2 \; , \; a = dm^3$

$\therefore \frac{a-d}{a+b+c} = \frac{dm^3-d}{dm^3+dm^2+dm}$

$= \frac{d(m^3-1)}{dm(m^2+m+1)} = \frac{(m-1)(m^2+m+1)}{m(m^2+m+1)}$

$= \frac{m-1}{m}$ (1)

$, \frac{a-2b+c}{a-b} = \frac{dm^3-2dm^2+dm}{dm^3-dm^2}$

$= \frac{dm(m^2-2m+1)}{dm^2(m-1)} = \frac{(m-1)^2}{m(m-1)} = \frac{m-1}{m}$ (2)

From (1) and (2) : $\therefore \frac{a-d}{a+b+c} = \frac{a-2b+c}{a-b}$

5 Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = dm \; , \; b = dm^2 \; , \; a = dm^3$

$\frac{c^2-d^2}{a-c} = \frac{d^2m^2-d^2}{dm^3-dm} = \frac{d^2(m^2-1)}{dm(m^2-1)} = \frac{d}{m}$ (1)

$, \frac{bd}{a} = \frac{d^2m^2}{dm^3} = \frac{d}{m}$ (2)

From (1) and (2) $\therefore \frac{c^2-d^2}{a-c} = \frac{bd}{a}$

6 Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$c = dm \; , \; b = dm^2 \; , \; a = dm^3$

$\frac{a^2-3c^2}{b^2-3d^2} = \frac{d^2m^6-3d^2m^2}{d^2m^4-3d^2} = \frac{d^2m^2(m^4-3)}{d^2(m^4-3)}$

$= m^2$ (1)

$, \frac{b}{d} = \frac{dm^2}{d} = m^2$ (2)

From (1) and (2) $\therefore \frac{a^2-3c^2}{b^2-3d^2} = \frac{b}{d}$

7 Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = dm \; , \; b = dm^2 \; , \; a = dm^3$

$\frac{ab-cd}{b^2-c^2} = \frac{dm^3 \times dm^2 - dm \times d}{d^2m^4-d^2m^2} = \frac{d^2m^5-d^2m}{d^2m^2(m^2-1)}$

$= \frac{d^2m(m^4-1)}{d^2m^2(m^2-1)} = \frac{(m^2-1)(m^2+1)}{m(m^2-1)} = \frac{m^2+1}{m}$ (1)

$, \frac{a+c}{b} = \frac{dm^3+dm}{dm^2} = \frac{dm(m^2+1)}{dm^2} = \frac{m^2+1}{m}$ (2)

From (1) and (2) : $\therefore \frac{ab-cd}{b^2-c^2} = \frac{a+c}{b}$

8 Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = dm \; , \; b = dm^2 \; , \; a = dm^3$

$\therefore \frac{a}{b+d} = \frac{dm^3}{dm^2+d} = \frac{dm^3}{d(m^2+1)} = \frac{m^3}{m^2+1}$ (1)

$, \frac{c^3}{c^2d+d^3} = \frac{d^3m^3}{d^2m^2 \times d + d^3} = \frac{d^3m^3}{d^3(m^2+1)} = \frac{m^3}{m^2+1}$ (2)

From (1) and (2) $\therefore \frac{a}{b+d} = \frac{c^3}{c^2d+d^3}$

9 Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = dm \; , \; b = dm^2 \; , \; a = dm^3$

$\therefore \frac{a^2+b^2+c^2}{b^2+c^2+d^2} = \frac{d^2m^6+d^2m^4+d^2m^2}{d^2m^4+d^2m^2+d^2}$

$= \frac{d^2m^2(m^4+m^2+1)}{d^2(m^4+m^2+1)} = m^2$ (1)

$, \frac{ac}{bd} = \frac{dm^3 \times dm}{dm^2 \times d} = m^2$ (2)

From (1) and (2) : $\therefore \frac{a^2+b^2+c^2}{b^2+c^2+d^2} = \frac{ac}{bd}$

10 Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = dm \; , \; b = dm^2 \; , \; a = dm^3$

$\therefore \frac{2a+3d}{3a-4d} = \frac{2dm^3+3d}{3dm^3-4d}$

$= \frac{d(2m^3+3)}{d(3m^3-4)} = \frac{2m^3+3}{3m^3-4}$ (1)

$, \frac{2a^3+3b^3}{3a^3-4b^3} = \frac{2d^3m^9+3d^3m^6}{3d^3m^9-4d^3m^6}$

$= \frac{d^3m^6(2m^3+3)}{d^3m^6(3m^3-4)} = \frac{2m^3+3}{3m^3-4}$ (2)

From (1) and (2) $\frac{2a+3d}{3a-4d} = \frac{2a^3+3b^3}{3a^3-4b^3}$

11 Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$ where $m > 0$

$\therefore c = dm \;,\; b = dm^2 \;,\; a = dm^3$

$\because \frac{a+5b}{b+5c} = \frac{dm^3+5dm^2}{dm^2+5dm} = \frac{dm^2(m+5)}{dm(m+5)} = m$ (1)

$, \sqrt{\frac{b}{d}} = \sqrt{\frac{dm^2}{d}} = \sqrt{m^2} = m$ (2)

From (1) and (2) $\therefore \frac{a+5b}{b+5c} = \sqrt{\frac{b}{d}}$

12 Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = dm \;,\; b = dm^2 \;,\; a = dm^3$

$\sqrt[3]{\frac{5a^3-3c^3}{5b^3-3d^3}} = \sqrt[3]{\frac{5d^3m^9-3d^3m^3}{5d^3m^6-3d^3}}$

$= \sqrt[3]{\frac{d^3m^3(5m^6-3)}{d^3(5m^6-3)}} = \sqrt[3]{m^3} = m$ (1)

$, \frac{a+c}{b+d} = \frac{dm^3+dm}{dm^2+d} = \frac{dm(m^2+1)}{d(m^2+1)} = m$ (2)

From (1) and (2) $\sqrt[3]{\frac{5a^3-3c^3}{5b^3-3d^3}} = \frac{a+c}{b+d}$

13 Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

where m is a positive real number

$\therefore c = dm \;,\; b = dm^2 \;,\; a = dm^3$

$\therefore \left(\frac{a+b}{b+c}\right)^3 = \left(\frac{dm^3+dm^2}{dm^2+dm}\right)^3 = \left(\frac{dm^2(m+1)}{dm(m+1)}\right)^3$

$= m^3$ (1)

$, \frac{a}{d} = \frac{dm^3}{d} = m^3$ (2)

From (1) and (2) $\therefore \left(\frac{a+b}{b+c}\right)^3 = \frac{a}{d}$

14 Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = dm \;,\; b = dm^2 \;,\; a = dm^3$

$\frac{a^2+d^2}{c(a+c)} = \frac{d^2m^6+d^2}{dm(dm^3+dm)} = \frac{d^2(m^6+1)}{d^2m^2(m^2+1)}$

$= \frac{(m^2+1)(m^4-m^2+1)}{m^2(m^2+1)} = \frac{m^4-m^2+1}{m^2}$ (1)

$, \frac{b}{d} + \frac{d}{b} - 1 = \frac{dm^2}{d} + \frac{d}{dm^2} - 1 = m^2 + \frac{1}{m^2} - 1$

$= \frac{m^4+1-m^2}{m^2} = \frac{m^4-m^2+1}{m^2}$ (2)

From (1) and (2) $\frac{a^2+d^2}{c(a+c)} = \frac{b}{d} + \frac{d}{b} - 1$

5

1 c	2 c	3 c	4 c	5 b	6 a
7 b	8 c	9 c	10 d	11 c	12 c

6

$\because a \,,\, 3 \,,\, 9 \,,\, b$ are in continued proportion

$\therefore \frac{a}{3} = \frac{3}{9} = \frac{9}{b}$

$\therefore a = \frac{3 \times 3}{9} = 1 \;,\; b = \frac{9 \times 9}{3} = 27$

7

$\because 3 \,,\, \ell \,,\, 12 \,,\, m$ are in continued proportion

$\therefore \frac{3}{\ell} = \frac{\ell}{12} = \frac{12}{m}$ $\therefore \ell = \pm\sqrt{3 \times 12} = \pm\sqrt{36} = \pm 6$

$, m = \frac{12 \times 12}{\pm 6} = \pm 24$

8

1 $4(x-1)^2$ 2 $\pm(3x+5y)$ 3 $\frac{4}{25}$

4 m^4 5 a 6 3 7 -10

9

Let the number be x

$\therefore \frac{3-x}{7-x} = \frac{7-x}{19-x}$

$\therefore (3-x)(19-x) = (7-x)^2$

$\therefore 57 - 22x + x^2 = 49 - 14x + x^2$

$\therefore 57 - 49 = -14x + 22x$

$\therefore 8 = 8x$ $\therefore x = 1$

$\therefore$ The number is 1

10

$a = 4c = 4$ $\therefore a = 4 \;,\; c = 1$

$\because$ b is the middle proportional between a and c

$\therefore b^2 = ac$ $\therefore b^2 = 4 \times 1 = 4$

$\therefore a^2 + b^2 + c^2 = 4^2 + 4 + 1^2 = 16 + 4 + 1 = 21$

11

Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = d\,m \;,\; b = d\,m^2 \;,\; a = d\,m^3$

$\therefore \frac{a^2}{b^2} + \frac{b^2}{c^2} + \frac{c^2}{d^2} = m^2 + m^2 + m^2 = 3\,m^2$ (1)

$, \frac{a}{c} + \frac{b}{d} + \frac{a\,c}{b\,d} = \frac{d\,m^3}{d\,m} + \frac{d\,m^2}{d} + \frac{d\,m^3 \times d\,m}{d\,m^2 \times d}$

$= m^2 + m^2 + m^2 = 3\,m^2$ (2)

From (1) and (2) $\therefore \frac{a^2}{b^2} + \frac{b^2}{c^2} + \frac{c^2}{d^2} = \frac{a}{c} + \frac{b}{d} + \frac{a\,c}{b\,d}$

12

$\because y^2 = x\,z$

$\therefore x \,,\, y \,,\, z$ are proportional

$\therefore \frac{x}{y} = \frac{y}{z} = m \qquad \therefore y = z\,m \,,\, x = z\,m^2$

$\therefore \frac{x\,(x - y)}{y\,(y - z)} = \frac{z\,m^2\,(z\,m^2 - z\,m)}{z\,m\,(z\,m - z)} = \frac{z\,m^2 \times z\,m\,(m - 1)}{z\,m \times z\,(m - 1)}$

$= \frac{z^2\,m^3\,(m - 1)}{z^2\,m\,(m - 1)} = m^2$ (1)

$, \frac{y^2}{z^2} = \frac{z^2\,m^2}{z^2} = m^2$ (2)

From (1) and (2) $\therefore \frac{x\,(x - y)}{y\,(y - z)} = \frac{y^2}{z^2}$

13

$\because b^2 = a\,c \qquad \therefore \frac{a}{b} = \frac{b}{c} \qquad \because c^2 = b\,d \qquad \therefore \frac{b}{c} = \frac{c}{d}$

$\therefore \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = d\,m \,,\, b = d\,m^2 \,,\, a = d\,m^3$

$, \frac{2\,a + 3\,d}{3\,a - 4\,d} = \frac{2\,d\,m^3 + 3\,d}{3\,d\,m^3 - 4\,d} = \frac{d\,(2\,m^3 + 3)}{d\,(3\,m^3 - 4)} = \frac{2\,m^3 + 3}{3\,m^3 - 4}$ (1)

$, \frac{2\,a^3 + 3\,b^3}{3\,a^3 - 4\,b^3} = \frac{2\,d^3m^9 + 3\,d^3m^6}{3\,d^3m^9 - 4\,d^3m^6}$

$= \frac{d^3m^6\,(2\,m^3 + 3)}{d^3m^6\,(3\,m^3 - 4)} = \frac{2\,m^3 + 3}{3\,m^3 - 4}$ (2)

From (1) and (2) $\therefore \frac{2\,a + 3\,d}{3\,a - 4\,d} = \frac{2\,a^3 + 3\,b^3}{3\,a^3 - 4\,b^3}$

14

$\because \frac{a^2 + b^2}{b^2} = \frac{b^2 + c^2}{c^2} \qquad \therefore a^2\,c^2 + b^2\,c^2 = b^4 + b^2\,c^2$

$\therefore a^2\,c^2 = b^4 \qquad \therefore a\,c = b^2$

$\therefore$ b is the middle proportional between a and c

15

Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = d\,m \;,\; b = d\,m^2 \;,\; a = d\,m^3$

$\therefore (b + c)^2 = (d\,m^2 + d\,m)^2 = \left(d\,m\,(m + 1)\right)^2$

$= d^2\,m^2\,(m + 1)^2$ (1)

$, (a + b)\,(c + d) = (d\,m^3 + d\,m^2)\,(d\,m + d)$

$= d\,m^2\,(m + 1) \times d\,(m + 1) = d^2\,m^2\,(m + 1)^2$ (2)

From (1) and (2) $\therefore (b + c)^2 = (a + b)(c + d)$

$\therefore$ (b + c) is the middle proportional between (a + b) and (c + d)

16

Let $\frac{5\,a}{6\,b} = \frac{6\,b}{7\,c} = \frac{7\,c}{8\,d} = m$ where $m > 0$

$\therefore 7\,c = 8\,d\,m \,,\, 6\,b = 8\,d\,m^2 \,,\, 5\,a = 8\,d\,m^3$

$\therefore \sqrt[3]{\frac{5\,a}{8\,d}} = \sqrt[3]{\frac{8\,d\,m^3}{8\,d}} = \sqrt[3]{m^3} = m$ (1)

$, \sqrt{\frac{5\,a + 6\,b}{7\,c + 8\,d}} = \sqrt{\frac{8\,d\,m^3 + 8\,d\,m^2}{8\,d\,m + 8\,d}}$

$= \sqrt{\frac{8\,d\,m^2\,(m + 1)}{8\,d\,(m + 1)}} = \sqrt{m^2} = m$ (2)

From (1) and (2) : $\therefore \sqrt[3]{\frac{5\,a}{8\,d}} = \sqrt{\frac{5\,a + 6\,b}{7\,c + 8\,d}}$

17

$\because \frac{a}{b} = \frac{b}{c} = m \qquad \therefore b = c\,m \,,\, a = c\,m^2$

$\therefore \frac{a^4 + b^4 + c^4}{a^{-4} + b^{-4} + c^{-4}} = \frac{c^4\,m^8 + c^4\,m^4 + c^4}{c^{-4}\,m^{-8} + c^{-4}\,m^{-4} + c^{-4}}$

$= \frac{c^4\,m^8\,(1 + m^{-4} + m^{-8})}{c^{-4}\,(m^{-8} + m^{-4} + 1)} = c^8\,m^8$

$\because b^8 = c^8\,m^8 \qquad \therefore \frac{a^4 + b^4 + c^4}{a^{-4} + b^{-4} + c^{-4}} = b^8$

18

Let $\frac{x}{y} = \frac{y}{z} = m \qquad \therefore y = z\,m \,,\, x = z\,m^2$

$\because x + y = 15 \qquad \therefore z\,m + z\,m^2 = 15$

$\therefore z\,m\,(1 + m) = 15 \qquad \therefore 1 + m = \frac{15}{z\,m}$ (1)

$\because y + z = 22.5 \qquad \therefore z\,m + z = 22.5$

$\therefore z\,(m + 1) = 22.5 \qquad \therefore m + 1 = \frac{22.5}{z}$ (2)

From (1) and (2) $\therefore \frac{15}{z\,m} = \frac{22.5}{z}$

$\therefore 22.5\,m = 15 \qquad \therefore m = \frac{15}{22.5} = \frac{2}{3}$

$\therefore x : y = 2 : 3$

19

Let $\frac{m(\angle A)}{m(\angle B)} = \frac{m(\angle B)}{m(\angle C)} = e$

$\therefore m(\angle B) = m(\angle C) \times e$

, $m(\angle A) = m(\angle C) \times e^2$

$\because m(\angle A) + m(\angle B) + m(\angle C) = 180^\circ$

$\therefore m(\angle C) \times e^2 + m(\angle C) \times e + m(\angle C) = 180^\circ$

$\therefore 60^\circ e^2 + 60^\circ e + 60^\circ = 180^\circ$

$\therefore e^2 + e + 1 = 3$

$\therefore e^2 + e - 2 = 0$

$\therefore (e+2)(e-1) = 0$

$\therefore e = -2$ (refused) or $e = 1$

$\therefore m(\angle A) = 60^\circ \times 1^2 = 60^\circ$

, $m(\angle B) = 60^\circ \times 1 = 60^\circ$

20

$\because \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = 2$

$\therefore c = 2d$, $b = 4d$, $a = 8d$

$\because a x^2 - 2 b x + c = 0$

$\therefore 8 d x^2 - 8 d x + 2 d = 0$

dividing by 2 d $\qquad \therefore 4x^2 - 4x + 1 = 0$

$\therefore (2x-1)^2 = 0 \qquad \therefore 2x - 1 = 0$

$\therefore x = \frac{1}{2} \qquad \therefore$ The S.S $= \left\{\frac{1}{2}\right\}$

21

$\because$ The number 5 is the middle proportional between x and y

$\therefore xy = 25 \qquad (1)$

Let the middle proportional between $\left(x+\frac{1}{y}\right)$ and $\left(y+\frac{1}{x}\right)$ be z

$\therefore z^2 = \left(x+\frac{1}{y}\right)\left(y+\frac{1}{x}\right) = xy + 1 + 1 + \frac{1}{xy}$

$= xy + \frac{1}{xy} + 2$

and from (1) : $\therefore z^2 = 25 + \frac{1}{25} + 2 = 27.04$

$\therefore z = \pm\sqrt{27.04} = \pm 5.2$

Answers of Exercise 8

1

1 m y where m ≠ 0 $\qquad$ 2 $\frac{1}{x^2}$ $\qquad$ 3 $\frac{y}{y_2}$

4 $\frac{x_2}{x_1}$ $\qquad$ 5 x $\qquad$ 6 x

7 y $\qquad$ 8 $\frac{1}{y}$ $\qquad$ 9 3

10 5 $\qquad$ 11 $\frac{1}{2}$ $\qquad$ 12 $\frac{3}{2}$

13 directly , inversely

2

$\because y \propto x \qquad \therefore \frac{y_1}{y_2} = \frac{x_1}{x_2}$

$\therefore \frac{20}{40} = \frac{7}{x_2} \qquad \therefore x_2 = \frac{40 \times 7}{20} = 14$

3

$\because a \propto \frac{1}{b} \qquad \therefore \frac{a_1}{a_2} = \frac{b_2}{b_1}$

1 $\frac{12}{a_2} = \frac{1.5}{8} \qquad \therefore a_2 = \frac{8 \times 12}{1.5} = 64$

2 $\frac{12}{2} = \frac{b_2}{8} \qquad \therefore b_2 = \frac{8 \times 12}{2} = 48$

4

1 $\because y \propto x \qquad \therefore y = m x$

$\therefore 14 = 42\,m \qquad \therefore m = \frac{1}{3}$

$\therefore y = \frac{1}{3} x$ (The relation between x , y)

2 As $x = 60 \qquad \therefore y = \frac{1}{3} \times 60 = 20$

5

1 $\because y \propto \frac{1}{x} \qquad \therefore x y = m$

$\therefore 3 \times 2 = m \qquad \therefore m = 6$

$\therefore x y = 6$ (The relation between x , y)

2 As $x = 1.5 \qquad \therefore y = \frac{6}{1.5} = 4$

6

$\because y \propto \frac{1}{x} \qquad \therefore x y = m$

$\therefore m = 3 \times 10 = 30 \qquad \therefore x y = 30 \qquad \therefore y = \frac{30}{x}$

As $x = 1 \qquad \therefore y = \frac{30}{1} = 30$

As $x = 2 \qquad \therefore y = \frac{30}{2} = 15$

As $x = 3 \qquad \therefore y = \frac{30}{3} = 10$

As $x = 4 \qquad \therefore y = \frac{30}{4} = 7.5$

As $x = 5 \qquad \therefore y = \frac{30}{5} = 6$

7

$y \propto x^2$ $\quad$ ∵ $y = m x^2$

$4 = m(3)^2$ $\quad$ ∴ $m = \frac{4}{9}$

∴ $y = \frac{4}{9} x^2$ (The relation between x and y)

As $x = 9$ $\quad$ ∴ $y = \frac{4}{9} \times 9^2 = 36$

8

∵ $y \propto x^3$ $\quad$ $y = m x^3$

∴ $64 = m(2)^3$ $\quad$ ∴ $m = 8$

∴ $y = 8 x^3$ (The relation between x and y)

As $x = \frac{1}{2}$ $\quad$ ∴ $y = 8\left(\frac{1}{2}\right)^3 = 1$

9

∵ $y \propto \frac{1}{\sqrt{x}}$ $\quad$ ∴ $\frac{y_1}{y_2} = \frac{\sqrt{x_2}}{\sqrt{x_1}}$

∴ $\frac{2}{y_2} = \frac{\sqrt{32}}{\sqrt{16}}$ $\quad$ ∴ $y_2 = \frac{2 \times 4}{4\sqrt{2}} = \sqrt{2}$

10

$y^2 \propto x^3$ $\quad$ ∴ $y^2 = m x^3$ $\quad$ $y = 3$ as $x = 2$

∴ $9 = 8m$ $\quad$ ∴ $m = \frac{9}{8}$ $\quad$ ∴ $y^2 = \frac{9}{8} x^3$

11

$y^2 \propto \frac{1}{\sqrt[3]{x}}$ $\quad$ ∴ $\left(\frac{y_1}{y_2}\right)^2 = \sqrt[3]{\frac{x_2}{x_1}}$

∴ $\left(\frac{3}{1.5}\right)^2 = \sqrt[3]{\frac{x_2}{8}}$ $\quad$ ∴ $4 = \frac{\sqrt[3]{x_2}}{2}$

$\sqrt[3]{x_2} = 8$ $\quad$ ∴ $x_2 = 512$

12

∵ $y \propto (x+1)$ $\quad$ ∴ $y = m(x+1)$

∵ $y = 2$, $x = 3$ $\quad$ ∴ $2 = m(3+1)$

∴ $m = \frac{1}{2}$ $\quad$ ∴ $y = \frac{1}{2}(x+1)$

13

∵ $\frac{5x - 3y}{3x + 5y} = 1$ $\quad$ ∴ $5x - 3y = 3x + 5y$

$2x = 8y$

∴ $y = \frac{1}{4} x$ $\quad$ ∴ $y \propto x$

14

∵ $\frac{a + 2b}{6} = \frac{b + 3c}{3}$ $\quad$ ∴ $3a + 6b = 6b + 18c$

$3a = 18c$ $\quad$ ∴ $a = 6c$ $\quad$ ∴ $a \propto c$

15

∵ $\frac{21x - y}{7x - z} = \frac{y}{z}$ $\quad$ ∴ $21xz - zy = 7xy - zy$

∴ $21xz = 7xy$ $\quad$ ∴ $3z = y$ $\quad$ ∴ $y \propto z$

16

∵ $x^2y^2 - 6xy + 9 = 0$ $\quad$ ∴ $(xy - 3)^2 = 0$

∴ $xy = 3$ $\quad$ ∴ $y \propto \frac{1}{x}$

17

∵ $4a^2 - 12ab + 9b^2 = 0$

∴ $(2a - 3b)^2 = 0$ $\quad$ ∴ $2a - 3b = 0$ $\quad$ ∴ $2a = 3b$

$a = \frac{3}{2} b$ $\quad$ ∴ $a \propto b$

18

$x^4y^2 - 14x^2y + 49 = 0$

∴ $(x^2y - 7)^2 = 0$ $\quad$ ∴ $x^2y - 7 = 0$

∴ $x^2y = 7$ $\quad$ $y \propto \frac{1}{x^2}$

19

$(4x + 7y) \propto (x + 2y)$

∴ $4x + 7y = m(x + 2y)$

$4x + 7y = mx + 2my$

∴ $7y - 2my = mx - 4x$

∴ $y(7 - 2m) = x(m - 4)$

∴ $y = \frac{m - 4}{7 - 2m} x$

putting $\frac{m - 4}{7 - 2m} = k \in \mathbb{R}^*$

∴ $y = kx$ $\quad$ ∴ $y \propto x$

20

∵ $\left(\frac{a}{y} - \frac{a}{x}\right) \propto x - y$ $\quad$ ∴ $\frac{a}{y} - \frac{a}{x} = m(x - y)$

$\frac{ax - ay}{xy} = m(x - y)$ $\quad$ ∴ $\frac{a(x - y)}{xy} = m(x - y)$

$xy = \frac{a}{m}$ (constant) $\quad$ ∴ x varies inversely as y

21

The first table represents an inverse variation

because , $3 \times 20 = 60$, $5 \times 12 = 60$, $4 \times 15 = 60$

, $6 \times 10 = 60$ i.e. $x\,y = m$

The second table represents a direct variation

because : $\frac{9}{2} = \frac{18}{4} = \frac{54}{12} = \frac{72}{16}$ i.e. $\frac{y}{x} = m$

The third table represents a direct variation

because : $\frac{9}{5} = \frac{18}{10} = \frac{27}{15} = \frac{45}{25}$ i.e. $\frac{y}{x} = m$

The fourth table does not represent a direct variation nor an inverse variation because

$3 \times 6 \neq -18 \times 1$ or $\frac{6}{3} \neq \frac{-9}{-2}$

i.e. $x\,y \neq m$ ∴ The variation is not inverse

or $\frac{y}{x} \neq m$ ∴ The variation is not direct

22

[1] The variation is inverse

[2] $\because y \propto \frac{1}{x}$ $\therefore y\,x = m$ $\therefore m = 12$

[3] As $x = 3$ $\therefore 3\,y = 12$ $\therefore y = 4$

[4] As $y = 2\frac{2}{5}$ $\therefore \left(2\frac{2}{5}\right) x = 12$

$\therefore \frac{12}{5}\,x = 12$ $\therefore x = 12 \times \frac{5}{12} = 5$

23

[1] Direct variation

[2] $\because y \propto x$ $\therefore \frac{y}{x} = m$ $\therefore m = 12$

$\therefore \frac{a}{2} = 12$ $\therefore a = 24$

, $\frac{36}{b} = 12$ $\therefore b = 3$

24

$\because y = z + 5$, $z \propto \frac{1}{x}$ $\therefore z = \frac{m}{x}$

$\therefore y = \frac{m}{x} + 5$

At $y = 6$, $x = 2$

$\therefore 6 = \frac{m}{2} + 5$ $\therefore 1 = \frac{m}{2}$ $\therefore m = 2$

$\therefore y = \frac{2}{x} + 5$

At $x = 1$ $\therefore y = \frac{2}{1} + 5 = 7$

25

$\because y = a + b$, $b \propto x$ $\therefore b = m\,x$

$\therefore y = a + m\,x$ At $y = 3$, $x = 0$

$\therefore 3 = a + m \times 0$ $\therefore a = 3$

$\therefore y = 3 + m\,x$ At $y = 5$, $x = 3$

$\therefore 5 = 3 + m \times 3$ $\therefore m = \frac{2}{3}$

$\therefore y = 3 + \frac{2}{3}\,x$

At $x = 7$ $\therefore y = 3 + \frac{2}{3} \times 7 = 7\frac{2}{3}$

26

$\because y = a - 9$, $y \propto \frac{1}{x^2}$ $\therefore y = \frac{m}{x^2}$

$\therefore \frac{m}{x^2} = a - 9$ $\therefore m = x^2 (a - 9)$

$\because a = 18$ as $x = \frac{2}{3}$ $\therefore m = \frac{4}{9}(18 - 9)$

$\therefore m = \frac{4}{9} \times 9 = 4$ $\therefore y = \frac{4}{x^2}$

As $x = 1$ $\therefore y = 4$

27

[1] $\because y = 2 + a$, $a \propto \frac{1}{x}$ $\therefore a = \frac{m}{x}$

At $a = 5$, $x = 2$

$\therefore 5 = \frac{m}{2}$ $\therefore m = 10$

$\therefore a = \frac{10}{x}$ $\therefore y = 2 + \frac{10}{x}$

[2] At $x = 5$ $\therefore y = 2 + \frac{10}{5} = 4$

28

$\because x = \ell + 9$, $\ell \propto y$ $\therefore \ell = m\,y$

$\therefore x = m\,y + 9$ As $x = 24$, $y = 5$

$\therefore 24 = 5\,m + 9$ $\therefore 5\,m = 15$

$\therefore m = 3$ $\therefore \ell = 3\,y$

As $\ell = 12$ $\therefore 12 = 3\,y$ $\therefore y = 4$

29

[1] d	[2] d	[3] d	[4] d
[5] c	[6] b	[7] d	[8] c
[9] b	[10] b	[11] a	[12] d
[13] c	[14] a	[15] c	[16] d

30

$h \propto \frac{1}{r^2}$ $\qquad \frac{h_1}{h_2} = \frac{r_2^2}{r_1^2}$

$\therefore \frac{27}{h_2} = \frac{(15.75)^2}{(10.5)^2}$ $\qquad \therefore h_2 = \frac{27 \times (10.5)^2}{(15.75)^2} = 12$ cm.

31

$d \propto t$ $\qquad \therefore \frac{d_1}{d_2} = \frac{t_1}{t_2}$

$\frac{150}{d_2} = \frac{6}{10}$ $\qquad \therefore d_2 = \frac{150 \times 10}{6} = 250$ km

32

$W \propto R$ $\qquad \therefore \frac{W_1}{W_2} = \frac{R_1}{R_2}$

$\therefore \frac{14}{W_2} = \frac{84}{144}$ $\qquad W_2 = \frac{14 \times 144}{84} = 24$ kg

33

$n \propto \frac{1}{X}$ $\qquad \frac{n_1}{n_2} = \frac{X_2}{X_1}$

$\therefore \frac{4}{n_2} = \frac{8}{6}$ $\qquad \therefore n_2 = \frac{4 \times 6}{8} = 3$ hours.

34

$\therefore d \propto t^2$ $\qquad \therefore \frac{d_1}{d_2} = \frac{t_1^2}{t_2^2}$

$\frac{\frac{81}{16}}{144} = \frac{\frac{1}{16}}{t_2^2}$ $\qquad \therefore t_2^2 = \frac{144 \times \frac{1}{16}}{\frac{81}{16}} = \frac{16}{9}$

$\therefore t_2 = \frac{4}{3} = 1\frac{1}{3}$ hour

35

$v \propto \frac{1}{r^2}$ $\qquad \therefore \frac{v_1}{v_2} = \frac{r_2^2}{r_1^2}$

$\frac{5}{v_2} = \frac{(2.5)^2}{3^2}$ $\qquad \therefore v_2 = \frac{5 \times 3^2}{(2.5)^2} = 7.2$ cm./s

36

Let the weight of the body = w , and the distance from the centre of the earth = d

$\therefore w \propto \frac{1}{d^2}$ $\qquad \therefore \frac{w_1}{w_2} = \frac{d_2^2}{d_1^2}$

$\therefore \frac{500}{w_1} = \frac{(640 + 6390)^2}{(6390)^2}$ $\qquad \therefore w_2 = 413$ w.kg

37

$\because X \propto y$, $z \propto \ell$ $\quad \therefore X = m\,y$, $z = k\,\ell$

$(X + y)(z + \ell) = (m\,y + y)(k\,\ell + \ell)$

$= y\,\ell\,(m + 1)(k + 1)$ $\qquad (1)$

, $(X - y)(z - \ell) = (m\,y - y)(k\,\ell - \ell)$

$= y\,\ell\,(m - 1)(k - 1)$ $\qquad (2)$

Dividing (1) by (2)

$\frac{(X + y)(z + \ell)}{(X - y)(z - \ell)} = \frac{y\,\ell\,(m + 1)(k + 1)}{y\,\ell\,(m - 1)(k - 1)} = \frac{(m + 1)(k + 1)}{(m - 1)(k - 1)}$

= constant

$\therefore (X + y)(z + \ell) \propto (X - y)(z - \ell)$

38

$\because (a + b) \propto \frac{a}{b}$ $\qquad \therefore (a + b) = \frac{m\,a}{b}$ $\qquad (1)$

, $\because (a^2 - a\,b + b^2) \propto \frac{b}{a}$ $\qquad \therefore (a^2 - a\,b + b^2) = \frac{\ell\,b}{a}$ $\quad (2)$

, multiplying (1) by (2) :

$\therefore (a + b)(a^2 - a\,b + b^2) = \frac{m\,a}{b} \times \frac{\ell\,b}{a}$

$\therefore a^3 + b^3 = \ell\,m =$ constant

Answers of exams on unit two

Model 1

1

1 c	2 d	3 a
4 d	5 b	6 c

2

[a] $\frac{28}{3}$

[b] Prove by yourself

3

[a] 1 $x^2 y = 36$ 2 $x = \pm 2$

[b] Prove by yourself

4

[a] 3

[b] $y = 1 + \frac{2}{x}$, $y = 4$

5

[a] Prove by yourself

[b] 4

Model 2

1

1 a	2 c	3 a
4 c	5 c	6 c

2

[a] Prove by yourself

[b] 1 $y = \frac{16}{x^2} - 1$ 2 $x = \pm \frac{4}{3}$

3

[a] Prove by yourself

[b] Prove by yourself

4

[a] 3

[b] Prove by yourself

5

[a] 1 Prove by yourself, 2 $x = 4$

[b] Prove by yourself

Answers of unit three

Answers of Exercise 9

1

1 c	2 c	3 d
4 b	5 b	6 b

2

The primary sources : 1 and 2

The secondary sources : 3 , 4 and 5

3

Side of comparison / The method	Mass population	Samples
Its definition	It is setup collecting data related to the phenomenon from all the individuals of the statistical society	It is setup collecting data about the phenomenon under study from some individuals of the statistical society not all the individuals , this by selecting a sample representing all statistical society
Advantages	Accuracy , perfect representation of all statistical society	1 It is faster and less cost 2 It is the unique method for collecting data from the large societies (infinite) 3 It is the unique method for collecting data from some limited societies
Disadvantages	Sometimes it needs a long time and more costs	The results are not accurated specially if the sample does not represent the statistical society very well

4

The method of mass population : 1 and 5

The method of samples : 2 , 3 and 4

5 : **7** Answer by yourself

8

The total number of students

$= 4\,000 + 3\,000 + 2\,000 + 1\,000 = 10\,000$ students

The number of the individuals of the first layer in the sample $= \frac{4\,000}{10\,000} \times 500 = 200$ students

The number of the individuals of the second layer in the sample $= \frac{3\,000}{10\,000} \times 500 = 150$ students

The number of the individuals of the third layer in the sample $= \frac{2\,000}{10\,000} \times 500 = 100$ students

The number of the individuals of the fourth layer in the sample $= \frac{1\,000}{10\,000} \times 500 = 50$ students

9

The total number of cars $= 300 + 500 + 200$

$= 1000$ cars

The number of individuals of the sample $= 1000 \times 5\%$

$= 50$ cars

The number of the first model in the sample

$= \frac{300}{1000} \times 50 = 15$ cars

The number of the second model in the sample

$= \frac{500}{1000} \times 50 = 25$ cars

The number of the third model in the sample

$= \frac{200}{1000} \times 50 = 10$ cars

10

The number of the second layer

$= 5\,000 - 1\,500 = 3\,500$ individuals

The number of individuals of all the sample

$= \frac{5\,000 \times 140}{3\,500} = 200$ individuals

11

The size of the whole sample $= \frac{40\,000 \times 240}{12\,000}$

$= 800$ individuals

12

Number of the layer	1	2	3	4	Total
Number of individuals of the layer	500	700	350	450	2000
Number of individuals of the layer in the sample	10	14	7	9	40

Answers of Exercise 10

1

1 c	2 a	3 a	4 c	5 b
6 a	7 b	8 c	9 c	10 b
11 d	12 c	13 c	14 b	15 d
16 c	17 c	18 a	19 c	

2

1 The mean $(\overline{x}) = \frac{16+32+5+20+27}{5} = 20$

x	$x-\overline{x}$	$(x-\overline{x})^2$
16	$16-20=-4$	16
32	$32-20=12$	144
5	$5-20=-15$	225
20	$20-20=0$	0
27	$27-20=7$	49
Total		434

The standard deviation $(\sigma) = \sqrt{\frac{434}{5}} \simeq 9.3$

2 The mean $(\overline{x}) = \frac{72+53+61+70+59}{5} = 63$

x	$x-\overline{x}$	$(x-\overline{x})^2$
72	$72-63=9$	81
53	$53-63=-10$	100
61	$61-63=-2$	4
70	$70-63=7$	49
59	$59-63=-4$	16
Total		250

The standard deviation $(\sigma) = \sqrt{\frac{250}{5}} \simeq 7.1$

3 The mean $(\overline{x}) = \frac{15+(-12)+(-9)+27+(-6)}{5} = 3$

x	$x-\overline{x}$	$(x-\overline{x})^2$
15	$15-3=12$	144
-12	$-12-3=-15$	225
-9	$-9-3=-12$	144
27	$27-3=24$	576
-6	$-6-3=-9$	81
Total		1170

The standard deviation $(\sigma) = \sqrt{\frac{1170}{5}} \simeq 15.3$

4 The mean $(\overline{x}) = \frac{22+20+20+20+18}{5} = 20$

x	$x-\overline{x}$	$(x-\overline{x})^2$
22	$22-20=2$	4
20	$20-20=0$	0
20	$20-20=0$	0
20	$20-20=0$	0
18	$18-20=-2$	4
Total		8

The standard deviation $(\sigma) = \sqrt{\frac{8}{5}} \simeq 1.3$

3

- The mean of the set (A) $= \frac{7+8+9+10+11}{5} = 9$

x	$x-\overline{x}$	$(x-\overline{x})^2$
7	$7-9=-2$	4
8	$8-9=-1$	1
9	$9-9=0$	0
10	$10-9=1$	1
11	$11-9=2$	4
Total		10

The standard deviation (σ) of the set (A) $= \sqrt{\frac{10}{5}} \simeq 1.4$

- The mean of the set (B) $= \frac{21+20+11+19}{4} = 17.75$

x	$x-\overline{x}$	$(x-\overline{x})^2$
21	$21-17.75=3.25$	10.5625
20	$20-17.75=2.25$	5.0625
11	$11-17.75=-6.75$	45.5625
19	$19-17.75=1.25$	1.5625
Total		62.75

The standard deviation of the set (B) $= \sqrt{\frac{62.75}{4}} \simeq 4$

- The mean of the set (C) $= \frac{29+30+30+35}{4} = 31$

x	$x-\overline{x}$	$(x-\overline{x})^2$
29	$29-31=-2$	4
30	$30-31=-1$	1
30	$30-31=-1$	1
35	$35-31=4$	16
Total		22

The standard deviation of the set (C) $= \sqrt{\frac{22}{4}} \simeq 2.3$

$\therefore$ The set B has more dispersion

4

1 The mean $(\overline{x}) = \frac{73+54+62+71+60}{5} = 64$

x	$x-\overline{x}$	$(x-\overline{x})^2$
73	$73-64=9$	81
54	$54-64=-10$	100
62	$62-64=-2$	4
71	$71-64=7$	49
60	$60-64=-4$	16
Total		250

The standard deviation $(\sigma) = \sqrt{\frac{250}{5}} \simeq 7.07$

2 The mean $(\overline{x}) = \frac{13+14+17+19+22}{5} = 17$

x	$x-\overline{x}$	$(x-\overline{x})^2$
13	$13-17=-4$	16
14	$14-17=-3$	9
17	$17-17=0$	0
19	$19-17=2$	4
22	$22-17=5$	25
Total		54

The standard deviation $(\sigma) = \sqrt{\frac{54}{5}} = 3.286$

3 The mean $(\overline{x})$

$= \frac{65+61+70+64+70+76+70}{7} = 68$

x	$x-\overline{x}$	$(x-\overline{x})^2$
65	$65-68=-3$	9
61	$61-68=-7$	49
70	$70-68=2$	4
64	$64-68=-4$	16
70	$70-68=2$	4
76	$76-68=8$	64
70	$70-68=2$	4
Total		150

The standard deviation $(\sigma) = \sqrt{\frac{150}{7}} \simeq 4.6$

4 The mean $(\overline{x})$

$= \frac{23+12+17+13+15+16+8+9+37+10}{10} = 16$

x	$x-\overline{x}$	$(x-\overline{x})^2$
23	$23-16=7$	49
12	$12-16=-4$	16
17	$17-16=1$	1
13	$13-16=-3$	9
15	$15-16=-1$	1
16	$16-16=0$	0
8	$8-16=-8$	64
9	$9-16=-7$	49
37	$37-16=21$	441
10	$10-16=-6$	36
Total		666

The standard deviation $(\sigma) = \sqrt{\frac{666}{10}} = 8.2$

5

1 The mean of the marks of pupils

$= \frac{8+9+6+12+10}{5} = 9$

2

x	$x-\overline{x}$	$(x-\overline{x})^2$
8	$8-9=-1$	1
9	$9-9=0$	0
6	$6-9=-3$	9
12	$12-9=3$	9
10	$10-9=1$	1
Total		20

The standard deviation (σ) of the marks of pupils $= \sqrt{\frac{20}{5}} = 2$

6

1 The mean of the maximum degrees $(\overline{x})$

$= \frac{25+26+24+24+22+26+27+26}{8} = 25$ degrees

x	$x-\overline{x}$	$(x-\overline{x})^2$
25	$25-25=0$	0
26	$26-25=1$	1
24	$24-25=-1$	1
24	$24-25=-1$	1
22	$22-25=-3$	9
26	$26-25=1$	1
27	$27-25=2$	4
26	$26-25=1$	1
Total		18

The standard deviation $(\sigma) = \sqrt{\frac{18}{8}} = 1.5$ degrees

6 The mean of the minimum degrees $(\overline{x})$

$$= \frac{11+12+10+6+7+16+15+11}{8} = 11 \text{ degrees}$$

x	$x - \overline{x}$	$(x-\overline{x})^2$
11	$11 - 11 = 0$	0
12	$12 - 11 = 1$	1
10	$10 - 11 = -1$	1
6	$6 - 11 = -5$	25
7	$7 - 11 = -4$	16
16	$16 - 11 = 5$	25
15	$15 - 11 = 4$	16
11	$11 - 11 = 0$	0
Total		84

The standard deviation $(\sigma) = \sqrt{\frac{84}{8}} \simeq 3.2$ degrees

7

Number of children (X)	Number of families (k)	$X \times k$
0	8	0
1	16	16
2	50	100
3	20	60
4	6	24
Total	100	200

The mean $(\overline{x}) = \frac{200}{100} = 2$ children

x	k	$x - \overline{x}$	$(x-\overline{x})^2$	$(x-\overline{x})^2 \times k$
0	8	$0 - 2 = -2$	4	32
1	16	$1 - 2 = -1$	1	16
2	50	$2 - 2 = 0$	0	0
3	20	$3 - 2 = 1$	1	20
4	6	$4 - 2 = 2$	4	24
Total	100			92

The standard deviation $(\sigma) = \sqrt{\frac{92}{100}} \simeq 1$ child

8

Number of defective units (X)	Number of boxes (k)	$X \times k$
0	3	0
1	16	16
2	17	34
3	25	75
4	20	80
5	19	95
Total	100	300

The mean $(\overline{x}) = \frac{300}{100} = 3$ units

x	k	$x - \overline{x}$	$(x-\overline{x})^2$	$(x-\overline{x})^2 \times k$
0	3	$0 - 3 = -3$	9	27
1	16	$1 - 3 = -2$	4	64
2	17	$2 - 3 = -1$	1	17
3	25	$3 - 3 = 0$	0	0
4	20	$4 - 3 = 1$	1	20
5	19	$5 - 3 = 2$	4	76
Total	100			204

The standard deviation $(\sigma) = \sqrt{\frac{204}{100}} \simeq 1.4$ units

9

Number of goals (X)	Number of players (k)	$X \times k$
0	2	0
1	4	4
2	5	10
3	8	24
4	7	28
5	4	20
Total	30	86

The mean $(\overline{x}) = \frac{86}{30} \simeq 2.9$ goals

x	k	$x - \overline{x}$	$(x-\overline{x})^2$	$(x-\overline{x})^2 \times k$
0	2	$0 - 2.9 = -2.9$	8.41	16.82
1	4	$1 - 2.9 = -1.9$	3.61	14.44
2	5	$2 - 2.9 = -0.9$	0.81	4.05
3	8	$3 - 2.9 = 0.1$	0.01	0.08
4	7	$4 - 2.9 = 1.1$	1.21	8.47
5	4	$5 - 2.9 = 2.1$	4.41	17.64
Total	30			61.5

The standard deviation $(\sigma) = \sqrt{\frac{61.5}{30}} \simeq 1.4$ goals

10

Age (X)	Number of children (k)	X × k
5	1	5
8	2	16
9	3	27
10	3	30
12	1	12
Total	10	90

The mean $\left(\overline{x}\right) = \frac{90}{10} = 9$ years

x	k	$x-\overline{x}$	$(x-\overline{x})^2$	$(x-\overline{x})^2 \times k$
5	1	$5-9=-4$	16	16
8	2	$8-9=-1$	1	2
9	3	$9-9=0$	0	0
10	3	$10-9=1$	1	3
12	1	$12-9=3$	9	9
Total	10			30

The standard deviation $(\sigma) = \sqrt{\frac{30}{10}} \simeq 1.7$ years

11

Number of students (X)	Number of classes (k)	X × k
0	1	0
1	3	3
2	5	10
3	6	18
4	3	12
5	2	10
Total	20	53

The mean $\left(\overline{x}\right) = \frac{53}{20} = 2.65$ students

x	k	$x-\overline{x}$	$(x-\overline{x})^2$	$(x-\overline{x})^2 \times k$
0	1	$0-2.65=-2.65$	7.0225	7.0225
1	3	$1-2.65=-1.65$	2.7225	8.1675
2	5	$2-2.65=-0.65$	0.4225	2.1125
3	6	$3-2.65=0.35$	0.1225	0.735
4	3	$4-2.65=1.35$	1.8225	5.4675
5	2	$5-2.65=2.35$	5.5225	11.045
Total	20			34.55

The standard deviation $(\sigma) = \sqrt{\frac{34.55}{20}} \simeq 1.3$ student

12

Sets	Centres of sets (X)	Frequency (k)	X × k
5 –	10	7	70
15 –	20	9	180
25 –	30	11	330
35 –	40	15	600
45 –	50	8	400
Total		50	1580

The mean $\left(\overline{x}\right) = \frac{1580}{50} = 31.6$

x	k	$x-\overline{x}$	$(x-\overline{x})^2$	$(x-\overline{x})^2 \times k$
10	7	$10-31.6=-21.6$	466.56	3265.92
20	9	$20-31.6=-11.6$	134.56	1211.04
30	11	$30-31.6=-1.6$	2.56	28.16
40	15	$40-31.6=8.4$	70.56	1058.4
50	8	$50-31.6=18.4$	338.56	2708.48
Total	50			8272

The standard deviation $(\sigma) = \sqrt{\frac{8272}{50}} \simeq 12.9$

13

Sets	Centres of sets (X)	Frequency (k)	X × k
0 –	2	3	6
4 –	6	4	24
8 –	10	7	70
12 –	14	2	28
16 – 20	18	9	162
Total		25	290

The mean $\left(\overline{x}\right) = \frac{290}{25} = 11.6$

x	k	$x-\overline{x}$	$(x-\overline{x})^2$	$(x-\overline{x})^2 \times k$
2	3	$2-11.6=-9.6$	92.16	276.48
6	4	$6-11.6=-5.6$	31.36	125.44
10	7	$10-11.6=-1.6$	2.56	17.92
14	2	$14-11.6=2.4$	5.76	11.52
18	9	$18-11.6=6.4$	40.96	368.64
Total	25			800

The standard deviation $(\sigma) = \sqrt{\frac{800}{25}} \simeq 5.7$

14

Sets	Centres of sets (X)	Frequency (k)	X × k
20 –	25	10	250
30 –	35	12	420
40 –	45	8	360
50 –	55	6	330
60 –	65	3	195
70 –	75	1	75
Total		40	1630

The mean $\left(\overline{X}\right) = \frac{1630}{40} = 40.75$ pounds

X	k	$X - \overline{X}$	$\left(X - \overline{X}\right)^2$	$\left(X - \overline{X}\right)^2 \times k$
25	10	25 − 40.75 = − 15.75	248.0625	2480.625
35	12	35 − 40.75 = − 5.75	33.0625	396.75
45	8	45 − 40.75 = 4.25	18.0625	144.5
55	6	55 − 40.75 = 14.25	203.0625	1218.375
65	3	65 − 40.75 = 24.25	588.0625	1764.1875
75	1	75 − 40.75 = 34.25	1173.0625	1173.0625
Total	40			7177.5

The standard deviation $(\sigma) = \sqrt{\frac{7177.5}{40}} = 13.4$ pounds

15

Sets	Centres of sets (X)	Frequency (k)	X × k
5 –	6	3	18
7 –	8	6	48
9 –	10	10	100
11 –	12	12	144
13 –	14	5	70
15 – 17	16	4	64
Total		40	444

The mean $\left(\overline{X}\right) = \frac{444}{40} = 11.1$ km/litre

X	k	$X - \overline{X}$	$\left(X - \overline{X}\right)^2$	$\left(X - \overline{X}\right)^2 \times k$
6	3	6 − 11.1 = − 5.1	26.01	78.03
8	6	8 − 11.1 = − 3.1	9.61	57.66
10	10	10 − 11.1 = − 1.1	1.21	12.1
12	12	12 − 11.1 = 0.9	0.81	9.72
14	5	14 − 11.1 = 2.9	8.41	42.05
16	4	16 − 11.1 = 4.9	24.01	96.04
Total	40			295.6

The standard deviation $(\sigma) = \sqrt{\frac{295.6}{40}} = 2.7$ km/litre

16

Sets	Centres of sets (X)	Frequency (k)	X × k
5 –	10	19	190
15 –	20	50	1000
25 –	30	85	2550
35 –	40	25	1000
45 –	50	15	750
55 –	60	6	360
Total		200	5850

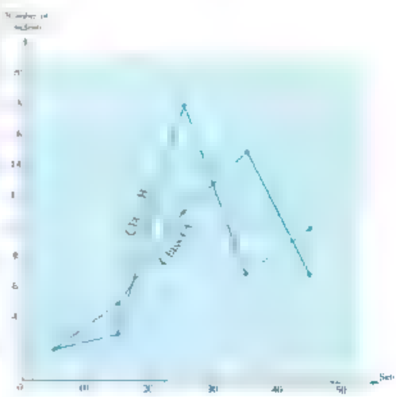
The mean of $\left(\overline{X}\right) = \frac{5850}{200} = 29.25$ pounds

X	k	$X - \overline{X}$	$\left(X - \overline{X}\right)^2$	$\left(X - \overline{X}\right)^2 \times k$
10	19	10 − 29.25 = − 19.25	370.5625	7040.6875
20	50	20 − 29.25 = − 9.25	85.5625	4278.125
30	85	30 − 29.25 = 0.75	0.5625	47.8125
40	25	40 − 29.25 = 10.75	115.5625	2889.0625
50	15	50 − 29.25 = 20.75	430.5625	6458.4375
60	6	60 − 29.25 = 30.75	945.5625	5673.375
Total	200			26387.5

The standard deviation $(\sigma) = \sqrt{\frac{26387.5}{200}} = 11.5$ pounds

17

1



[2] With respect to class (A)

Sets	Centres of sets (X)	Frequency (k)	X × k
0	5	2	10
10 –	15	5	75
20 –	25	11	275
30 –	35	15	525
40 50	45	7	315
Total		40	1200

The mean $(\overline{X})$ of class (A) = $\frac{1200}{40}$ = 30 marks

X	k	$X - \bar{X}$	$(X-\bar{X})^2$	$(X-\bar{X})^2 \times k$
5	2	5 30 = – 25	625	1250
15	5	15 30 = 15	225	1125
25	11	25 30 = 5	25	275
35	15	35 30 = 5	25	375
45	7	45 30 = 15	225	1575
Total	40			[illegible]

The standard deviation (σ) of class (A)

$= \sqrt{\frac{4600}{40}} \simeq 10.7$ marks

With respect to class (B)

Sets	Centres of sets (X)	Frequency (k)	X × k
0 –	5	2	10
10 –	15	3	45
20 –	25	18	450
30 –	35	7	245
40 – 50	45	10	450
Total		40	1200

The mean $(\overline{X})$ of class (B) = $\frac{1200}{40}$ = 30 marks

X	k	$X - \bar{X}$	$(X-\bar{X})^2$	$(X-\bar{X})^2 \times k$
5	2	5 – 30 = – 25	625	1250
15	3	15 – 30 = – 15	225	675
25	18	25 - 30 = – 5	25	450
35	7	35 - 30 = 5	25	175
45	10	45 30 = 15	225	2250
Total	40			4800

The standard deviation (σ) of class (B)

$\sqrt{\frac{4800}{40}} \simeq 11$ marks

[3] Class A is the most homogeneous in getting marks.

Answers of exam on unit three

1

[1] a [2] a [3] b

[4] b [5] d [6] a

2 $\bar{x} = 20$, $\sigma \simeq 9.32$

3 $\bar{x} = 4.88$, $\sigma \simeq 1.7$

4 $\sigma \simeq 2.32$

Answers of accumulative basic skills

[1] d [2] c [3] b [4] d [5] c

[6] d [7] c [8] d [9] b [10] c

[11] b [12] b [13] c [14] a [15] b

[16] d [17] c [18] c [19] a [20] a

[21] c [22] d [23] d [24] c [25] c

[26] c [27] c [28] c [29] b [30] a

[31] d [32] c [33] a [34] c [35] c

GUIDE ANSWERS

# of Trigonometry and Geometry Exercises

Answers of unit four

Answers of Exercise 1

1

1. $\frac{15}{17}$, $\frac{8}{17}$ 2. $\frac{8}{17}$, $\frac{15}{17}$ 3. $\frac{15}{8}$, $\frac{8}{15}$

2

1. c 2. b 3. c 4. d 5. c
6. a 7. d 8. d 9. c 10. b
11. b

3

Let the measures of the two angles be $3X$ and $5X$

$\therefore 3X + 5X = 180°$ $\therefore 8X = 180°$

$\therefore X = \frac{180°}{8} = 22.5°$

The measure of the first angle $= 3 \times 22.5° = 67.5°$
$= 67° 30'$

The measure of the second angle $= 5 \times 22.5°$
$= 112.5° = 112° 30'$

4

Let the measures of the two angles be $3X$ and $4X$

$3X + 4X = 90°$ $7X = 90°$

$\therefore X = \frac{90°}{7} = 12\frac{6°}{7}$

The measure of the greater angle

$= 4 \times 12\frac{6°}{7} = 51° 25' 43''$

5

Let the measures of the interior angles of the triangle be $3X$, $4X$, $7X$

$\therefore 3X + 4X + 7X = 180°$ $\therefore 14X = 180°$

$\therefore X = \frac{180°}{14} = 12\frac{6°}{7}$

The measure of the first angle

$= 3 \times 12\frac{6°}{7} = 38° 34' 17''$

The measure of the second angle

$= 4 \times 12\frac{6°}{7} = 51° 25' 43''$

The measure of the third angle $= 7 \times 12\frac{6°}{7} = 90°$

6

$m(\angle A) = 90°$ $(BC)^2 = (20)^2 + (15)^2 = 625$

$\therefore BC = 25$ cm

$\therefore \cos C \cos B - \sin C \sin B = \frac{15}{25} \times \frac{20}{25} - \frac{20}{25} \times \frac{15}{25} = 0$

7

$m(\angle Z) = 90°$

$(ZY)^2 = (25)^2 - (7)^2$
$= 576$

$ZY = 24$ cm

1. $\tan X \times \tan Y = \frac{24}{7} \times \frac{7}{24} = 1$

2. $\sin^2 X + \sin^2 Y = \left(\frac{24}{25}\right)^2 + \left(\frac{7}{25}\right)^2 = \frac{625}{625} = 1$

8

$m(\angle B) = 90°$

$\therefore (AB)^2 = (5)^2 - (4)^2 = 9$

$AB = 3$ cm

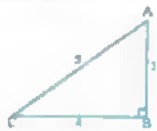
$\sin^2 A - \cos^2 A$

$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

$2\sin^2 A - 1 = 2 \times \left(\frac{4}{5}\right)^2 - 1 = 2 \times \frac{16}{25} - 1 = \frac{7}{25}$

$\sin^2 A - \cos^2 A = 2\sin^2 A$

9



$\frac{AB}{AC} = \frac{3}{5}$

$\therefore$ Let $AB = 3$ length unit

, $AC = 5$ length unit

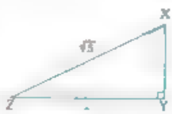
$\because m(\angle B) = 90°$

$(BC)^2 = 5^2 - 3^2 = 16$ $\therefore BC = 4$ length unit

$\sin A = \frac{BC}{AC} = \frac{4}{5}$, $\cos A = \frac{AB}{AC} = \frac{3}{5}$

, $\tan A = \frac{BC}{AB} = \frac{4}{3}$

10



$YZ = 2\,XY$ $\therefore \frac{YZ}{XY} = \frac{2}{1}$

Let $YZ = 2$ length unit

, $XY = 1$ length unit

$XZ = \sqrt{5}$ length unit

$\therefore \tan Z = \frac{1}{2}$, $\tan X = 2$

, $\cos Z = \frac{2}{\sqrt{5}}$, $\cos X = \frac{1}{\sqrt{5}}$

$\because 2\ AB = \sqrt{3}\ AC$ $\qquad \therefore \frac{AB}{AC} = \frac{\sqrt{3}}{2}$

Let $AB = \sqrt{3}$ length unit

, $AC = 2$ length unit $\therefore BC = 1$ length unit

$\therefore \sin C = \frac{\sqrt{3}}{2}$, $\cos C = \frac{1}{2}$, $\tan C = \sqrt{3}$

12

1. $\because \tan C = \frac{AB}{BC}$ $\qquad \therefore \frac{3}{4} = \frac{6}{BC}$

$\therefore BC = 8$ cm

, $\because (AC)^2 = (AB)^2 + (BC)^2$

$\therefore (AC)^2 = 36 + 64 = 100$ $\qquad \therefore AC = 10$ cm.

2. $\sin A + \cos A = \frac{8}{10} + \frac{6}{10} = \frac{14}{10} = \frac{7}{5}$

13

Δ ACD is right-angled at D

$(AD)^2 = (17)^2 - (15)^2 = 64$ $\qquad \therefore AD = 8$ cm

$\therefore 3 \tan C + \sin B = 3 \times \frac{8}{15} + \frac{8}{10} = \frac{12}{5}$

14

1. In Δ ABC : $\because m(\angle BAC) = 90°$

$\therefore (BC)^2 = 36 + 64 = 100$ $\therefore BC = 10$ cm.

, $\because \overline{AD} \perp \overline{BC}$ $\qquad \therefore AD = \frac{6 \times 8}{10} = 4.8$ cm.

, $(AB)^2 = BD \times BC$ $\qquad 36 = BD \times 10$

$\therefore BD = 3.6$ cm

$\therefore \tan(\angle BAD) = \frac{3.6}{4.8} = \frac{3}{4}$

2. $\cos(\angle DAC) + \cos(\angle DAB)$

$= \frac{4.8}{8} + \frac{4.8}{6} = \frac{7}{5}$

15

From Δ ABD : $\because \cos B = \frac{BD}{AB}$

, from Δ ACD : $\because \cos C = \frac{CD}{AC}$

$\therefore AB \cos B + AC \cos C = AB \times \frac{BD}{AB} + AC \times \frac{CD}{AC}$

$= BD + CD = 8$ cm

16

In Δ ABD , $\because m(\angle A) = 90°$

$(AD)^2 = (BD)^2 - (AB)^2 = 100 - 36 = 64$

$AD = 8$ cm.

$\therefore \tan(\angle ADB) = \frac{6}{8} = \frac{3}{4}$

, $\because \overline{AD} \parallel \overline{BC}$, $\overrightarrow{BD}$ is a transversal to them

$\therefore m(\angle ADB) = m(\angle DBC)$ (alternate angles)

$\therefore \tan(\angle ADB) = \tan(\angle DBC)$

$\therefore \tan(\angle DBC) = \frac{3}{4}$ $\qquad \frac{DC}{10} = \frac{3}{4}$

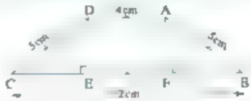
$\therefore DC = \frac{10 \times 3}{4} = 7.5$ cm

17

Draw $\overline{AF} \perp \overline{BC}$,

$\overline{DE} \perp \overline{BC}$

, $\quad \overline{AD} \parallel \overline{BC}$,



AFED is a rectangle , FE = 4 cm.

$\therefore BF + EC = 8$ cm

, $\because BF = EC = 4$ cm

(Δ ABF and Δ DCE are congruent)

$\therefore$ from Δ ABF which is right-angled at F :

$(AF)^2 = (5)^2 - (4)^2 = 9$

$\therefore AF = 3$ cm. $\qquad \therefore DE = AF = 3$ cm.

(AFED is a rectangle)

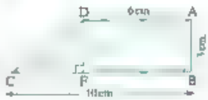
$\therefore \frac{5 \tan B \cos C}{\sin^2 C + \cos^2 B} = \frac{5 \times \frac{3}{4} \times \frac{4}{5}}{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2} = 3$

18

Draw $\overline{DF} \perp \overline{BC}$

$\therefore \overline{AD} \parallel \overline{BC}$, $\overline{AB} \perp \overline{BC}$

, $\overline{DF} \perp \overline{BC}$



ABFD is a rectangle

BF = AD = 6 cm

FC = 4 cm , DF = AB = 3 cm

$\therefore$ From Δ DFC which is right-angled at F :

$(DC)^2 = 3^2 + 4^2 = 25$ $\qquad \therefore DC = 5$ cm

$\therefore \cos(\angle DCB) - \tan(\angle ACB) = \frac{4}{5} - \frac{3}{10} = \frac{1}{2}$

19

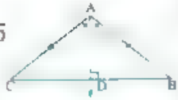
$4\ DE = 3\ EC$ $\qquad \frac{DE}{EC} = \frac{3}{4}$

$\tan C = \frac{DE}{EC} = \frac{3}{4}$, $\tan C = \frac{AB}{BC}$

$\frac{AB}{BC} = \frac{3}{4}$ $\qquad \frac{9}{BC} = \frac{3}{4}$ $\qquad \therefore BC = 12$ cm

$\therefore$ The area of Δ ABC $= \frac{1}{2} \times 12 \times 9 = 54\ cm^2$

20



Bisect $\angle A$ by the bisector $\overline{AD}$

$\because \triangle ABC$ is an isosceles triangle

$\overline{AD} \perp \overline{BC}$

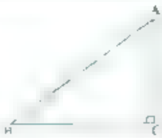
$\sin \frac{A}{2} = \sin (\angle BAD)$

$\sin (\angle BAD) = \frac{4}{5}$

$\angle B$, $\angle BAD$ are acute angles

$\therefore \cos B = \sin (\angle BAD)$ $\quad \therefore \cos B = \frac{4}{5}$

21



$\sin B = \frac{AC}{AB}$

$, \cos B = \frac{BC}{AB}$

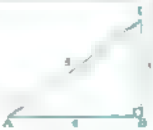
$\sin B + \cos B = \frac{AC}{AB} + \frac{BC}{AB}$

$= \frac{AC + BC}{AB}$

From triangle inequality

$AC + BC > AB$ $\quad \therefore \sin B + \cos B > 1$

22



$\because \sin A = \frac{6}{10} = \frac{3}{5}$

$\frac{BC}{AC} = \frac{3}{5}$

Assuming that

$BC = 3$ length unit , $AC = 5$ length unit

$\therefore AB = 4$ length unit

$\sin A \cos C + \cos A \sin C$

$= \frac{3}{5} \times \frac{3}{5} + \frac{4}{5} \times \frac{4}{5} = \frac{9}{25} + \frac{16}{25} = 1$

23

$7 \tan A - 24 = 0$

$\therefore 7 \tan A = 24$

$\tan A = \frac{24}{7}$

$\therefore \frac{BC}{AB} = \frac{24}{7}$

Assuming that $BC = 24$ length unit

$\therefore AB = 7$ length unit $\qquad AC = 25$ length unit

$\therefore 1 - \tan A \sin C = 1 - \frac{24}{7} \times \frac{7}{25} = \frac{1}{25}$

24

1 $\because m (\angle 1) = X$ (corresponding angles)

$\because \tan (\angle 1) = \frac{2}{5}$

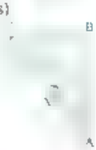
$\tan X = \frac{2}{5}$

2 $m (\angle 1) = X$ (alternate angles)

$\because \tan (\angle 1) = \frac{3}{2}$

$\tan X = \frac{3}{2}$

3 $m (\angle 1) = X$ (corresponding angles)



$\triangle ABC$ is right-angled at B

$(AC)^2 = 3^2 + 4^2 = 25$

$AC = 5$ length units

$\cos (\angle 1) = \frac{4}{5}$

$\cos X = \frac{4}{5}$

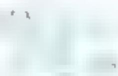
4 $m (\angle 1) = X$ (corresponding angles)

$\because \tan (\angle 1) = \frac{2}{5}$

$\tan X = \frac{2}{5}$

5 $\because m (\angle 1) = X$ (corresponding angles)

$, m (\angle 2) = y$ (corresponding angles)



$\because \tan (\angle 1) = \frac{3}{2}$, $\tan (\angle 2) = \frac{5}{2}$

$\tan X + \frac{1}{\tan y} = \frac{3}{2} + \frac{2}{5} = \frac{19}{10}$

6 $\because m (\angle 1) = y$ (corresponding angles)

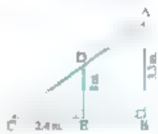
$, m (\angle 2) = z$ (corresponding angles)

$\tan X + \tan y - \tan z$

$= \tan X + \tan (\angle 1) - \tan (\angle 2)$

$= \frac{3}{2} + \frac{2}{3} - \frac{3}{1} = -\frac{4}{3}$

25



$\tan C = \frac{AB}{BC} = \frac{DE}{BC}$

$\frac{3.3}{BC} = \frac{1.8}{2.4}$

$BC = 4.4$ m

$\therefore BE = 4.4 - 2.4 = 2$ m

The distance between the man and the base of the lamppost = 2 m

26

$(AB)^2 = (3.6)^2 + (4.8)^2 = 36$

$AB = 6$ km

$\because m (\angle BAD) + m (\angle DAC) = 90°$

$\therefore \sin (\angle BAD) = \cos (\angle DAC)$ $\quad \therefore \frac{BD}{AB} = \frac{AD}{AC}$

$\therefore \frac{3.6}{6} = \frac{4.8}{AC}$ $\quad \therefore AC = 8$ km

$\therefore (DC)^2 = (8)^2 - (4.8)^2 = 40.96$ $\quad \therefore DC = 6.4$ km

Try to solve this problem by using Euclidean theorem

27

From Δ ABC : $\cos B = \frac{18}{BC}$

, $\because$ H is the midpoint of $\overline{BC}$

$\therefore$ BC = 2 BH

$\therefore \cos B = \frac{18}{2\ BH} = \frac{9}{BH}$ (1)

, from Δ BDH $\cos B = \frac{BH}{13}$ (2)

From (1) and (2) $\therefore \frac{9}{BH} = \frac{BH}{13}$

$\therefore (BH)^2 = 9 \times 13$ $\therefore BH = 3\sqrt{13}$ cm.

, $\because$ m ($\angle$ BHD) = 90°

$\therefore (DH)^2 = (BD)^2 - (BH)^2 = 169 - 117 = 52$

$\therefore DH = 2\sqrt{13}$ cm $\therefore \tan B = \frac{2\sqrt{13}}{3\sqrt{13}} = \frac{2}{3}$

Another Solution :

Construction Draw : $\overline{CD}$

Proof : In Δ BCD

$\because \overline{DH} \perp \overline{BC}$, BH = CH

$\therefore$ BD = CD = 13 cm.

In Δ ACD : $\because$ m ($\angle$ A) = 90°

$\therefore (AC)^2 = (CD)^2 - (AD)^2 = 169 - 25 = 144$

$\therefore$ AC = 12 cm.

In Δ ABC : $\tan B = \frac{AC}{AB} = \frac{12}{18} = \frac{2}{3}$

28

Draw $\overline{CE} \perp \overline{BD}$

In Δ CBD : $\because$ CB = CD

, $\overline{CE} \perp \overline{BD}$

$\therefore$ E is the midpoint of $\overline{BD}$

$\therefore$ ED = 9 cm.

In Δ EDC which is right-angled at E

$(CE)^2 = (15)^2 - (9)^2 = 144$

$\therefore$ CE = 12 cm.

$\therefore \tan (\angle BAC) = \frac{EC}{AB} = \frac{12}{15} = \frac{4}{5}$

29

Let DE = ℓ cm. $\therefore$ AE = $(5 - \ell)$ cm.

, $\because$ m ($\angle$ AEB) = m ($\angle$ DCE)

$\therefore$ tan ($\angle$ AEB) = tan ($\angle$ DCE)

$\therefore \frac{2}{5 - \ell} = \frac{\ell}{2}$ $\therefore 5\ell - \ell^2 = 4$

$\therefore \ell^2 - 5\ell + 4 = 0$ $\therefore (\ell - 4)(\ell - 1) = 0$

$\therefore \ell = 4$ or $\ell = 1$

, $\because$ AE < ED $\therefore$ ED = 4 cm

$\therefore \tan (\angle CED) = \frac{2}{4} = \frac{1}{2}$

30

$\angle$ BAD and $\angle$ CAD are two acute angles

, sin ($\angle$ BAD) = cos ($\angle$ CAD) $\therefore$ m ($\angle$ CAB) = 90°

, $\because \sin (\angle BAD) = \frac{DB}{AB} = \frac{3}{5}$ $\therefore \frac{9}{AB} = \frac{3}{5}$

$\therefore$ AB = 15 cm

, $\because (AD)^2 = (15)^2 - (9)^2 = 144$

$\therefore$ AD = 12 cm.

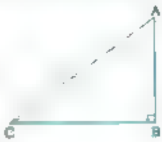
, $\overline{AD} \perp \overline{BC}$, $\overline{CA} \perp \overline{AB}$

$\therefore (AB)^2 = BD \times BC$ (Euclidean theorem)

$\therefore (15)^2 = 9 \times BC$ $\therefore$ BC = 25 cm

$\therefore$ The area of Δ ABC $= \frac{1}{2} \times 25 \times 12 = 150$ cm^2

31



$\because \Delta$ ABC is right-angled at B

$\therefore \sin A = \frac{BC}{AC}$

$\therefore \sin^2 A = \frac{(BC)^2}{(AC)^2}$

, $\sin C = \frac{AB}{AC}$ $\therefore \sin^2 C = \frac{(AB)^2}{(AC)^2}$

$\therefore \sin^2 A + \sin^2 C = \frac{(BC)^2}{(AC)^2} + \frac{(AB)^2}{(AC)^2} = \frac{(BC)^2 + (AB)^2}{(AC)^2}$

, $(BC)^2 + (AB)^2 = (AC)^2$ (Pythagoras)

$\therefore \sin^2 A + \sin^2 C = \frac{(AC)^2}{(AC)^2} = 1$

32

$\overline{AD} \parallel \overline{BO}$, $\overline{EZ} \parallel \overline{BO}$ $\therefore \overline{AD} \parallel \overline{EZ}$

$\therefore$ m ($\angle$ DAY) = m ($\angle$ AZE) (alternate angles)

, $\because$ DC = 12 cm. , CE = 4 cm.

$\therefore$ DE = 8 cm.

Let EY = X cm. $\therefore$ DY = $(8 - X)$ cm.

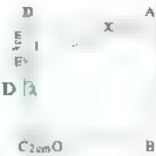
$\therefore$ tan ($\angle$ DAY) = tan ($\angle$ AZE)

$\therefore \frac{8 - X}{12} = \frac{X}{4}$ $\therefore \frac{8 - X}{3} = X$

$\therefore 8 - X = 3X$ $\therefore 8 = 4X$

$\therefore X = 2$ $\therefore \tan (\angle AZE) = \frac{EY}{EZ} = \frac{2}{4} = \frac{1}{2}$

33



∵ m (∠ AEO) = 90°

∴ m (∠ 1) + m (∠ 2) = 90°

In Δ ADE which is right-angled at D

∴ X + m (∠ 1) = 90°

X = m (∠ 2)

Let the side length of the square be ℓ

EC = $\ell - 3$ ∴ tan X = tan (∠ 2)

∴ $\frac{3}{\ell} = \frac{2}{\ell - 3}$ ∴ $2\ell = 3\ell - 9$

∴ $\ell = 9$ tan $X = \frac{DE}{AD} = \frac{3}{9} = \frac{1}{3}$

34

Draw : $\overline{BM} \perp \overline{AC}$

to intersect it at M

∵ AC = 8 cm

, M is the midpoint of $\overline{AC}$

BM = $\frac{1}{2}$ AC = 4 cm

(properties of the square)

, AM = $\frac{1}{2}$ AC = 4 cm

∴ EM = 2 cm , MC = 4 cm

∴ MO = 3 cm

∴ tan X + tan y = $\frac{4}{2} + \frac{4}{3} = 3\frac{1}{3}$

Answers of Exercise 2

1

1. $\sin 45° - \cos 45° = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 0$

2. $\cos 60° + \sin 30° = \frac{1}{2} + \frac{1}{2} = 1$

3. $\sin 30° + \cos 60° - \tan 45° = \frac{1}{2} + \frac{1}{2} - 1 = 0$

4. $\sin 60° + \cos 30° + \tan 60° = \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} + \sqrt{3} = 2\sqrt{3}$

5. $\sin^2 45° + \cos^2 45° = \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} + \frac{1}{2} = 1$

6. $4 \cos 30° \tan 60° = 4 \times \frac{\sqrt{3}}{2} \times \sqrt{3} = 6$

7. $\tan^2 60° - 2 \sin 45° \cos 45°$

$= (\sqrt{3})^2 - 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 3 - 1 = 2$

8. $\sin^2 60° - \tan 60° \cos 30° + \cos 60° \sin 30°$

$= \left(\frac{\sqrt{3}}{2}\right)^2 - \sqrt{3} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} - \frac{3}{2} + \frac{1}{4} = -\frac{1}{2}$

9. $2 \sin 30° \cos 60° + \sqrt{2} \sin 45°$

$= 2 \times \frac{1}{2} \times \frac{1}{2} + \sqrt{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2} + 1 = 1\frac{1}{2}$

10. $(\cos 30° - \cos 60°)(\sin 30° + \sin 60°)$

$= \left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) = \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2$

$= \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$

11. $\frac{\sin 30°}{\cos 60°} - \cos 30° \sin 60° = \frac{\frac{1}{2}}{\frac{1}{2}} - \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

$= 1 - \frac{3}{4} = \frac{1}{4}$

12. $\frac{\cos^2 60° + \cos^2 30° + \tan^2 45°}{\sin 60° \tan 60° - \sin 30°}$

$\frac{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + 1^2}{\frac{\sqrt{3}}{2} \times \sqrt{3} - \frac{1}{2}} = \frac{\frac{1}{4} + \frac{3}{4} + 1}{\frac{3}{2} - \frac{1}{2}} = 2$

2

1. The left side = $\sin 60° = \frac{\sqrt{3}}{2}$

The right side = $2 \sin 30° \cos 30°$

$= 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

The two sides are equal

2. The left side = $\cos 60° = \frac{1}{2}$

The right side = $2 \cos^2 30° - 1$

$= 2\left(\frac{\sqrt{3}}{2}\right)^2 - 1 = 2 \times \frac{3}{4} - 1 = \frac{1}{2}$

∴ The two sides are equal

3. The left side = $2 \cos^2 30° - 1 = 2 \times \left(\frac{\sqrt{3}}{2}\right)^2 - 1$

$= 2 \times \frac{3}{4} - 1 = \frac{1}{2}$

The right side = $1 - 2 \sin^2 30° = 1 - 2 \times \left(\frac{1}{2}\right)^2$

$= 1 - 2 \times \frac{1}{4} = \frac{1}{2}$

∴ The two sides are equal.

4. The left side = $\cos 60° = \frac{1}{2}$

The right side = $\cos^2 30° - \sin^2 30°$

$= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$

∴ The two sides are equal.

5 The left side $= \tan 60° = \sqrt{3}$

The right side $= \dfrac{2\tan 30°}{1 - \tan^2 30°}$

$$= \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}} = \sqrt{3}$$

∴ The two sides are equal.

6 The left side $= \cos^2 60° = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

The right side $= 5\sin^2 30° - \tan^2 45°$

$$= 5\left(\frac{1}{2}\right)^2 - 1^2 = 5 \times \frac{1}{4} - 1 = \frac{1}{4}$$

∴ The two sides are equal

7 The left side $= \sin^3 30° = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

The right side $= 9\cos^3 60° - \tan 45°$

$$= 9 \times \left(\frac{1}{2}\right)^3 - 1$$

$$= 9 \times \frac{1}{8} - 1 = \frac{9}{8} - \frac{8}{8} = \frac{1}{8}$$

∴ The two sides are equal

8 The left side $= \dfrac{\sin 30° \cos 45° + \sin 45° \cos 30°}{\sin 45° \cos 60° + \cos 45° \sin 60°}$

$$= \frac{\frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}} \times \frac{1}{2} + \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}}$$

$$= \frac{\frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}}{\frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}} = 1$$

The right side $= \tan^2 45° = 1^2 = 1$

∴ The two sides are equal

9 The left side $= \sin 30° = \frac{1}{2}$

The right side $= \sqrt{\dfrac{1 - \cos 60°}{2}} = \sqrt{\dfrac{1 - \frac{1}{2}}{2}} = \sqrt{\dfrac{\frac{1}{2}}{2}}$

$$= \sqrt{\frac{1}{4}} = \frac{1}{2}$$

∴ The two sides are equal

1 b	2 d	3 c	4 b	5 c
6 d	7 a	8 b	9 d	10 c
11 c	12 d	13 d	14 b	15 c
16 d	17 a			

1 $X \times \left(\frac{1}{\sqrt{2}}\right)^2 = \left(\sqrt{3}\right)^2$ ∴ $\frac{1}{2} X = 3$ ∴ $X = 6$

2 $X \times \frac{1}{2} \times \left(\frac{1}{\sqrt{2}}\right)^2 = \left(\frac{\sqrt{3}}{2}\right)^2$

∴ $\frac{1}{4} X = \frac{3}{4}$ ∴ $X = 3$

3 $X \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times \sqrt{3} = (1)^2 - \left(\frac{1}{2}\right)^2$

$\frac{\sqrt{3}}{2} X = \frac{3}{4}$ ∴ $X = \frac{\sqrt{3}}{2}$

4 $4X = \left(\frac{\sqrt{3}}{2}\right)^2 \times \left(\frac{1}{\sqrt{3}}\right)^2 \times (1)^2$

$4X = \frac{1}{4}$ ∴ $X = \frac{1}{16}$

1 $\tan X = 4 \times \frac{1}{2} \times \frac{1}{2}$ ∴ $\tan X = 1$ ∴ $X = 45°$

2 $\sin X = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2}$ ∴ $\sin X = \frac{1}{2}$

∴ $X = 30°$

3 $2\sin X = \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$ ∴ $2\sin X = 1$

$\sin X = \frac{1}{2}$ ∴ $X = 30°$

4 $6 \times \sin X \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 1 - \frac{1}{4}$

∴ $6 \times \frac{1}{2} \times \sin X = \frac{3}{4}$ ∴ $3 \times \sin X = \frac{3}{4}$

$\sin X = \frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$ ∴ $X \approx 14° 28' 39''$

5 ∴ $\cos X = \dfrac{\frac{\sqrt{3}}{2} \times \frac{1}{2}}{1 \times \left(\frac{1}{\sqrt{2}}\right)^2}$ ∴ $\cos X = \frac{\sqrt{3}}{2}$ ∴ $X = 30°$

6 $\cos(3X + 6°) = \frac{1}{2}$ ∴ $3X + 6° = 60°$

$3X = 54°$ ∴ $X = \frac{54}{3} = 18°$

7 $\sqrt{3} \times \sin X \times \frac{1}{\sqrt{3}} = 1 \times \cos 2X$

∴ $\sin X = \cos 2X$ ∴ $X + 2X = 90°$

∴ $3X = 90°$ ∴ $X = 30°$

1 ∴ $\left(\frac{1}{\sqrt{2}}\right)^2 = \cos E \times \frac{1}{\sqrt{3}}$

$\cos E = \frac{1}{2} \div \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{2}$ ∴ $E = 30°$

2 $\sin E \times \left(\frac{\sqrt{3}}{2}\right)^2 = 3 \times \left(\frac{1}{\sqrt{2}}\right)^2 \times \left(\frac{1}{\sqrt{2}}\right)^2 \times \frac{1}{2}$

∴ $\sin E \times \frac{3}{4} = \frac{3}{8}$ ∴ $\sin E = \frac{1}{2}$

∴ $E = 30°$

[3] $3 \tan E \quad 4 \times \left(\frac{1}{2}\right)^2 = 8 \times \left(\frac{1}{2}\right)^2$

$\therefore 3 \tan E \quad 1 \quad 2$

$3 \tan E = 3 \qquad \tan E \quad 1 \qquad E = 45°$

7

$\tan X = \frac{1}{\sqrt{3}} \qquad X = 30°$

$\sin X \tan\left(\frac{3X}{2}\right) + \cos(2X)$

$= \sin 30° \tan\left(\frac{3 \times 30°}{2}\right) + \cos(2 \times 30°)$

$= \sin 30° \tan 45° + \cos 60° = \frac{1}{2} \times 1 + \frac{1}{2} \quad 1$

8

$\because \sin X = \tan 30° \sin 60° \qquad \sin X = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{2}$

$\sin X = \frac{1}{2} \qquad X = 30°$

$\therefore 4 \cos X \sin X = 4 \cos 30° \sin 30°$

$= 4 \times \frac{\sqrt{3}}{2} \times \frac{1}{2} = \sqrt{3}$

9

Angle / Ratio	30°	60°	45°
sin	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$
tan	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	1

Angle / Ratio	34° 12	51° 33 36	65° 45 13	36° 52 12
sin	0.562	0.7833	0.9118	0.6
cos	0.827	0.6217	0.4107	0.8
tan	0.6796	1.2599	2.2204	0.75

10

[1] $\because \tan 32° = \frac{6}{BC}$

$\therefore BC = 6 \div \tan 32° \simeq 9.6$ cm

[2] $\cos 50° = \frac{AB}{8}$

$AB = 8 \times \cos 50° \simeq 5.14$ cm

[3] $\sin 65° = \frac{BC}{12}$

$BC = 12 \times \sin 65° \simeq 10.88$ cm

11

[1] $\sin A = \frac{4}{6} \qquad m(\angle A) \simeq 41° 48' 37''$

[2] $\tan C = \frac{10}{6} \qquad m(\angle C) \simeq 59° 2' 10''$

[3] $\cos C = \frac{5}{7} \qquad m(\angle C) \simeq 44° 24' 55''$

12

In ΔACD

$m(\angle DAC) = 90°$, $m(\angle D) = 30°$

$\therefore AC = \frac{1}{2} CD = \frac{1}{2} \times 8 = 4$ cm

$\therefore \tan B = \frac{AC}{BC} = \frac{4}{3}$ (First req.)

In ΔABC

$\because \tan(\angle BAC) = \frac{3}{4} \qquad \therefore m(\angle BAC) \simeq 36° 52' 12''$

$\therefore m(\angle BAD) \simeq 90° + 36° 52' 12''$

$\simeq 126° 52' 12''$ (Second req.)

13

Draw $\overline{AD} \perp \overline{BC}$ to cut it at D

$\because \overline{AD} \perp \overline{BC}$, $AB = AC$

$\therefore BD = DC = 5$ cm

In ΔABD : $\cos B = \frac{5}{7}$

$m(\angle B) \simeq 44° 24' 55''$ (First req.)

In ΔABD : $\because (AD)^2 = (AB)^2 - (BD)^2$

$(AD)^2 = 49 - 25 = 24 \qquad AD = 2\sqrt{6}$ cm

The area of $\Delta ABC = \frac{1}{2} \times 10 \times 2\sqrt{6}$

$= 10\sqrt{6}$ cm^2 (Second req.)

14

Draw $\overline{AD} \perp \overline{BC}$ to cut it at D

$\because \overline{AD} \perp \overline{BC}$, $AB = AC$

$BD = DC$

In ΔADC , $\cos C = \frac{DC}{AC}$

$\therefore \cos 84° 24' = \frac{DC}{12.6}$

$DC = 12.6 \times \cos 84° 24' \simeq 1.23$ cm

$BC = 2 \times 1.23 = 2.46 \simeq 2.5$ cm (The req.)

15

ΔABC is a right-angled triangle at B

$m(\angle A) + m(\angle C) = 90°$

$\because m(\angle A) = 2\, m(\angle C)$

$\therefore 2\, m(\angle C) + m(\angle C) = 90°$

$\therefore 3\, m(\angle C) = 90° \qquad \therefore m(\angle C) = 30°$

$\therefore m(\angle A) = 60°$

$\therefore \cos^2 A + \tan^2 C = \cos^2 60° + \tan^2 30°$

$= \frac{1}{4} + \frac{1}{3} = \frac{3+4}{12} = \frac{7}{12}$

16

$\because$ ABCD is a rectangle $\therefore$ m $(\angle B) = 90°$

In Δ ABC : $\therefore \sin (\angle ACB) = \frac{15}{25}$

m $(\angle ACB) \simeq 36° \, 52' \, 12''$ (First req.)

In Δ ABC

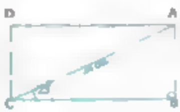
$\because (BC)^2 = (AC)^2 - (AB)^2$

$(BC)^2 = 625 - 225 = 400$ $\therefore$ BC = 20 cm

The area of the rectangle = $15 \times 20 = 300 \text{ cm}^2$

(Second req.)

17



$\because$ ABCD is a rectangle

$\therefore$ m $(\angle B) = 90°$

In Δ ABC :

$\because \cos (\angle ACB) = \frac{BC}{AC}$

$\cos 25° = \frac{BC}{24}$

$\therefore$ BC = 24 cos 25° $\simeq$ 21.8 cm. (The req.)

18

BC = 96 ÷ 8 = 12 cm

$\because$ AD = BC $\therefore$ AD = 12 cm. (First req.)

$\because$ BC = 12 cm. , BE = $\frac{1}{4}$BC $\therefore$ BE = 3 cm

In Δ ABE

$\because$ m $(\angle AEB) = 90°$ $\therefore \tan B = \frac{AE}{BE} = \frac{8}{3}$

m $(\angle B) = 69° \, 26' \, 38''$ (Second req.)

In Δ AEB

$\sin B = \frac{AE}{AB}$ $\sin 69° \, 26' \, 38'' = \frac{8}{AB}$

$AB = \frac{8}{\sin 69° \, 26' \, 38''} \simeq 8.5$ cm. (Third req.)

Another solution :

In Δ AEB $\cos B = \frac{BE}{AB}$

$\therefore \cos 69° \, 26' \, 38'' = \frac{3}{AB}$

$AB = \frac{3}{\cos 69° \, 26' \, 38''} \simeq 8.5$ cm. (Third req.)

A third solution :

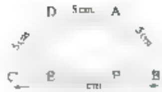
Δ ABE is right-angled at E

$\therefore (AB)^2 = (AE)^2 + (BE)^2$

$= 8^2 + 3^2 = 64 + 9 = 73$

$\therefore AB = \sqrt{73} \simeq 8.5$ cm. (Third req.)

19



Draw $\overline{AF} \perp \overline{BC}$, $\overline{DE} \perp \overline{BC}$

$\overline{AD} \,//\, \overline{BC}$, $\overline{AF} \perp \overline{BC}$,

$\overline{DE} \perp \overline{BC}$

$\therefore$ AFED is a rectangle $\therefore$ FE = 5 cm

$\because$ BF + EC = 6 cm. $\therefore$ BF = 3 cm

In Δ ABF

$\cos B = \frac{BF}{AB}$ $\cos B = \frac{3}{5}$

$\therefore$ m $(\angle B) \simeq 53° \, 7' \, 48''$

m $(\angle A) = 180° - 53° \, 7' \, 48''$

$= 126° \, 52' \, 12''$ (First req.)

In Δ ABF

$(AF)^2 = (AB)^2 - (BF)^2$

$\therefore (AF)^2 = (5)^2 - (3)^2 = 16$ $\therefore$ AF = 4 cm

The area of the trapezium = $\frac{1}{2}(5 + 11) \times 4 = 32 \text{ cm}^2$

(Second req.)

20

Draw $\overline{DE} \perp \overline{BC}$

$\because \overline{AD} \,//\, \overline{BC}$, $\overline{AB} \perp \overline{BE}$

, $\overline{DE} \perp \overline{BE}$

$\therefore$ ABED is a rectangle.

BE = 16 cm

, EC = 25 − 16 = 9 cm , DE = 12 cm

In Δ DEC : $\because$ m $(\angle DEC) = 90°$

$(DC)^2 = (12)^2 + (9)^2 = 225$

DC = 15 cm. (First req.)

$\because \tan C = \frac{12}{9}$

$\therefore$ m $(\angle C) \simeq 53° \, 7' \, 48''$ (Second req.)

$\sin (\angle DCB) - \tan (\angle ACB) = \frac{12}{15} - \frac{12}{25} = \frac{8}{25}$

(Third req.)

21

In Δ ABC :

m $(\angle C) = 90°$

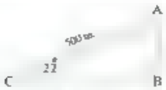
$\therefore \sin B = \frac{AC}{AB}$

$\therefore \sin 60° = \frac{AC}{6}$

$\therefore AC = 6 \sin 60° = 3\sqrt{3}$ m

(The req.)

In Δ ABC



$m(\angle B) = 90°$

$\sin C = \frac{AB}{AC}$ $\quad \sin 22° = \frac{AB}{500}$

$AB = 500 \sin 22° \simeq 187$ m (The req.)

In Δ ABC

$m(\angle B) = 90°$

$\cos A = \frac{AB}{AC}$

$\cos 60° = \frac{4}{AC}$

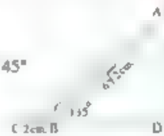
$\therefore AC = \frac{4}{\cos 60°} = 8$ m

$\because (BC)^2 = 8^2 - 4^2 = 48$ $\quad \therefore BC = 4\sqrt{3}$ m

$\therefore$ The height of the tree $= 8 + 4\sqrt{3} \simeq 15$ m

$\cos A \times \tan A = \frac{1}{2}$ $\quad \cos A \times \frac{\sin A}{\cos A} = \frac{1}{2}$

$\sin A = \frac{1}{2}$ $\quad A = 30°$



Draw $\overline{AD} \perp \overline{BC}$

$\therefore m(\angle ABD) = 180° - 135° = 45°$

$\because \tan(\angle ABD) = \frac{AD}{DB}$

$\tan 45° = \frac{AD}{DB} = 1$

$AD = DB = X$ cm

$\because$ In Δ ADB which is right-angled at D

$(AB)^2 = X^2 + X^2$ $\quad \therefore 72 = 2X^2$

$\therefore X^2 = 36$ $\quad \therefore X = 6$ cm

$\therefore \tan C = \frac{AD}{DC} = \frac{6}{8} = \frac{3}{4}$

Answers of exams on unit four

Model (1)

1

1 c $\quad$ 2 c $\quad$ 3 c

4 a $\quad$ 5 c $\quad$ 6 a

2 [a] Prove by yourself

[b] 1 $X = 7$ $\quad$ 2 $X = 30°$

3 [a] $\frac{3}{5}$

[b] 1

4 [a] 1 13 cm $\quad$ 2 7

[b] The height of the tree $\simeq 5$ m

5 [a] 30°

[b] $\frac{2\sqrt{3}}{3}$

Model (2)

1

1 c $\quad$ 2 c $\quad$ 3 d

4 d $\quad$ 5 d $\quad$ 6 c

2 [a] $X = 30°$

[b] $33° 45'$, $56° 15'$

3 [a] 1 $\cos B = \frac{3}{5}$

2 $m(\angle B) \simeq 53° 7' 48''$

3 $\sin(90° - B) = \frac{3}{5}$

[b] Prove by yourself

4 [a] 1 1 , $\sqrt{2}$

2 $\tan B = 1$, $\sin A = \frac{1}{\sqrt{2}}$

[b] 1 $\frac{56}{15}$ $\quad$ 2 $\frac{56}{65}$

5 [a] 1 Prove by yourself

2 $m(\angle C) \simeq 22° 37' 12''$

[b] $\frac{1}{2}$

Answers of unit five

Answers of Exercise 3

1

1. $AB = \sqrt{(4-1)^2 + (6-2)^2} = \sqrt{9+16} = 5$ length unit
2. $AB = \sqrt{(5-2)^2 + (-5+1)^2} = \sqrt{9+16} = 5$ length unit
3. $AB = \sqrt{(3+2)^2 + (-5-7)^2}$
   $= \sqrt{25+144} = 13$ length unit.
4. $AB = \sqrt{(3+2)^2 + (0-5)^2}$
   $= \sqrt{25+25} = 5\sqrt{2}$ length unit
5. $AB = \sqrt{(15-6)^2 + (0-0)^2} = \sqrt{9^2} = 9$ length unit.
6. $AB = \sqrt{(6-0)^2 + (0+8)^2}$
   $= \sqrt{36+64} = 10$ length unit.

2

1 d	2 c	3 c
4 a	5 b	6 c
7 c	8 d	9 c
10 d	11 a	12 c

3

$AB = \sqrt{(1-3)^2 + (2-1)^2} = \sqrt{4+1} = \sqrt{5}$ length unit

, $BC = \sqrt{(5-1)^2 + (4-2)^2} = \sqrt{16+4} = \sqrt{20}$

$= 2\sqrt{5}$ length unit.

$\because BC = 2\,AB$

4

$AB = \sqrt{(1-4)^2 + (1-3)^2} = \sqrt{9+4} = \sqrt{13}$ length unit

, $BC = \sqrt{(-5-1)^2 + (-3-1)^2} = \sqrt{36+16} = \sqrt{52}$

$= 2\sqrt{13}$ length unit

, $AC = \sqrt{(4+5)^2 + (3+3)^2} = \sqrt{81+36} = \sqrt{117}$

$= 3\sqrt{13}$ length unit

$\therefore AC = AB + BC$

$\because$ A , B and C are collinear

5

$\because CA = \sqrt{(3+2)^2 + (4-2)^2} = \sqrt{25+4} = \sqrt{29}$ length unit

, $CB = \sqrt{(3-1)^2 + (4+1)^2} = \sqrt{4+25} = \sqrt{29}$ length unit

$CA = CB$

$\therefore$ C lies on the axis of symmetry of $\overline{AB}$

6

1. $AB = \sqrt{(3-1)^2 + (-2-4)^2}$
   $= \sqrt{4+36} = \sqrt{40} = 2\sqrt{10}$ length unit.
   , $BC = \sqrt{(-3-3)^2 + (16+2)^2}$
   $= \sqrt{36+324} = \sqrt{360} = 6\sqrt{10}$ length unit.
   , $AC = \sqrt{(-3-1)^2 + (16-4)^2}$
   $= \sqrt{16+144} = \sqrt{160} = 4\sqrt{10}$ length unit.
   $BC = AB + AC$
   A , B and C are collinear points
2. $AB = \sqrt{(-3-7)^2 + (6-0)^2} = \sqrt{100+36}$
   $= 2\sqrt{34}$ length unit.
   , $BC = \sqrt{(22+3)^2 + (9-6)^2} = \sqrt{625+9}$
   $= \sqrt{634}$ length unit.
   , $AC = \sqrt{(22-7)^2 + (9-0)^2} = \sqrt{225+81}$
   $= 3\sqrt{34}$ length unit.
   $\because BC \neq AB + AC$
   $\therefore$ A , B and C are non-collinear points
3. $AB = \sqrt{(3+1)^2 + (-14-4)^2}$
   $= \sqrt{16+324} = 2\sqrt{85}$ length unit.
   , $BC = \sqrt{(-5-3)^2 + (-6+14)^2}$
   $= \sqrt{64+64} = 8\sqrt{2}$ length unit.
   , $AC = \sqrt{(-5+1)^2 + (-6-4)^2} = \sqrt{16+100}$
   $= 2\sqrt{29}$ length unit
   $\because AB \neq BC + AC$
   $\therefore$ A , B and C are non-collinear points

7

$\because AB = \sqrt{(-2-3)^2 + (4+1)^2} = \sqrt{25+25} = \sqrt{50}$

$= 5\sqrt{2}$ length unit

, $BC = \sqrt{(3-4)^2 + (-1-5)^2} = \sqrt{1+36}$

$= \sqrt{37}$ length unit

, $AC = \sqrt{(-2-4)^2 + (4-5)^2} = \sqrt{36+1}$

$= \sqrt{37}$ length unit

$BC = AC$

ΔABC is an isosceles triangle

8

1. $AB = \sqrt{(4-2)^2 + (-2-1)^2}$
   $= \sqrt{4+9} = \sqrt{13}$ length unit.

, $BC = \sqrt{(7-4)^2 + (5+2)^2} = \sqrt{9+49}$

$= \sqrt{58}$ length unit

, $AC = \sqrt{(7-2)^2 + (5-1)^2}$

$= \sqrt{25+16} = \sqrt{41}$ length unit.

, $\because (BC)^2 > (AB)^2 + (AC)^2$

$\therefore$ A , B and C are the vertices of an obtuse-angled triangle at A

2 $AB = \sqrt{(-1-3)^2 + (1-5)^2} = \sqrt{16+16} = \sqrt{32}$ length unit.

, $BC = \sqrt{(5+1)^2 + (-5-1)^2} = \sqrt{36+36}$

$= \sqrt{72}$ length unit.

, $AC = \sqrt{(5-3)^2 + (-5-5)^2} = \sqrt{4+100}$

$= \sqrt{104}$ length unit

, $\because (AC)^2 = (AB)^2 + (BC)^2$

$\therefore$ A , B and C are the vertices of a right-angled triangle at B

3 $AB = \sqrt{(3-4)^2 + (-1-4)^2} = \sqrt{1+25} = \sqrt{26}$ length unit.

, $BC = \sqrt{(-2-3)^2 + (4+1)^2}$

$= \sqrt{25+25} = \sqrt{50}$ length unit.

, $AC = \sqrt{(-2-4)^2 + (4-4)^2} = \sqrt{36} = 6$ length units.

, $\because \overline{BC}$ is the longest side ,

$\therefore (BC)^2 < (AB)^2 + (AC)^2$

$\therefore$ A , B and C are the vertices of an acute-angled triangle

4 $AB = \sqrt{(6-0)^2 + (0-0)^2} = \sqrt{36+0} = 6$ length units.

, $BC = \sqrt{(0-6)^2 + (8-0)^2} = \sqrt{36+64} = \sqrt{100}$

$= 10$ length units

, $AC = \sqrt{(0-0)^2 + (8-0)^2} = \sqrt{0+64} = 8$ length units

, $\because (BC)^2 = (AB)^2 + (AC)^2$

$\therefore$ A , B and C are the vertices of a right-angled triangle at A

6 $AB = \sqrt{(2-1)^2 + (1+1)^2} = \sqrt{1+4} = \sqrt{5}$ length unit.

, $BC = \sqrt{(-3-2)^2 + (-2-1)^2} = \sqrt{25+9}$

$= \sqrt{34}$ length unit

, $AC = \sqrt{(1+3)^2 + (-1+2)^2} = \sqrt{16+1}$

$= \sqrt{17}$ length unit.

, $\because (BC)^2 > (AB)^2 + (AC)^2$

$\therefore$ A , B and C are the vertices of an obtuse-angled triangle at A

9

$AB = \sqrt{(-1-5)^2 + (7+5)^2} = \sqrt{36+144}$

$= \sqrt{180}$ length unit

, $BC = \sqrt{(15+1)^2 + (15-7)^2} = \sqrt{256+64}$

$= \sqrt{320}$ length unit

and $CA = \sqrt{(5-15)^2 + (-5-15)^2} = \sqrt{100+400}$

$= \sqrt{500}$ length unit

, $\because (CA)^2 = (AB)^2 + (BC)^2$

$\therefore \Delta ABC$ is right angled at B

$\therefore$ The area of $\Delta ABC = \frac{1}{2} \times AB \times BC$

$= \frac{1}{2} \times \sqrt{180} \times \sqrt{320}$

$= 120$ square units

10

$\because AB = \sqrt{(7-5)^2 + (2\sqrt{3}-0)^2}$

$= \sqrt{4+12} = 4$ length unit

, $BC = \sqrt{(3-7)^2 + (2\sqrt{3}-2\sqrt{3})^2}$

$= \sqrt{16} = 4$ length unit

, $AC = \sqrt{(3-5)^2 + (2\sqrt{3}-0)^2}$

$= \sqrt{4+12} = 4$ length unit

$\therefore \Delta ABC$ is an equilateral triangle

Let M be the midpoint of the base $\overline{AB}$

$\therefore$ The height $MC = \sqrt{(4)^2 - (2)^2} = \sqrt{16-4} = \sqrt{12}$

$= 2\sqrt{3}$ length unit.

$\therefore$ The are of $\Delta ABC = \frac{1}{2} \times AB \times MC$

$= \frac{1}{2} \times 4 \times 2\sqrt{3}$

$= 4\sqrt{3}$ square unit

ABCO is a rectangle

AC = BO (properties of the rectangle)

$\therefore AC = \sqrt{(9-0)^2 + (12-0)^2} = \sqrt{81+144}$

$= 15$ length units

1 $AB = \sqrt{(0+1)^2 + (5-1)^2} = \sqrt{1+16} = \sqrt{17}$ length unit.

, $BC = \sqrt{(5-0)^2 + (6-5)^2} = \sqrt{25+1} = \sqrt{26}$ length unit.

, $CD = \sqrt{(4-5)^2 + (2-6)^2} = \sqrt{1+16} = \sqrt{17}$ length unit

, $DA=\sqrt{(-1-4)^2+(1-2)^2}=\sqrt{25+1}=\sqrt{26}$ length unit.

$\therefore AB = CD$, $BC = DA$

$\therefore$ ABCD is a parallelogram.

E $AB=\sqrt{(5+2)^2+(-3-4)^2}=\sqrt{49+49}$

$=\sqrt{98}=7\sqrt{2}$ length unit.

, $BC=\sqrt{(7-5)^2+(1+3)^2}=\sqrt{4+16}$

$=\sqrt{20}=2\sqrt{5}$ length unit.

, $CD=\sqrt{(0-7)^2+(8-1)^2}=\sqrt{49+49}$

$=\sqrt{98}=7\sqrt{2}$ length unit.

, $DA=\sqrt{(0+2)^2+(8-4)^2}=\sqrt{4+16}$

$=\sqrt{20}=2\sqrt{5}$ length unit.

$\therefore AB = CD$, $BC = DA$

$\therefore$ ABCD is a parallelogram.

13

$\because AB=\sqrt{(0-4)^2+(1-5)^2}=\sqrt{16+16}$

$=\sqrt{32}=4\sqrt{2}$ length unit.

, $BC=\sqrt{(4-1)^2+(5-8)^2}=\sqrt{9+9}$

$=\sqrt{18}=3\sqrt{2}$ length unit.

, $CD=\sqrt{(1+3)^2+(8-4)^2}=\sqrt{16+16}$

$=\sqrt{32}=4\sqrt{2}$ length unit.

, $AD=\sqrt{(0+3)^2+(1-4)^2}=\sqrt{9+9}$

$=\sqrt{18}=3\sqrt{2}$ length unit.

$\because AB = CD$, $BC = AD$

$\therefore$ The figure ABCD is a parallelogram

, $\because AC=\sqrt{(0-1)^2+(1-8)^2}=\sqrt{1+49}$

$=\sqrt{50}=5\sqrt{2}$ length unit.

, $BD=\sqrt{(4+3)^2+(5-4)^2}=\sqrt{49+1}$

$=\sqrt{50}=5\sqrt{2}$ length unit.

$\therefore AC = BD = 5\sqrt{2}$ length unit.

$\therefore$ The figure ABCD is a rectangle its diagonal length $= 5\sqrt{2}$ length unit.

14

$\because AB=\sqrt{(3-0)^2+(3-3)^2}=\sqrt{9+0}=3$ length unit.

, $BC=\sqrt{(0-0)^2+(3-0)^2}=\sqrt{0+9}=3$ length unit.

, $CD=\sqrt{(0-3)^2+(0-0)^2}=\sqrt{9+0}=3$ length unit

, $DA=\sqrt{(3-3)^2+(0-3)^2}=\sqrt{0+9}=3$ length unit.

$\because AB = BC = CD = DA$ $\therefore$ ABCD is a rhombus

$\because AC=\sqrt{(3-0)^2+(3-0)^2}=\sqrt{9+9}=\sqrt{18}$

$=3\sqrt{2}$ length unit.

, $BD=\sqrt{(0-3)^2+(3-0)^2}=\sqrt{9+9}=\sqrt{18}$

$=3\sqrt{2}$ length unit.

$\therefore AC = BD$

The figure ABCD is a square , the length of its diagonal $= 3\sqrt{2}$ length unit ,

its area $= 3 \times 3 = 9$ square unit

15

$AB=\sqrt{(6-5)^2+(-2-3)^2}=\sqrt{1+25}=\sqrt{26}$ length unit.

, $BC=\sqrt{(1-6)^2+(-1+2)^2}$

$=\sqrt{25+1}=\sqrt{26}$ length unit.

, $CD=\sqrt{(0-1)^2+(4+1)^2}=\sqrt{1+25}=\sqrt{26}$ length unit

, $DA=\sqrt{(5-0)^2+(3-4)^2}=\sqrt{25+1}=\sqrt{26}$ length unit

$\therefore AB = BC = CD = DA$

$\therefore$ The figure ABCD is a rhombus

, $\because AC=\sqrt{(1-5)^2+(-1-3)^2}$

$=\sqrt{16+16}=\sqrt{32}=4\sqrt{2}$ length unit.

, $BD=\sqrt{(0-6)^2+(4+2)^2}$

$=\sqrt{36+36}=\sqrt{72}=6\sqrt{2}$ length unit.

$\therefore$ The area of the rhombus $= \frac{1}{2} AC \times BD$

$= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$

$= 24$ square units.

16

$AB=\sqrt{(3+2)^2+(3-5)^2}=\sqrt{25+4}=\sqrt{29}$ length unit.

, $BC=\sqrt{(-4-3)^2+(2-3)^2}=\sqrt{49+1}=\sqrt{50}$ length unit.

, $CA=\sqrt{(-2+4)^2+(5-2)^2}=\sqrt{4+9}=\sqrt{13}$ length unit.

, $\because$ BC is the greatest distance

, $BC < AB + CA$

$\therefore$ A , B , C are non-collinear points

$\because AD=\sqrt{(-2+9)^2+(5-4)^2}$

$=\sqrt{49+1}=\sqrt{50}$ length unit.

, $CD=\sqrt{(-9+4)^2+(4-2)^2}$

$=\sqrt{25+4}=\sqrt{29}$ length unit

$\therefore AB = CD$, $BC = AD$

$\therefore$ ABCD is a parallelogram

17

$AB = \sqrt{(-3-2)^2 + (0-4)^2} = \sqrt{25+16} = \sqrt{41}$ length unit

, $BC = \sqrt{(-7+3)^2 + (5-0)^2} = \sqrt{16+25} = \sqrt{41}$ length unit.

, $CD = \sqrt{(-2+7)^2 + (9-5)^2} = \sqrt{25+16} = \sqrt{41}$ length unit.

, $DA = \sqrt{(2+2)^2 + (4-9)^2} = \sqrt{16+25} = \sqrt{41}$ length unit.

$\therefore$ ABCD is a rhombus

, $\because AC = \sqrt{(-7-2)^2 + (5-4)^2}$

$= \sqrt{81+1} = \sqrt{82}$ length unit.

, $BD = \sqrt{(-2+3)^2 + (9-0)^2} = \sqrt{1+81}$

$= \sqrt{82}$ length unit

$\therefore AC = BD$ $\therefore$ The figure ABCD is a square

$\because MA = \sqrt{(-1-3)^2 + (2+1)^2} = \sqrt{16+9}$

$= \sqrt{25} = 5$ length units

, $MB = \sqrt{(-1+4)^2 + (2-6)^2} = \sqrt{9+16}$

$= \sqrt{25} = 5$ length units

and $MC = \sqrt{(-1-2)^2 + (2+2)^2} = \sqrt{9+16}$

$= \sqrt{25} = 5$ length units

$MA = MB = MC$

$\therefore$ A , B and C lie on the circle M which its radius length is 5 length units

$\therefore$ The circumference of the circle $= 2\pi r$

$= 2 \times 3.14 \times 5$

$= 31.4$ length units

$\because \sqrt{(x-6)^2 + (5-1)^2} = 2\sqrt{5}$

$\sqrt{(x-6)^2 + 16} = 2\sqrt{5}$ "squaring the two sides"

$(x-6)^2 + 16 = 20$

$\therefore x^2 - 12x + 36 + 16 - 20 = 0$

$\therefore x^2 - 12x + 32 = 0$

$\therefore (x-4)(x-8) = 0$

$x = 4$ or $x = 8$

[1] $\because \sqrt{(a+2)^2 + (7-3)^2} = 5$ "squaring the two sides"

$\therefore (a+2)^2 + (4)^2 = 25$ $\therefore a^2 + 4a + 4 + 16 = 25$

$\therefore a^2 + 4a - 5 = 0$ $\therefore (a-1)(a+5) = 0$

$a = 1$ or $a = -5$

[2] $\sqrt{(3a-1-a)^2 + (-5-7)^2} = 13$ "squaring the two sides"

$\therefore (2a-1)^2 + (-12)^2 = 169$

$\therefore 4a^2 - 4a + 1 + 144 = 169$

$\therefore 4a^2 - 4a - 24 = 0$ "dividing by 4 for both sides"

$a^2 - a - 6 = 0$ $\therefore (a+2)(a-3) = 0$

$\therefore a = -2$ or $a = 3$

21

$BC = \sqrt{(5-3)^2 + (1-2)^2} = \sqrt{4+1} = \sqrt{5}$ length unit

$AB = \sqrt{5}$ length unit

$\therefore \sqrt{(x-3)^2 + (3-2)^2} = \sqrt{5}$ "squaring the two sides"

$\therefore (x-3)^2 + (1)^2 = 5$ $\therefore x^2 - 6x + 9 + 1 = 5$

$x^2 - 6x + 5 = 0$ $\therefore (x-5)(x-1) = 0$

$\therefore x = 5$ or $x = 1$

$\because AB = \sqrt{(-6-9)^2 + (0-0)^2} = \sqrt{225}$

$= 15$ length units.

Let $C = (0 , y)$

$\therefore AC = \sqrt{(0-9)^2 + (y-0)^2} = \sqrt{81+y^2}$

, $\because AB = AC$

$15 = \sqrt{81+y^2}$ "squaring the two sides"

$\therefore 225 = 81 + y^2$ $\therefore y^2 = 225 - 81 = 144$

$\therefore y = 12$

or $y = -12$ (refused because the point C lies on the positive part of y-axis)

$\therefore C (0 , 12)$

$\therefore CO = \sqrt{(0-0)^2 + (12-0)^2} = \sqrt{144}$

$= 12$ length units

23

$\because$ The axis of symmetry of $\overline{CD}$ passes through the point A

$\therefore CA = DA$

$\sqrt{(6-3)^2 + (m-1)^2} = \sqrt{(6+3)^2 + (m-7)^2}$

"squaring the two sides"

$\therefore 3^2 + (m-1)^2 = 9^2 + (m-7)^2$

$\therefore 9 + m^2 - 2m + 1 = 81 + m^2 - 14m + 49$

$\therefore -2m + 14m = 81 + 49 - 9 - 1$

$\therefore 12m = 120$ $\therefore m = \frac{120}{12} = 10$

24

$A \in$ the x-axis

Let $A = (x , 0)$

$\because AO = AB$ $\therefore x = \sqrt{(x+9)^2 + (0-15)^2}$

$x = \sqrt{x^2 + 18x + 81 + 225}$

$x^2 = x^2 + 18x + 306$ $\therefore 18x = -306$

$\therefore x = -17$ $\therefore AB = 17$ length units

25

$\because f(x) = x$ $\therefore C$ is (x , x)

$, \because AO = 4$ length units $\therefore A(4 , 0)$

$, \because OB = 6$ length units $\therefore B(0 , 6)$

$, \because AC = BC$

$\therefore \sqrt{(x-4)^2 + (x-0)^2} = \sqrt{(x-0)^2 + (x-6)^2}$

"squaring the two sides"

$\therefore x^2 - 8x + 16 + x^2 = x^2 + x^2 - 12x + 36$

$\therefore 4x = 20$ $\therefore x = 5$ $\therefore C(5 , 5)$

26

The point that represents Basem's house is $(1 , 9)$

The point that represents Eslam's house is $(3 , 10)$

The point that represents the school is $(10 , 2)$

The point that represents the railway station is $(4 , 0)$

[1] The distance between Basem's house and the school $= \sqrt{(10-1)^2 + (2-9)^2}$

$= \sqrt{81 + 49} = \sqrt{130}$ km

, the distance between Eslam's house and the school $= \sqrt{(10-3)^2 + (2-10)^2}$

$= \sqrt{49 + 64} = \sqrt{113}$ km

$\therefore$ Eslam's house is nearer to the school

[2] The distance between Basem's house and the school $= \sqrt{130}$ km.

, the distance between Basem's house and railway station $= \sqrt{(1-4)^2 + (9-0)^2}$

$= \sqrt{9 + 81} = \sqrt{90}$ km.

, the distance between the school and railway station $= \sqrt{(10-4)^2 + (2-0)^2} = \sqrt{36 + 4} = \sqrt{40}$ km.

$\therefore$ The square of the distance between Basem's house and the school equals the sum of the squares of the distance between Basem's house and railway , and the distance between the school and railway.

$\therefore$ Basem's house , the school and railway station make a right-angled triangle at the railway station.

$\therefore$ The way (School - railway station) is perpendicular to the way (Basem's house - railway station)

27

$AB = \sqrt{(x-4)^2 + (2+2)^2} = \sqrt{(x-4)^2 + 16}$ length unit

$, BC = \sqrt{(x-3)^2 + (2-5)^2} = \sqrt{(x-3)^2 + 9}$ length unit

$, CA = \sqrt{(4-3)^2 + (-2-5)^2} = \sqrt{1 + 49} = \sqrt{50}$ length unit.

$\because \Delta ABC$ is right-angled at B

$\therefore (CA)^2 = (AB)^2 + (BC)^2$

$\therefore (x-4)^2 + 16 + (x-3)^2 + 9 = 50$

$\therefore x^2 - 8x + 16 + 16 + x^2 - 6x + 9 + 9 = 50$

$2x^2 - 14x = 0$ "dividing by 2"

$x^2 - 7x = 0$ $\therefore x(x-7) = 0$

$\therefore x = 0$ or $x = 7$ At $x = 0$

$\therefore AB = \sqrt{32} = 4\sqrt{2}$ length unit

$, BC = \sqrt{18} = 3\sqrt{2}$ length unit

$\therefore$ The area of $\Delta ABC = \frac{1}{2} AB \times BC$

$= \frac{1}{2} \times 4\sqrt{2} \times 3\sqrt{2}$

$= 12$ square units.

At $x = 7$ $AB = 5$ length units

$, BC = 5$ length units

$\therefore$ The area of $\Delta ABC = \frac{1}{2} AB \times BC = \frac{1}{2} \times 5 \times 5$

$= 12.5$ square units

[illegible]

[1] The midpoint of $\overline{AB} = \left(\frac{3+7}{2} , \frac{5+\ }{2}\right) = (5 , 3)$

[2] The midpoint of $\overline{AB} = \left(\frac{5-1}{2} , \frac{-3+3}{2}\right) = (2 , 0)$

[3] The midpoint of $\overline{AB} = \left(\frac{-5+5}{2} , \frac{4-4}{2}\right) = (0 , 0)$

[4] The midpoint of $\overline{AB} = \left(\frac{0+8}{2} , \frac{4+0}{2}\right) = (4 , 2)$

[5] The midpoint of $\overline{AB} = \left(\frac{2+6}{2} , \frac{4+0}{2}\right) = (4 , 2)$

[6] The midpoint of $\overline{AB} = \left(\frac{7-1}{2} , \frac{6+0}{2}\right) = (3 , -3)$

2

$(x , 0) = \left(\frac{1+2}{2} , \frac{5+5}{2} \right) = \left(\frac{3}{2} , 0 \right)$

$x = \frac{3}{2}$

3

$\because (5 , 3) = \left(\frac{5\ \ 5}{2} , \frac{y\ \ 2}{2} \right)$ $\therefore \frac{y\ \ 2}{2} = 3$

$y\ \ 2 = 6$ $y = 8$

4

Let $B\ (x , y)$

$\because (6 , \ 4) \left(\frac{5+x}{2} , \frac{3+y}{2} \right)$

$\frac{5+x}{2} = 6$ $\therefore 5 + x = 12$

$\therefore x = 7$, $\frac{3+y}{2} = \ 4$

$\therefore -3 + y = \ 8$

$\therefore y = \ 5$ $\therefore B (7 , -5)$

5

[1] $\because (x , y) = \left(\frac{1+3}{2} , \frac{5+7}{2} \right)$

$x = \frac{1+3}{2} = 2$, $y = \frac{5+7}{2} = 6$

[2] $(x , -3) = \left(\frac{3+9}{2} , \frac{y+11}{2} \right)$

$\therefore x = \frac{3+9}{2} = 3$, $\frac{y+11}{2} = -3$

$y + 11 = \ 6$ $\therefore y = -17$

[3] $(-3 , y) = \left(\frac{x+9}{2} , \frac{6\ \ 11}{2} \right)$

$\therefore \frac{x+9}{2} = -3$ $\therefore x + 9 = \ 6$

$\therefore x = -15$, $y = \frac{6\ \ 11}{2} = \ 8.5$

[4] $(4 , 6) = \left(\frac{x+6}{2} , \frac{3+y}{2} \right)$

$\therefore \frac{x+6}{2} = 4$ $x + 6 = 8$ $\therefore x = 2$

, $\frac{3+y}{2} = 6$ $3 + y \ \ 12$ $y = 9$

6

[1] d	[2] d	[3] d	[4] a
[5] c	[6] b	[7] a	[8] c

7

[1] 4 , 6 [2] (2 , 2)

[3] (3 , 2) , (7 , 0) [4] (0 , 4) , (0 , 6)

8

Let D be the midpoint of $\overline{AB}$

$D = \left(\frac{1+9}{2} , \frac{6+2}{2} \right) = (5 , -2)$

Let E be the midpoint of $\overline{AD}$

$E = \left(\frac{1+5}{2} , \frac{6\ \ 2}{2} \right) = (3 , \ 4)$

Let X be the midpoint of $\overline{BD}$

$\therefore X = \left(\frac{9+5}{2} , \frac{2\ \ 2}{2} \right) = (7 , 0)$

9

$(0 , 0) = \left(\frac{x\ \ 2\ \ 2}{2} , \frac{y+2}{2} \right)$ $\frac{x\ \ 4}{2} = 0$

$\therefore x - 4 = 0$ $\therefore x = 4$

, $\frac{y+2}{2} = 0$ $\therefore y + 2 = 0$

$y = -2$ $(x , y) = (4 , -2)$

10

$(2a - 3 , a \ \ b) = \left(\frac{7+3}{2} , \frac{-1+7}{2} \right) = (5 , 3)$

$2a - 3 = 5$ $\therefore 2a = 8$ $\therefore a = 4$

, $a - b = 3$ $4 - b = 3$ $\therefore b = 1$

11

Let $A (x , y)$

$(5 , 7) = \left(\frac{x+8}{2} , \frac{y+11}{2} \right)$

$\frac{x+8}{2} = 5$ $x + 8 = 10$

$\therefore x = 2$, $\frac{y+11}{2} \ \ 7$ $\therefore y + 11 = 14$

$y = 3$ $\therefore A (2 , 3)$

$r = MA = \sqrt{(5\ \ 2)^2 + (7\ \ 3)^2}$

$= \sqrt{9+16} = 5$ length unit.

The circumference of the circle $= 2 \pi r$

$= 2 \times 3.14 \times 5 = 31.4$ length unit

12

$\because$ D is the midpoint of $\overline{AB}$

D $\left(\frac{1+5}{2}, \frac{3+1}{2}\right) = (3, 2)$

, $\because$ E is the midpoint of $\overline{AC}$

E $=\left(\frac{1+3}{2}, \frac{3+7}{2}\right) = (2, 5)$

$\therefore DE = \sqrt{(3-2)^2 + (2-5)^2} = \sqrt{10}$ length unit (1)

, $BC = \sqrt{(5-3)^2 + (1-7)^2}$

$= \sqrt{40} = 2\sqrt{10}$ length unit (2)

From (1) and (2)

$\therefore DE = \frac{1}{2} BC$ (Q.E.D.)

13

Let A (X , 0) , B (0 , y)

$(3, 4) = \left(\frac{X+0}{2}, \frac{y+0}{2}\right)$

$\frac{X}{2} = 3$ $\therefore X = 6$

A (6 , 0) $\therefore$ OA = 6 length unit

, $\frac{y}{2} = 4$ $\therefore y = 8$

B (0 , 8) $\therefore$ OB = 8 length unit

, $AB = \sqrt{(6-0)^2 + (0-8)^2} = \sqrt{36+64} = \sqrt{100}$

= 10 length unit

$\therefore$ The perimeter of Δ OAB = 6 + 8 + 10

= 24 length unit

14

Let D $(X_1, 0)$ and B $(0, y_1)$

$\because$ D is the midpoint of $\overline{AB}$

$\therefore (X_1, 0) = \left(\frac{2+0}{2}, \frac{4+y_1}{2}\right)$

$\therefore X_1 = \frac{2}{2} = 1$, $\frac{4+y_1}{2} = 0$

$4 + y_1 = 0$ $\therefore y_1 = -4$ $\therefore$ B (0 , −4)

Let E $(0, y_2)$ and C $(X_2, 0)$

, $\because$ E is the midpoint of $\overline{AC}$

$(0, y_2) = \left(\frac{2+X_2}{2}, \frac{4+0}{2}\right)$

$\frac{2+X_2}{2} = 0$ $\therefore 2 + X_2 = 0$ $X_2 = -2$

, $y_2 = 2$ $\therefore$ C (−2 , 0)

$\therefore BC = \sqrt{(-2-0)^2 + (0+4)^2} = \sqrt{4+16} = \sqrt{20}$

$= 2\sqrt{5}$ length units

In Δ ABC

D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$

$DE = \frac{1}{2} BC = \frac{1}{2} \times 2\sqrt{5} = \sqrt{5}$ length unit

15

AD is a median in Δ ABC

D is the midpoint of $\overline{BC}$

$D = \left(\frac{3-3}{2}, \frac{2+6}{2}\right) = (0, 4)$

$\because$ M is the midpoint of AD and let A (X , y)

$(0, 6) = \left(\frac{X+0}{2}, \frac{y+4}{2}\right)$

$\frac{X}{2} = 0$ $\therefore X = 0$, $\frac{y+4}{2} = 6$

$y + 4 = 12$ $y = 8$ A (0 , 8)

16

The midpoint of $\overline{AC} = \left(\frac{-1+6}{2}, \frac{1+0}{2}\right)$

$= \left(\frac{5}{2}, \frac{1}{2}\right)$

, the midpoint of $\overline{BD} = \left(\frac{2+3}{2}, \frac{3-4}{2}\right)$

$= \left(\frac{5}{2}, -\frac{1}{2}\right)$

The midpoint of $\overline{AC}$ is the same midpoint of $\overline{BD}$

$\overline{AC}$ and $\overline{BD}$ bisect each other

17

$\because$ The midpoint of $\overline{AC} = \left(\frac{3+0}{2}, \frac{-2-7}{2}\right) = \left(\frac{3}{2}, \frac{-9}{2}\right)$

, the midpoint of $\overline{BD} = \left(\frac{-5+8}{2}, \frac{0-9}{2}\right) = \left(\frac{3}{2}, \frac{-9}{2}\right)$

$\therefore$ The midpoint of $\overline{AC}$ is the same midpoint of $\overline{BD}$

The two diagonals bisect each other

$\therefore$ The points A , B , C and D are the vertices of a parallelogram

18

1 Let E be the point of intersection of the two diagonals

$\therefore E = \left(\frac{3-1}{2}, \frac{2-2}{2}\right) = (1, 0)$

2 $AC = \sqrt{(-1-3)^2 + (-2-2)^2} = \sqrt{16+16} = \sqrt{32}$

$= 4\sqrt{2}$ length units

, $BD = \sqrt{(-2-4)^2 + (3+3)^2} = \sqrt{36+36} = \sqrt{72}$

$= 6\sqrt{2}$ length units

The area of the rhombus $= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$

= 24 square unit

19

The midpoint of $AC = \left(\frac{3+0}{2}, \frac{2-3}{2}\right) = \left(1\frac{1}{2}, -\frac{1}{2}\right)$

The point of intersection of the two diagonals is $\left(1\frac{1}{2}, -\frac{1}{2}\right)$ (First req.)

and let $D(x, y)$

The midpoint of AC = the midpoint of BD

$\left(1\frac{1}{2}, -\frac{1}{2}\right) = \left(\frac{x+4}{2}, \frac{y-5}{2}\right)$ $\frac{x+4}{2} = 1\frac{1}{2}$

$x + 4 = 3$ $x = -1$

, $\frac{y-5}{2} = -\frac{1}{2}$ $y - 5 = -1$ $y = 4$

$D(-1, 4)$ (Second req.)

20

$AB = \sqrt{(2-6)^2 + (-4-0)^2} = \sqrt{16+16}$

$= \sqrt{32} = 4\sqrt{2}$ length unit

, $BC = \sqrt{(-4-2)^2 + (2+4)^2} = \sqrt{36+36} = \sqrt{72}$

$= 6\sqrt{2}$ length unit

, $CA = \sqrt{(6+4)^2 + (0-2)^2} = \sqrt{100+4} = \sqrt{104}$

$= 2\sqrt{26}$ length unit

$\because (AB)^2 + (BC)^2 = (4\sqrt{2})^2 + (6\sqrt{2})^2$

$= 32 + 72 = 104 = (CA)^2$

$\therefore \Delta ABC$ is right-angled at B (First req.)

Let E be the midpoint of $\overline{AC}$

$E = \left(\frac{6-4}{2}, \frac{0+2}{2}\right) = (1, 1)$

$\because$ In the rectangle the two diagonals bisect each other

$\therefore$ E is the midpoint of $\overline{BD}$

Let $D(x, y)$

$\therefore (1, 1) = \left(\frac{x+2}{2}, \frac{y-4}{2}\right)$ $\frac{x+2}{2} = 1$

$\therefore x + 2 = 2$ $x = 0$

, $\frac{y-4}{2} = 1$ $y - 4 = 2$

$\therefore y = 6$ $D(0, 6)$ (Second req.)

21

$AB = \sqrt{(3-5)^2 + (-2-3)^2} = \sqrt{4+25}$

$= \sqrt{29}$ length unit

, $BC = \sqrt{(-2-3)^2 + (-4+2)^2} = \sqrt{25+4}$

$= \sqrt{29}$ length unit

, $CA = \sqrt{(5+2)^2 + (3+4)^2} = \sqrt{49+49}$

$= \sqrt{98}$ length unit

, $\because (AB)^2 + (BC)^2 = 29 + 29 = 58$, $(AC)^2 = 98$

$\therefore (AC)^2 > (AB)^2 + (BC)^2$

$\therefore \Delta ABC$ is obtuse-angled at B (First req.)

Let E be the midpoint of $\overline{AC}$

$E = \left(\frac{5-2}{2}, \frac{3-4}{2}\right) = (1.5, -0.5)$

$\because$ In the rhombus the two diagonals bisect each other

$\therefore$ E is the midpoint of $\overline{BD}$

Let $D(x, y)$

$\therefore (1.5, -0.5) = \left(\frac{x+3}{2}, \frac{y-2}{2}\right)$

$\therefore \frac{x+3}{2} = 1.5$ $x + 3 = 3$ $x = 0$

, $\frac{y-2}{2} = -0.5$ $\therefore y - 2 = -1$

$y = 1$ $\therefore D(0, 1)$

, $AC = \sqrt{98} = 7\sqrt{2}$

, $BD = \sqrt{(3-0)^2 + (-2-1)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

The area of the rhombus ABCD

$= \frac{1}{2} \times 7\sqrt{2} \times 3\sqrt{2} = 21$ square unit (Second req.)

22

$AB = \sqrt{(3+3)^2 + (4-0)^2} = \sqrt{36+16} = \sqrt{52}$

$= 2\sqrt{13}$ length unit

, $BC = \sqrt{(1-3)^2 + (-6-4)^2} = \sqrt{4+100} = \sqrt{104}$

$= 2\sqrt{26}$ length unit

, $CA = \sqrt{(-3-1)^2 + (0+6)^2} = \sqrt{16+36} = \sqrt{52}$

$= 2\sqrt{13}$ length unit

$\therefore AB = AC$

$\therefore \Delta ABC$ is an isosceles triangle and its vertex is A (First req.)

Let D be the midpoint of $\overline{BC}$ (the base of ΔABC)

$D = \left(\frac{3+1}{2}, \frac{4-6}{2}\right) = (2, -1)$

$AD = \sqrt{(2+3)^2 + (-1-0)^2} = \sqrt{25+1} = \sqrt{26}$ length unit

$\therefore$ The length of the drawn line segment perpendicular to $\overline{BC}$ from $A = \sqrt{26}$ length unit (Second req.)

23

$\because AB = \sqrt{(3-1)^2 + (1-1)^2} = \sqrt{4+0} = 2$ length units

, $BC = \sqrt{(1-3)^2 + (3-1)^2} = \sqrt{4+4}$

$= \sqrt{8} = 2\sqrt{2}$ length units.

, $CA = \sqrt{(1-1)^2 + (1-3)^2} = \sqrt{0+4} = 2$ length units

∵ AB = CA

Δ ABC is an isosceles triangle and its vertex is A (First req.)

Let the point E be the midpoint of $\overline{BC}$

∴ $E = \left(\frac{1+3}{2}, \frac{1+3}{2}\right) = (2, 2)$

∴ $AE = \sqrt{(2-1)^2 + (2-1)^2} = \sqrt{1+1} = \sqrt{2}$ length unit

, ∵ Δ ABC is an isosceles triangle
and E is the midpoint of BC

∴ $\overline{AE} \perp \overline{BC}$

∴ The area of Δ ABC $= \frac{1}{2}$ BC × AE

$= \frac{1}{2} \times 2\sqrt{2} \times \sqrt{2}$

= 2 square units. (Second req.)

24

In the parallelagram the two diagonals bisect each other

Let M be the point of intersection of the two diagonals

$\therefore M = \left(\frac{3-4}{2}, \frac{4-3}{2}\right) = \left(-\frac{1}{2}, \frac{1}{2}\right)$

Let D (x_1, y_1)

$\therefore \left(-\frac{1}{2}, \frac{1}{2}\right) = \left(\frac{x_1+2}{2}, \frac{y_1-1}{2}\right)$

∴ $\frac{x_1+2}{2} = -\frac{1}{2}$ $\qquad x_1 + 2 = -1$

$x_1 = -3$ $\qquad , \frac{y_1 - 1}{2} = \frac{1}{2}$

$y_1 - 1 = 1$ $\qquad \therefore y_1 = 2$

D = (− 3 , 2)

AE = 2 AD $\qquad$ ∴ D is the midpoint of $\overline{AE}$

Let E (x_2, y_2)

$\therefore (-3, 2) = \left(\frac{x_2+3}{2}, \frac{y_2+4}{2}\right)$

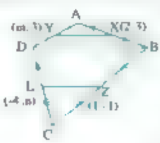
$\therefore \frac{x_2+3}{2} = -3$ $\qquad \therefore x_2 + 3 = -6$

$x_2 = -9$ $\qquad , \frac{y_2+4}{2} = 2$

$y_2 + 4 = 4$ $\qquad y_2 = 0$

E = (− 9 , 0)

25



In Δ ABD

X , Y are the midpoints of $\overline{AB}$, $\overline{AD}$

$\overline{XY} \parallel \overline{BD}$, $XY = \frac{1}{2}$ BD (1)

, similarly in Δ CBD ,

$\overline{ZL} \parallel \overline{BD}$, $ZL = \frac{1}{2}$ BD (2)

From (1) , (2) : $\qquad$ ∴ $\overline{XY} \parallel \overline{ZL}$, XY = ZL

The figure XYLZ is a parallelogram

∴ The midpoint of $\overline{XL}$ is the same midpoint of $\overline{ZY}$

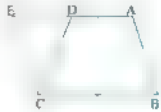
$\left(\frac{2-4}{2}, \frac{3+n}{2}\right) = \left(\frac{m+1}{2}, \frac{3-1}{2}\right)$

$\therefore \frac{2-4}{2} = \frac{m+1}{2}$ $\qquad \therefore m = -3$

$, \frac{3+n}{2} = \frac{3-1}{2}$ $\qquad \therefore n = -1$

∴ m + n = − 3 + (− 1) = − 4

26



Let E $(x, y) \in \overleftrightarrow{AD}$

such that ABCE

is a parallelogram

The midpoint of $\overline{AC}$ = the midpoint of $\overline{BE}$

$\left(\frac{6-2}{2}, \frac{4-4}{2}\right) = \left(\frac{4+x}{2}, \frac{2+y}{2}\right)$

$\frac{4+x}{2} = 2$ $\qquad \therefore x = 0$, $\frac{2+y}{2} = 0$

y = 2 $\qquad$ ∴ E (0 , 2)

∵ AE = BC (properties of the parallelogram)

, BC = 2 AD $\qquad$ ∴ AE = 2 AD

∴ D is the midpoint of $\overline{AE}$

$D = \left(\frac{6+0}{2}, \frac{4+2}{2}\right) = (3, 3)$

Answers of Exercise 5

1

1 b	2 a	3 b	4 d
5 a	6 b	7 c	8 b
9 c	10 a	11 b	12 d
13 c	14 d	15 a	16 c
17 b	18 a	19 c	20 b
21 a	22 d	23 d	

2

1 $\frac{1}{\sqrt{3}}$ $\qquad$ 2 $\frac{1}{\sqrt{3}}$ $\qquad$ 3 $-\sqrt{3}$

3

1 zero 2 $\frac{1}{\sqrt{3}}$ 3 1 4 1.5399

5 $\sqrt{3}$ 6 undefined 7 17.3432 8 1

4

1 $16° \, 4\bar{1} \, 5\bar{7}$ 2 $20° \, 1\bar{0} \, \bar{6}$

3 $45° \, 4\bar{1} \, 4\bar{6}$ 4 $38° \, 3\bar{9} \, 3\bar{5}$

5

$\because m_1 = \frac{6 - 2}{5 - 4} = 4$, $m_2 = \frac{1 - 5}{1 - 0} = 4$ $\therefore m_1 = m_2$

$\therefore$ The two straight lines are parallel

6

The slope of $\overleftrightarrow{AC} = \frac{2 - 4}{3 + 3} = \frac{6}{zero}$

The straight line $\overleftrightarrow{AC}$ // y-axis ,

The slope of $\overleftrightarrow{BD} = \frac{2 - 2}{3 - 1} =$ zero

The straight line $\overleftrightarrow{BD}$ // X-axis

$\therefore \overleftrightarrow{AC} \perp \overleftrightarrow{BD}$

7

$\because m_1 = \frac{3 + 1}{6 - 2} = 1$, $m_2 = \tan 45° = 1$

$m_1 = m_2$

The two straight lines are parallel

8

$\therefore m_1 = \frac{2\sqrt{3} - 3\sqrt{3}}{5 - 4} = -\sqrt{3}$, $m_2 = \tan 30° = \frac{1}{\sqrt{3}}$

$\therefore m_1 \times m_2 = -1$

The two straight lines are perpendicular

9

$\because$ The slope of $\overleftrightarrow{AD} = \frac{1 - 5}{2 - 1} = -4$

, the slope of $\overleftrightarrow{BC} = \frac{7 - 3}{4 - (X - 1)} = \frac{4}{5 - X}$

The slope of $\overleftrightarrow{AD}$ = the slope of $\overleftrightarrow{BC}$

$-4 = \frac{4}{5 - X}$ $\therefore 5 - X = -1$ $X = 6$

10

$\because \Delta XYZ$ is right-angled at Y

$\therefore \overleftrightarrow{XY} \perp \overleftrightarrow{YZ}$, the slope of $\overleftrightarrow{XY} = \frac{5 - 2}{3 - 4} = -3$

$\therefore$ The slope of $\overleftrightarrow{YZ} = \frac{1}{3}$

, $\because$ the slope of $\overleftrightarrow{YZ} = \frac{a - 2}{-5 - 4} = \frac{a - 2}{-9} = \frac{1}{3}$

$\therefore a - 2 = -3$ $\therefore a = -1$

11

$m = \frac{7 - 5}{X - 3}$, m is undefined

$\therefore X - 3 = 0$ $\therefore X = 3$

12

$m = \frac{y - 2}{5 - 4}$, m = zero

$\therefore y - 2 = 0$ $\therefore y = 2$

13

1 $m_1 = \frac{k - 1}{2 - 3} = \frac{k - 1}{-1}$, $m_2 = \tan 45° = 1$

$\because L_1 // L_2$ $\therefore m_1 = m_2$

$\frac{k - 1}{-1} = 1$ $\therefore k - 1 = -1$

k = zero

2 $m_1 = \frac{k - 1}{-1}$, $m_2 = 1$

$\because L_1 \perp L_2$ $\therefore m_1 \, m_2 = -1$

$\frac{k - 1}{-1} \times 1 = -1$ $\therefore k - 1 = 1$ $\therefore k = 2$

14

Let the measure of the positive angle be θ

$m = \frac{-5 - 3}{2 - 4} = 4$ $\therefore \tan \theta = 4$

$\therefore \theta = 75° \, 5\bar{7} \, 5\bar{0}$

15

Let the measure of the positive angle be θ

$m = \frac{-2 - 0}{2 - 0} = -1$

$\therefore$ tan (supplementary of θ) = 1

supplementary of θ = 45°

$\therefore \theta = 180° - 45° = 135°$

16

Let the measure of the positive angle be θ

The slope of the given straight line $= \frac{-1 - 5}{4 + 2} = -1$

The two straight lines are perpendicular

$\therefore$ The slope of the required straight line = 1

$\tan \theta = 1$ $\therefore \theta = 45°$

17

The slope of $\overleftrightarrow{AB} = \frac{3 - 1}{2 - 1} = \frac{2}{1} = 2$

, the slope of $\overleftrightarrow{BC} = \frac{1 - 3}{0 - 2} = \frac{-4}{-2} = 2$

The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$

∴ $\overleftrightarrow{AB} \parallel \overleftrightarrow{BC}$

B is a common point between the two straight lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$

∴ A , B , C are collinear points

18

Let X (0 , 1) , Y (a , 3) and Z (2 , 5)

, ∵ the three points are collinear

∴ The slope of $\overleftrightarrow{XY}$ = the slope of $\overleftrightarrow{XZ}$

∴ $\frac{3-1}{a-0} = \frac{5-1}{2-0}$ ∴ $\frac{2}{a} = 2$ ∴ $a = 1$

19

The slope of $\overleftrightarrow{AB} = \frac{5-7}{1-3} = 1$

, the slope of $\overleftrightarrow{BC} = \frac{2-5}{4+1} = \frac{-3}{5}$

∴ The slope of $\overleftrightarrow{AB}$ ≠ the slope of $\overleftrightarrow{BC}$

∴ $C \notin \overleftrightarrow{AB}$

20

The slope of $\overleftrightarrow{AB} = m_1 = \frac{3+1}{2+1} = \frac{4}{3}$

, the slope of $\overleftrightarrow{BC} = m_2 = \frac{0-3}{6-2} = \frac{-3}{4}$

, ∵ $m_1 \times m_2 = \frac{4}{3} \times \frac{-3}{4} = -1$

∴ $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$ ∴ Δ ABC is right-angled at B

21

The slope of $\overleftrightarrow{AB} = \frac{5-1}{0+1} = 4$

, the slope of $\overleftrightarrow{CD} = \frac{6-2}{5-4} = 4$

∵ The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{CD}$

∴ $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ (1)

The slope of $\overleftrightarrow{AC} = \frac{2-1}{4+1} = \frac{1}{5}$

, the slope of $\overleftrightarrow{BD} = \frac{6-5}{5-0} = \frac{1}{5}$

∵ The slope of $\overleftrightarrow{AC}$ = the slope of $\overleftrightarrow{BD}$

∴ $\overleftrightarrow{AC} \parallel \overleftrightarrow{BD}$ (2)

From (1) and (2)

∴ The figure ABDC is a parallelogram

22

The slope of $\overleftrightarrow{AB} = \frac{1-3}{5+1} = -\frac{1}{3}$

, the slope of $\overleftrightarrow{CD} = \frac{6-4}{0-6} = -\frac{1}{3}$

∴ $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ (1)

The slope of $\overleftrightarrow{AD} = \frac{6-3}{0+1} = 3$

, the slope of $\overleftrightarrow{BC} = \frac{4-1}{6-5} = 3$

∴ $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$ (2)

From (1) and (2) , we deduce that

ABCD is a parallelogram

∵ the slope of $\overleftrightarrow{AB}$ × the slope of $\overleftrightarrow{BC} = -\frac{1}{3} \times 3 = -1$

∴ $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$

∴ The figure ABCD is a rectangle

23

The slope of $\overleftrightarrow{AB} = \frac{4-3}{6-1} = \frac{1}{5}$

, the slope of $\overleftrightarrow{CD} = \frac{8-9}{2-7} = \frac{1}{5}$

∴ $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ (1)

The slope of $\overleftrightarrow{AD} = \frac{8-3}{2-1} = 5$

, the slope of $\overleftrightarrow{BC} = \frac{9-4}{7-6} = 5$

∴ $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$ (2)

From (1) and (2) , we deduce that

ABCD is a parallelogram

∵ The slope of $\overleftrightarrow{AC} = \frac{9-3}{7-1} = 1$

, the slope of $\overleftrightarrow{BD} = \frac{8-4}{2-6} = -1$

∴ The slope of $\overleftrightarrow{AC}$ × the slope of $\overleftrightarrow{BD} = 1 \times -1 = -1$

∴ $\overleftrightarrow{AC} \perp \overleftrightarrow{BD}$

∴ The figure ABCD is a rhombus

24

∵ The slope of $\overleftrightarrow{AB} = \frac{3+1}{2+1} = \frac{4}{3}$

, the slope of $\overleftrightarrow{CD} = \frac{4-0}{3-6} = \frac{4}{3}$

∴ $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ (1)

∵ The slope of $\overleftrightarrow{AD} = \frac{-4+1}{3+1} = -\frac{3}{4}$

, the slope of $\overleftrightarrow{BC} = \frac{0-3}{6-2} = -\frac{3}{4}$

∴ $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$ (2)

From (1) and (2) , we deduce that

ABCD is a parallelogram

The slope of $\overleftrightarrow{AB}$ × the slope of $\overleftrightarrow{BC} = \frac{4}{3} \times -\frac{3}{4} = -1$

∴ $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$

∴ The figure ABCD is a rectangle

, ∵ the slope of $\overleftrightarrow{AC} = \frac{0+1}{6+1} = \frac{1}{7}$

, the slope of $\overleftrightarrow{BD} = \frac{-4-3}{3-2} = -7$

∴ The slope of $\overleftrightarrow{AC}$ × The slope of $\overleftrightarrow{BD}$

$= \frac{1}{7} \times -7 = -1$

∴ $\overleftrightarrow{AC} \perp \overleftrightarrow{BD}$ ∴ ABCD is a square

25

∵ The slope of $\overleftrightarrow{AB} = \frac{2+2}{3-9} = -\frac{2}{3}$

The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{CD}$

$\frac{-X+3}{X-4} = -\frac{2}{3}$ ∴ $3(-X+3) = 2(X-4)$

∴ $-3X+9 = 2X-8$ ∴ $X = 1$

∴ C = (,)

26

The slope of $\overleftrightarrow{AB} = \frac{0-3}{7-4} = -1$

, the slope of $\overleftrightarrow{BC} = \frac{-2-0}{1-7} = \frac{1}{3}$

∴ The slope of $\overleftrightarrow{AB}$ ≠ the slope of $\overleftrightarrow{BC}$

∴ A , B and C are not collinear

∴ A , B and C are vertices of a triangle (First req.)

∵ The slope of $\overleftrightarrow{CD} = \frac{2+2}{1-1} = \frac{4}{0}$ (undefined)

, the slope of $\overleftrightarrow{AD} = \frac{3-2}{4-1} = \frac{1}{3}$

∴ The slope of $\overleftrightarrow{AB}$ ≠ the slope of $\overleftrightarrow{CD}$

, the slope of $\overleftrightarrow{BC}$ = the slope of $\overleftrightarrow{AD}$

∴ $\overleftrightarrow{BC} \parallel \overleftrightarrow{AD}$

∴ The figure ABCD is a trapezoid (Second req.)

$AD = \sqrt{(4-1)^2+(3-2)^2} = \sqrt{9+1} = \sqrt{10}$ length unit

$BC = \sqrt{(7-1)^2+(0+2)^2}$

$= \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$ length unit

AD : BC = 1 : 2 (Third req.)

27

Let the measure of the positive angle be θ

$\sin\theta = \frac{3}{5}$

$\theta = 36° 52' 11.63''$ $\tan\theta = \frac{3}{4}$

∴ The slope of the straight line $= \frac{3}{4}$

28

∵ $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

∴ The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{CD}$

∴ $\frac{3-1}{3-1} = \frac{y+3X}{X-0}$ ∴ $y+3X = X$

∴ $y + 2X = 0$ (1)

∵ $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$

∴ The slope of $\overleftrightarrow{AB}$ × the slope of $\overleftrightarrow{BC} = -1$

∴ $\frac{3-1}{3-1} \times \frac{-3X-3}{0-3} = -1$ $\frac{3X+3}{3} = -1$

∴ $-3X-3 = 3$ ∴ $-3X = 6$ ∴ $X = -2$

And from (1) $y + 2 \times (-2) = 0$ ∴ $y = 4$

29

[1] The two diagonals of the rhombus bisect each other

∴ Let M be the intersection point of the diagonals

$M = \left(\frac{3-1}{2}, \frac{2-2}{2}\right) = (1, 0)$

, ∵ the slope of $\overleftrightarrow{MA} = \frac{2-0}{3-1} = \frac{2}{2} = 1$

, the slope of $\overleftrightarrow{MB} = \frac{k-0}{4-1} = \frac{k}{3}$

, ∵ the two diagonals of the rhombus are perpendicular

∴ $\overleftrightarrow{MA} \perp \overleftrightarrow{MB}$ ∴ $1 \times \frac{k}{3} = -1$

∴ $k = -3$

[2] Let : D (X , y) ∴ $(1, 0) = \left(\frac{4+X}{2}, \frac{-3+y}{2}\right)$

$\frac{4+X}{2} = 1$ ∴ $4 + X = 2$

$X = -2$, $\frac{-3+y}{2} = 0$

∴ $-3 + y = 0$ ∴ $y = 3$ ∴ D (- 2 , 3)

∴ $BD = \sqrt{(4+2)^2+(-3-3)^2}$

$= \sqrt{36+36} = 6\sqrt{2}$ length units

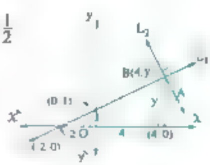
30

∵ The slope of the straight line $L_1 = \frac{1-0}{0+2} = \frac{1}{2}$

Let B (4 , y)

∴ The slope of the straight line $L_1 = \frac{y-0}{4+2}$

$= \frac{y}{6}$



$\therefore \frac{y}{6} = \frac{1}{2}$ $\therefore y = 3$ $\therefore B(4, 3)$

, $\because$ the straight line $L_1 \perp$ the straight line L_2

, the slope of the straight line L_1 $\frac{1}{2}$

. The slope of the straight line $L_2 = -2$

, A (5 m , m) , B (4 , 3) lie on the straight line L_2

. $\frac{m-3}{5m \quad 4}$ 2 $-10m+8$ m 3

$-11m =$ 11 $m = 1$

Answers of Exercise 6

1

[1] The slope = 5 and the intercepted part = 3 units from the negative part of y-axis

[2] $\because 2y = 4 - x$ (dividing by 2) $\therefore y = -\frac{1}{2}x + 2$

$\therefore$ The slope $= -\frac{1}{2}$ and the intercepted part = 2 units from the positive part of y-axis

[3] $\because 2x - 3y - 6 = 0$

$3y = 2x - 6$ (dividing by 3)

$\therefore y = \frac{2}{3}x - 2$

$\therefore$ The slope $= \frac{2}{3}$ and the intercepted part = 2 units from the negative part of y-axis

[4] $y + x \quad 1 = 0$ $\therefore y = \quad x + 1$

. The slope = − 1 and the intercepted part = one unit from the positive part of y-axis

[5] $\because \frac{x}{2} + 3y = 6$ (multiplying by 2)

$\therefore x + 6y = 12$ $\therefore 6y = -x + 12$

$y = -\frac{1}{6}x + 2$

$\therefore$ The slope $= -\frac{1}{6}$ and the intercepted part = 2 units from the positive part of y-axis

[6] $\frac{x}{2} + \frac{y}{3} = 1$ (multiplying by 3)

$\frac{3x}{2} + y = 3$ $\therefore y = \cdot \frac{3x}{2} + 3$

. The slope = $\frac{3}{2}$ and the intercepted part = 3 units from the positive part of y-axis

2

[1] $y = 2x + 7$

[2] $y \quad x + 3$

[3] $y = 2\frac{1}{2}x - 1$

[4] $y = -\frac{3}{4}x \quad 2\frac{1}{2}$

[5] $y = -2$

3

1 The slope = tan 45° = 1 $\therefore y = x + c$

The straight line passes through the point (3 , 2)

$\therefore 2 = 3 + c$ $\therefore c = \quad 1$

. The equation is . $y = x \quad 1$

[2] $\because$ The slope of the given straight line $= \frac{2}{3}$

The slope of the required straight line $= \frac{2}{3}$

and it intercepts from the negative part of y-axis 3 units

The equation of the required straight line is :

$y = \frac{2}{3}x - 3$

3 The slope of the given straight line $= \frac{3}{4}$

The slope of the required straight line $= -\frac{4}{3}$

and it intercepts from the positive part of y-axis 6 units

$\therefore$ The equation of the required straight line is

$y = -\frac{4}{3}x + 6$

4 $\because$ The slope of the given straight line $= \frac{7-1}{2+2} = \frac{3}{2}$

The slope of the required straight line $= \frac{2}{3}$

and intercepts from the positive part of y-axis 5 units

$\therefore$ The equation of the required straight line is

$y = -\frac{2}{3}x + 5$

5 The straight line passes through the two points (4 , 0) , (0 , 9)

. The slope of the straight line $= \frac{9 \quad 0}{0 \quad 4} = \quad \frac{9}{4}$

and the intercepted part = 9 units from the positive part of y-axis

$\therefore$ The equation of the straight line is :

$y = -\frac{9}{4}x + 9$

[6] $\because$ The slope = 2 $\therefore y = 2x + c$

, the straight line passes through the point (2 , − 1)

. $1 = 2 \times 2 + c$. $c = \cdot 5$

$y = 2x - 5$

[7] $\because$ The slope of the given straight line $= \frac{1}{2}$

. The slope of the required straight line = − 2

$\therefore$ The equation of the required straight line is :

$y = -2x + c$

$\because$ The straight line passes through the point (− 2 , 3)

. $3 = \quad 2 \times (\quad 2) + c$ $\therefore c = -1$

$\therefore$ The equation of the required straight line is :

$y = \quad 2x - 1$

[8] $\because$ The slope of the given straight line $= -\frac{1}{2}$

The slope of the required straight line $= -\frac{1}{2}$

The equation of the required straight line is

$y = -\frac{1}{2}X + c$

The straight line passes through the point (3 , 5)

$\therefore 5 = -\frac{1}{2} \times 3 + c \qquad \therefore c = -3\frac{1}{2}$

The equation of the required straight line is

$y = -\frac{1}{2}X - 3\frac{1}{2}$

[9] The slope of the given straight line

$= \frac{2-6}{-1-5} = \frac{2}{3}$

The slope of the required straight line $= \frac{2}{3}$

The equation of the required straight line is

$y = \frac{2}{3}X + c$

The straight line passes through the point (3 , 2)

$\therefore 2 = \frac{2}{3} \times 3 + c \qquad \therefore c = 0$

The equation of the required straight line is $y = \frac{2}{3}X$

[10] $\because$ The slope of $\overline{AB} = \frac{-4+3}{5-2} = -\frac{1}{3}$

The slope of the required straight line = 3

The equation of the required straight line is :

$y = 3X + c$

$\because$ The straight line passes through the point (1 , 2)

$2 = 3 \times 1 + c \qquad \therefore c = -1$

The equation of the required straight line is :

$y = 3X - 1$

[11] The slope of the given straight line = tan 45° = 1

$\therefore$ The slope of the required straight line $= -1$

The equation of the required straight line is :

$y = -X + c$

The straight line passes through the point (2 , −2)

$-2 = -2 + c \qquad \therefore c = 0$

The equation of the required straight line is :

$y = -X$

[12] The slope of the straight line $= \frac{1+1}{1-2} = -2$

The equation of the straight line

is $y = -2X + c$

The straight line passes through the point (1 , 1)

$\therefore 1 = -2 \times 1 + c \qquad \therefore c = 3$

The equation of the straight line is

$y = -2X + 3$

[13] $\because$ The slope of the straight line $= \frac{1-2}{2-4} = \frac{1}{2}$

$\therefore$ The equation of the straight line is $y = \frac{1}{2}X + c$

$\because$ The straight line passes through

the point (4 , 2)

$\therefore 2 = \frac{1}{2} \times 4 + c \qquad \therefore c =$ zero

$\therefore$ The equation of the straight line is $y = \frac{1}{2}X$

The intercepted part of y-axis = zero

$\therefore$ The straight line passes through the origin point

[14] $\frac{y-1}{X} = \frac{1}{3} \qquad y - 1 = \frac{1}{3}X$

$y = \frac{1}{3}X + 1$

$\therefore$ The slope of the given straight line $= \frac{1}{3}$

The slope of the required straight line $= \frac{1}{3}$

The equation of the required straight line is

$y = \frac{1}{3}X - 3$

[15] The slope of $\overline{AB} = \frac{1-6}{2+3} = -1$

The slope of the required straight line = 1

$\therefore$ The equation of the required straight line is

$y = X + c$

$\because$ The required straight line passes through

the point A (−3 , 6) $\quad \therefore 6 = -3 + c \quad \therefore c = 9$

$\therefore$ The equation of the required straight line is

$y = X + 9$

[16] The slope of $\overline{AB} = \frac{5-3}{3-1} = 1$

$\therefore$ The slope of the required straight line $= -1$

$\therefore$ The equation of the required straight line is

$y = -X + c$

$\because$ the midpoint of $\overline{AB} = \left(\frac{1+3}{2} , \frac{3+5}{2}\right) = (2 , 4)$

, $\because$ the required straight line passes through the midpoint of $\overline{AB}$

$\therefore 4 = -2 + c \qquad \therefore c = 6$

$\therefore$ The equation of the required straight line is

$y = -X + 6$

[17] $\because 2y = 4X - 5 \qquad \therefore y = 2X - \frac{5}{2} \qquad m = 2$

The slope of the required straight line = 2

$\therefore$ The equation of the required straight line is

$y = 2X + c$

The midpoint of $\overline{AB} = \left(\frac{4-2}{2}, \frac{8+4}{2}\right)$ (1 , 6)

∵ (1 , 6) satisfies its equation

∴ $6 = 2 \times 1 + c$ ∴ $c = 4$

∴ The equation of the required straight line is

$y = 2X + 4$

18 The slope of the given straight line = 2

∴ The slope of the required straight line $= -\frac{1}{2}$

The equation of the required straight line is

$y = -\frac{1}{2}X + c$

∵ The midpoint of $\overline{AB} = \left(\frac{3 - 1}{2}, \frac{6+4}{2}\right)$

$= (1 , 5)$

, the required straight line passes through the midpoint of $\overline{AB}$

$5 = -\frac{1}{2} \times 1 + c$ ∴ $c = 5\frac{1}{2}$

∴ The equation of the required straight line is

$y = -\frac{1}{2}X + 5\frac{1}{2}$

19 ∵ The required straight line intercepts from the positive part of X-axis 4 units

The required straight line passes through the point (4 , 0)

The slope of the required straight line

$= \frac{0 - 3}{4 - 2} = \frac{-3}{2}$

∴ The equation of the required straight line is

$y = \frac{-3}{2}X + c$

, ∵ the required straight line passes through the point (2 , 3)

∴ $3 = \frac{-3}{2} \times 2 + c$ ∴ $c = 6$

∴ The equation of the required straight line is

$y = \frac{-3}{2}X + 6$

4

1 d	2 c	3 b	4 d
5 d	6 a	7 d	8 c
9 d	10 a	11 a	12 a
13 c	14 d	15 a	16 c
17 c	18 d	19 b	20 a
21 b	22 d		

5

1 $\frac{3}{4}$, $-\frac{4}{3}$

2 (0 , 2)

3 $y = 0$, $X = 0$

4 $y = -5$

5 $y = 2X$

6 $y = 2X - 3$

7 (1) (4 , 6) (2) $\frac{3}{4}$

(3) $y = \frac{3}{4}X + 3$

6

The slope of $\overline{AB} = \frac{2}{1 - 3} = \frac{-1}{2}$

, the slope of the other straight line $= \frac{-2}{4} = \frac{-1}{2}$

∴ The slope of $\overline{AB}$ = the slope of the other straight line

∴ The two straight lines are parallel

7 The slope of the straight line whose equation is

$2X + y + 8 = 0$ is $\frac{-2}{1} = -2$

and the slope of $\overline{AB} = \frac{1 - 3}{2 - 2} = \frac{1}{2}$

, ∵ $-2 \times \frac{1}{2} = -1$

∴ The two straight lines are perpendicular

8 $y = 2$, $X = -3$

9

∵ The slope of the straight line $= \frac{-3}{-2} = \frac{3}{2}$

, the slope of the straight line = tan θ

∴ tan θ = $\frac{3}{2}$ ∴ θ ≈ 56° 18′ 36″

Put $X = 0$

∴ $3 \times 0 - 2y + 6 = 0$ ∴ $-2y = -6$ ∴ $y = 3$

∴ The intersection point with y-axis is (0 , 3)

10

1 Let A (X , 0)

At $y = 0$ ∴ $2X - 3 \times 0 - 6 = 0$

$2X = 6$ ∴ $X = 3$

The straight line cuts the X-axis at the point A (3 , 0)

Let B (0 , y)

∴ At $X = 0$ ∴ $2 \times 0 - 3y - 6 = 0$

∴ $-3y = 6$ ∴ $y = -2$

The straight line cuts the y-axis at the point B (0 , − 2)

2) Let D be the midpoint of AB

$D = \left(\frac{3+0}{2}, \frac{0-2}{2}\right) = \left(\frac{3}{2}, -1\right)$

, ∵ the straight line is parallel to the y-axis

Its slope is undefined

, ∵ the straight line passes through the point

$D\left(\frac{3}{2}, -1\right)$

The equation of the straight line is $X = \frac{3}{2}$

11

$m_1 = \frac{1-(-1)}{5-2} = \frac{2}{3}$, $m_2 = \frac{-a}{3}$

, ∵ the two straight lines are parallel

$m_1 = m_2$ $\therefore \frac{2}{3} = \frac{-a}{3}$

$a = -2$

12

$m_1 = \frac{3-2}{6-5} = 5$ $m_2 = \frac{1}{5}$

$m_2 = \frac{a}{5}$ $\therefore \frac{a}{5} = \frac{1}{5}$ $a = 1$

13

The slope of the straight line $L = \frac{4}{3}$

The slope of $\overline{AB} = \frac{4}{3}$

The slope of $\overline{AB} = \frac{y+3}{5-2}$ $\frac{y+3}{3} = \frac{4}{3}$

$y + 3 = 4$ $y = 1$

14

$m_1 = \frac{\text{Coefficient of } X}{\text{Coefficient of } y} = 2k - 1$

, $m_2 = \tan 45° = 1$

, ∵ the two straight lines are parallel

$m_1 = m_2$ $\therefore 2k - 1 = 1$

$2k = 2$ $k = 1$

15

The slope of $\overleftrightarrow{XY} = \frac{6+2}{-5-3} = -1$

The slope of the axis of symmetry of $\overline{XY} = 1$

The equation of the axis of symmetry of $\overline{XY}$ is

$y = X + c$

, ∵ the midpoint of $\overline{XY}$

$\left(\frac{3-5}{2}, \frac{-2+6}{2}\right) = (-1, 2)$

$(-1, 2)$ satisfies the equation $y = X + c$

$2 = -1 + c$ $c = 3$

∴ The equation of the axis of symmetry of XY is

$y = X + 3$

16

Let D be the midpoint of $\overline{BC}$

$D\left(\frac{3+1}{2}, \frac{7-3}{2}\right) = (2, 2)$

The slope of $\overleftrightarrow{AD} = \frac{6-2}{5-2} = \frac{8}{3}$

∴ The equation of $\overleftrightarrow{AD}$ is $y = \frac{8}{3}X + c$

$D \in \overline{AD}$

(2 , 2) satisfies its equation

$2 = \frac{8}{3} \times 2 + c$ $c = -\frac{22}{3}$

∴ The equation of $\overline{AD}$ is $y = \frac{8}{3}X - \frac{22}{3}$

17

The slope of $\overleftrightarrow{BC} = \frac{1+1}{-2-5} = -\frac{2}{7}$

The slope of the perpendicular straight line to it $= \frac{7}{2}$

The equation of the perpendicular to $\overline{BC}$ is

$y = \frac{7}{2}X + c$

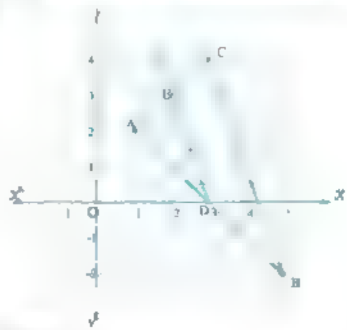
$A \in$ the perpendicular to $\overline{BC}$

(0 , 6) satisfies the equation

$6 = \frac{7}{2} \times 0 + c$ $\therefore c = 6$

∴ The equation of the perpendicular to $\overline{BC}$ from the point A is $y = \frac{7}{2}X + 6$

18



In △ABC

D is the midpoint of $\overline{AB}$, $\overline{DE} // \overline{BC}$

∴ E is the midpoint of $\overline{AC}$, $DE = \frac{1}{2}$ BC

$DE = \frac{1}{2}\sqrt{(5-3)^2+(-2-4)^2}$

$= \frac{1}{2}\sqrt{4+36} = \frac{1}{2}\sqrt{40}$

$= \frac{1}{2}\times 2\sqrt{10} = \sqrt{10}$ length unit

The slope of $\overline{BC} = \frac{4+2}{3-5} = -3$

The slope of $\overline{DE} = -3$

The equation of $\overleftrightarrow{DE}$ is $y = -3X + c$

$\because$ D is the midpoint of $\overline{AB} = \left(\frac{1+5}{2}, \frac{2-2}{2}\right) = (3, 0)$

$(3, 0)$ satisfies its equation

$\therefore 0 = -3\times 3 + c \qquad \therefore c = 9$

$\therefore$ The equation of $\overleftrightarrow{DE}$ is $y = -3X + 9$

19

The slope of $\overline{AC} = \frac{6-4}{-1-5} = -\frac{2}{6} = -\frac{1}{3}$

$\because$ the two diagonals of the square are perpendicular

$\therefore$ The slope of $\overline{BD} = 3$

$\therefore$ The equation of $\overleftrightarrow{BD}$ is $y = 3X + c$

The midpoint of $\overline{AC} = \left(\frac{5-1}{2}, \frac{6+4}{2}\right) = (2, 5)$

$(2, 5)$ satisfies the equation of $\overleftrightarrow{BD}$

$\therefore 5 = 2\times 3 + c \qquad \therefore c = -1$

The equation of $\overleftrightarrow{BD}$ is : $y = 3X - 1$

20

$\because$ The slope of $\overline{AC} = \frac{3-0}{1-6} = -\frac{3}{5}$

, $\overline{AC} \perp \overline{BD}$

$\therefore$ The slope of $\overline{BD} = \frac{5}{3}$

The equation of $\overleftrightarrow{BD}$ is $y = \frac{5}{3}X + c$

$\because$ The two diagonals of the rhombus bisect each other

$\therefore$ The midpoint of $\overline{AC} = \left(\frac{1+6}{2}, \frac{3-0}{2}\right) = (3.5, 1.5)$

$\therefore (3.5, 1.5)$ satisfies the equation of $\overleftrightarrow{BD}$

$1.5 = \frac{5}{3}\times 3.5 + c \qquad \therefore c = -4\frac{1}{3}$

$\therefore$ The equation of $\overleftrightarrow{BD}$ is $y = \frac{5}{3}X - 4\frac{1}{3}$

21

$\because$ The slope of $\overline{AB} = \frac{-3-3}{-1-2} = 2$

The equation of $\overleftrightarrow{AB}$ is : $y = 2X + c$

, $\because \overleftrightarrow{AB}$ passes through the point $(2, 3)$

$3 = 2\times 2 + c$

$\therefore c = -1$

$\therefore$ The equation of $\overleftrightarrow{AB}$ is : $y = 2X - 1$

by substituting in the equation of $\overleftrightarrow{AB}$ by $X = 2k + 1$

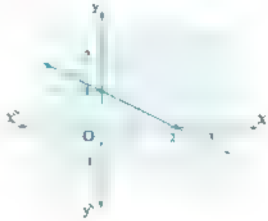
$y = 2(2k+1) - 1 = 4k + 2 - 1 = 4k + 1$

The point C $(2k+1, 4k+1)$ satisfies the equation of $\overleftrightarrow{AB}$

$C \in \overleftrightarrow{AB}$

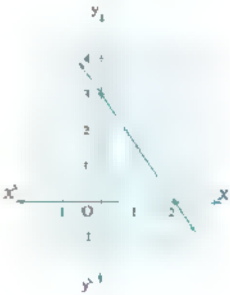
22

1



2]

3



23 The slope of the straight line = 2 and the length of the intercepted part from y-axis = 3 units

24

[1] The straight line passes through the two points (0 , 3) and (4 , 5)

$\because$ The slope $= \frac{5-3}{4-0} = \frac{1}{2}$

[2] 3 units from the positive part of y-axis

[3] The equation is $y = \frac{1}{2}X + 3$

[4] 6 units from the negative part of X-axis

[5] The area of the triangle $= \frac{1}{2} \times 3 \times 6 = 9$ square units

25

[1] $\because$ The slope of the straight line $= \frac{3-1}{2-1} = 2$

The equation of the straight line is $y = 2X + c$

$\because$ The point (1 , 1) $\in$ the straight line

$\therefore$ [illegible] $= 2 \times 1 + c$ $\qquad \therefore c =$ [illegible] 1

The equation of the straight line is $y = 2X - 1$

[2] One unit of the negative part of y-axis

[3] $\because$ The point (3 , a) satisfies the equation

a [illegible] 2×3 [illegible] 1 [illegible] 5

26

$\because \overleftrightarrow{AB}$ cuts two equal parts of the two axes

OA = OB

$\because$ In Δ AOB which is right-angled at O

m ($\angle$ ABO) = m ($\angle$ BAO) = 45°

$\because \overleftrightarrow{AB}$ makes with the positive direction of the X-axis an angle of measure 135°

The slope of $\overline{AB}$ = tan 135° = − 1

$\therefore$ k = the slope of $\overline{AB}$ [illegible] 1

[illegible] y [illegible] X + c

, $\because$ (2 , 3) $\in \overline{AB}$

[illegible] 3 [illegible] 2 + c $\qquad \therefore$ c = 5 $\qquad$ (First req.)

$\therefore$ OA = OB = 5 length units

$\therefore$ The area of Δ ABO $= \frac{1}{2} \times 5 \times 5$ [illegible] 12.5 square units

(Second req.)

27

$\because \Delta$ ABO is equilateral

, C is the midpoint of $\overline{AB}$

$\overline{OC} \perp \overline{AB}$

m ($\angle$ BOC) = 30°

tan ($\angle$ BOC) = tan 30° $= \frac{1}{\sqrt{3}}$

$\therefore$ The equation of $\overleftrightarrow{OC}$ is : $y = \frac{1}{\sqrt{3}}X + c$

, $\because O \in \overleftrightarrow{OC}$ $\quad \therefore c = 0$

$\therefore$ The equation of $\overleftrightarrow{OC}$ is : $y = \frac{1}{\sqrt{3}}X$

28

Let : D (X , y)

, $\because$ the midpoint of $\overline{AB}$ = The midpoint of $\overline{OD}$

, $\because$ the midpoint of $\overline{AB} = \left(\frac{6+2}{2}, \frac{6+2}{2}\right) = (4 , 4)$

, $\because$ the midpoint of $\overline{OD} = \left(\frac{0+X}{2}, \frac{0+y}{2}\right)$

$\therefore (4 , 4) = \left(\frac{X}{2}, \frac{y}{2}\right)$ $\qquad \therefore \frac{X}{2} = 4$ $\qquad X = 8$

, $\frac{y}{2} = 4$ $\qquad \therefore y = 8$

$\therefore$ D (8 , 8) $\qquad$ (First req.)

$\overleftrightarrow{OD}$ passes through the origin point

its equation is : y = m X (m is the slope)

, $\because$ the slope of $\overleftrightarrow{OD} = \frac{8-0}{8-0} = 1$

$\therefore$ The equation of $\overleftrightarrow{OD}$ is : y = X $\qquad$ (Second req.)

, $\because$ the slope of $\overleftrightarrow{OD}$ = tan ($\angle$ DOC)

$\therefore$ tan ($\angle$ DOC) = 1 $\qquad \therefore$ m ($\angle$ DOC) = 45°

m ($\angle$ DOE) = 180° − 45° = 135° $\qquad$ (Third req.)

29

Let the point A $(0 , y)$

The straight line L_1 passes through the point A $(0 , y)$

$\therefore 2 \times 0 - y + 2 = 0$ $\qquad y = 2 \qquad A(0 , 2)$

$, m_1 = \frac{-2}{-1} = 2 \quad , L_1 \perp L_2$

$m_1 m_2 = -1 \qquad 2 \times m_2 = -1$

$m_2 = \frac{-1}{2}$

The equation of L_2 is $y = -\frac{1}{2} X + c$

$, \because$ the straight line L_2 passes through the point A $(0 , 2)$

$2 = \frac{-1}{2} \times 0 + c \qquad \therefore c = 2$

$\therefore$ The equation of L_2 is : $y = -\frac{1}{2} X + 2$

30

1 **First :** $\tan(\angle ABO) = \frac{4}{3}$

$\therefore m(\angle ABO) \simeq 53^\circ\ 7'\ 48''$

From Δ ABO

$m(\angle BAO) = 180^\circ - (90^\circ + 53^\circ\ 7'\ 48'')$

$= 36^\circ\ 52'\ 12''$

Second : Let the point $(X , 0)$

$, \because \tan(\angle ABO) = \frac{4}{3}$

$\therefore \frac{8}{X} = \frac{4}{3} \qquad 4X = 24$

$\therefore X = 6 \qquad B(6 , 0)$

2 **First :** The slope of $\overleftrightarrow{AB} = \frac{0-8}{6-0} = \frac{-8}{6} = \frac{-4}{3}$

Second : The required straight line is perpendicular to $\overleftrightarrow{AB}$

$\because$ The slope of $\overleftrightarrow{AB} = -\frac{4}{3}$

The slope of the required straight line is $= \frac{3}{4}$

The equation of the required straight line is

$y = \frac{3}{4} X + c$

$, \because$ the straight line passes throught the point O $(0 , 0)$

$0 = \frac{3}{4} \times 0 + c \qquad \therefore c = 0$

$\therefore$ The equation of the required straight line is

$y = \frac{3}{4} X$

31

1 $O = (0 , 0)$

$, \because A = (X , 0) , B = (0 , y) , C = (4 , 3)$

where C is the midpoint of AB

$\frac{X + 0}{2} = 4 \qquad X = 8$

$, \frac{y + 0}{2} = 3 \qquad y = 6$

$A(8 , 0) , B(0 , 6)$

2 OA = 8 length units , OB = 6 length units

$, CA = \sqrt{(8-4)^2 + (0-3)^2} = 5$ length units

$, CB = CA = 5$ length units

$, CO = \sqrt{4^2 + 3^2} = 5$ length units

3 The slope of $\overleftrightarrow{AB} = \frac{6-0}{0-8} = -\frac{3}{4}$

, the slope of $\overleftrightarrow{OC} = \frac{0-3}{0-4} = \frac{3}{4}$

, the slope of $\overleftrightarrow{OA}$ = zero

, the slope of $\overleftrightarrow{OB}$ is undefined

4 The equation of $\overleftrightarrow{AB}$ is : $y = -\frac{3}{4} X + 6$

, the equation of $\overleftrightarrow{CO}$ is : $y = \frac{3}{4} X$

32

$\because$ The slope of $\overleftrightarrow{CD} = \frac{3-1}{4-3} = 2$

$\therefore$ The equation of $\overleftrightarrow{CD}$ is : $y = 2X + c$

$, \because C \in \overleftrightarrow{CD}$

$(3 , 1)$ satisfies the equation

$1 = 2 \times 3 + c \qquad \therefore 1 = 6 + c$

$\therefore c = -5$

The equation of $\overleftrightarrow{CD}$ is : $y = 2X - 5$

Let : $A(X , 0) \qquad , \because A \in \overleftrightarrow{CD}$

$\therefore (X , 0)$ satisfies the equation of $\overleftrightarrow{CD}$

$0 = 2X - 5 \qquad X = \frac{5}{2}$

$A\left(\frac{5}{2} , 0\right) \qquad \therefore AO = 2\frac{1}{2}$ length units

$, \because \overleftrightarrow{CD}$ intercepts from the negative side of y-axis 5 length units

OB = 5 length units

33

1 The slope of $\overleftrightarrow{AC} = \frac{-1}{-1} = 1$

, from the equation of $\overleftrightarrow{AC}$: $y = X - 3$

OH = 3 length unit

2 $\because$ The slope of $\overleftrightarrow{BC} = \sqrt{3}$

$\therefore \tan(\angle CBD) = \sqrt{3} \qquad \therefore m(\angle CBD) = 60^\circ$

$, \because$ the slope of $\overleftrightarrow{AC} = 1$

$\tan(\angle CAD) = 1 \qquad m(\angle CAD) = 45^\circ$

[3] $\angle$ CBD is an exterior angle of Δ ABC

m ($\angle$ CBD) = m ($\angle$ ACB) + m ($\angle$ CAD)

$\therefore$ m ($\angle$ ACB) = 60° − 45° = 15°

34

C (5 , 2)

OB = 5 length units , BC = 2 length units

, $\because$ ABCD is a square

$\therefore$ AB = BC = AD = 2 length units

$\therefore$ OA = OB − BA = 5 − 2 = 3 length units

$\therefore$ The slope of the straight line L = tan ($\angle$ AOD)

$= \frac{AD}{OA} = \frac{2}{3}$

$\therefore$ The equation of the straight line L is :

$y = \frac{2}{3} X + c$

, $\because$ the straight line L passes through the origin point

$\therefore$ c = 0

$\therefore$ The equation of the straight line L is : $y = \frac{2}{3} X$

35

Let the length of $\overline{OA}$ = ℓ length unit

ABCD is a square $\therefore$ AB = BC

, $\because$ OA = AB

$\therefore$ OA = AB = BC = ℓ length unit

In Δ OBC : $\because$ $\overline{BC} \perp \overline{BO}$ (properties of the square)

$\therefore$ tan ($\angle$ BOC) = $\frac{BC}{BO} = \frac{\ell}{2\ell} = \frac{1}{2}$

$\therefore$ The slope of $\overleftrightarrow{OC}$ = tan ($\angle$ BOC) = $\frac{1}{2}$

$\therefore$ The equation $\overleftrightarrow{OC}$ is : $y = \frac{1}{2} X + c$

, $\because$ $\overleftrightarrow{OC}$ passes through the origin point

$\therefore$ c = 0

$\therefore$ The equation of $\overleftrightarrow{OC}$ is : $y = \frac{1}{2} X$

36

The straight line L_1 passes through the origin point

$\therefore$ Its equation is : y = m X

, $\because$ the slope of the straight line L_1 = tan 45° = 1

$\therefore$ The equation of the straight line L_1 is : y = X (Q.E.D.1)

, $\because$ the straight line L_1 // the straight line L_2

$\therefore$ The slope of the straight line L_1 = the slope of the straight line L_2 = 1

$\therefore$ The equation of the straight line L_2 is : y = X + c

, $\because$ the straight line L_2 passes through the point A (1 , 5)

$\therefore$ 5 = 1 + c $\therefore$ c = 4

$\therefore$ The equation of the straight line L_2 is :

y = X + 4 (Q.E.D.2)

Let B (X , y)

$\because$ B belongs to the straight line L_1 $\therefore$ y = X

, $\overleftrightarrow{AB} \perp L_1$ $\therefore$ The slope of $\overleftrightarrow{AB}$ = −1

$\therefore$ $\frac{y-5}{X-1} = -1$ $\therefore$ y − 5 = −X + 1

, $\because$ X = y $\therefore$ y − 5 = −y + 1

$\therefore$ 2 y = 6 $\therefore$ y = 3

$\therefore$ X = 3 $\therefore$ B (3 , 3)

$\therefore$ The length of $\overline{AB} = \sqrt{(1-3)^2 + (5-3)^2}$

$= 2\sqrt{2}$ length unit (Q.E.D.3)

37

[1] 2 m

[2] The velocity of the particle = The slope of the straight line passing through the two points (0 , 2)

, (4 , 4) = $\frac{4-2}{4-0} = \frac{1}{2}$

$\therefore$ The velocity = $\frac{1}{2}$ m./sec

[3] d = $\frac{1}{2}$t + 2

[4] 2 metre

[5] 7 seconds

38

[1] 90 km. [2] 2.5 hours

[3] The velocity of the car = The slope of the straight line which passes through the two points (0.5 , 30)

, (2 , 120) = $\frac{120-30}{2-0.5}$ = 60 km./hr

[4] d = 60 t

39

[1] Yes , A and B start motion at the same time

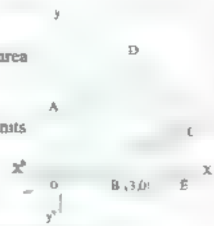
[2] 3.2 minutes approximately

3] The velocity of A = The slope of the straight line passing through the two points (0 , 9) , (5 , 0)

$= \frac{0-9}{5-0} = -1.8$ km./minutes

4] d = t

30



∵ ABCD is a square its area equals 25 square units

∴ AB = BC = 5 length units

, ∵ B (3 , 0)

∴ OB = 3 length units

In Δ AOB which is right-angled at O

AO = 4 length units (Pythagoras)

, ∵ Δ AOB ≡ Δ BEC (prove by yourself)

∴ EC = OB = 3 length units

, EB = AO = 4 length units

∴ OE = 7 length units ∴ The point C (7 , 3)

, ∵ $\overleftrightarrow{CO}$ passes through the origin point

∴ The equation of $\overleftrightarrow{CO}$ is :

y = m x (where m is the slope)

, ∵ the slope of $\overleftrightarrow{CO} = \frac{0-3}{0-7} = \frac{3}{7}$

∴ The equation of $\overleftrightarrow{CO}$ is : $y = \frac{3}{7} x$

i.e. $7y = 3x$ (The req.)

31

Let OB = OA = x

, ∵ Δ AOB is right angled at O

∴ $x^2 + x^2 = (2\sqrt{2})^2$ (Pythagoras)

∴ $2x^2 = 8$ ∴ $x^2 = 4$

∴ $x = 2$ ∴ OB = OA = 2

∴ The point B (0 , 2)

Let the equation of $\overleftrightarrow{AB}$ be : $y = m x + n$

, ∵ The slope of $\overleftrightarrow{AB} = \tan(\angle BAO) = \frac{BO}{AO} = 1$

, n = 2

∴ The equation of $\overleftrightarrow{AB}$ is : $y = x + 2$

, ∵ C = (1 , k) ∈ $\overleftrightarrow{AB}$ ∴ k = 1 + 2

∴ k = 3 ∴ C (1 , 3)

, ∵ $\overleftrightarrow{CD} \perp \overleftrightarrow{AB}$, the slope of $\overleftrightarrow{AB}$ = 1

∴ The slope of $\overleftrightarrow{CD}$ = − 1

∴ The equation of $\overleftrightarrow{CD}$ is : $y = -x + l$

, ∵ (1 , 3) ∈ $\overleftrightarrow{CD}$ ∴ $3 = -1 + l$

∴ $l = 4$

∴ The equation of $\overleftrightarrow{CD}$ is : $y = -x + 4$ (The req.)

Answers of exams on unit five

Model 1

1

1 c 2 a 3 a
4 c 5 c 6 d

2 [a] Prove by yourself

[b] The equation of the straight line is $5y + 2X - 26 = 0$

3 [a] 1 the area of the circle = 25π square units

2 $y = \frac{3}{4}X - \frac{7}{4}$

[b] 1 The point of intersection of its diagonals = (1 , 0)

2 The area of the rhombus = 24 square units.

4 [a] $X = 8$ or $X = 4$

[b] Prove by yourself , $\sqrt{26}$ length units

5 [a] $y = 2X + 1$

[b] $y = -X + 3$

Model 2

1

1 a 2 a 3 d
4 d 5 b 6 d

2 [a] 1 D (−2 , 9)

2 41 square units

[b] 1 BC = $2\sqrt{2}$ length units

2 The area of the figure OABC = 4 square units

3 m ($\angle$ OCB) = 45°

3 [a] Prove by yourself

[b] $X = 5$ or $X = 1$

4 [a] Prove by yourself

, the area of Δ ABC = 5 square units

[b] The equation of the straight line is : $y = \frac{1}{3}X + \frac{5}{3}$

5 [a] Prove by yourself

[b] m = 10

Answers of accumulative basic skills

1 d	2 a	3 c	4 b
5 b	6 a	7 a	8 a
9 d	10 d	11 b	12 a
13 d	14 a	15 a	16 d
17 c	18 c	19 d	20 c
21 c	22 c	23 b	24 d
25 c	26 c	27 d	28 c
29 c	30 d	31 b	32 d
33 a	34 c	35 c	36 b
37 c	38 b	39 c	

GUIDE ANSWERS

of The Notebook

- Accumulative tests.
- Final Examinations.

Answers of accumulative tests on algebra & statistics

Accumulative test 1

1
1 b 2 d 3 b 4 b
5 b 6 a 7 d 8 b

2
1 $X \times Y = \{(2, 3), (2, 4), (2, 5)\}$
2 $n(Y^2) = 9$ 3 $X^2 = \{(2, 2)\}$

3 $X + 2y = 11$

Accumulative test 2

1
1 c 2 d 3 a 4 a
5 a 6 d 7 c 8 c

2
1 $(X \cap Y) \times Z = \{(2, 7), (2, 2), (3, 7), (3, 2)\}$
2 $(X - Y) \times Z = \{(1, 7), (1, 2), (4, 7), (4, 2)\}$

3 $R = \{(\frac{1}{2}, 2), (1, 1), (-\frac{1}{2}, -2), (-1, -1)\}$
, represent by yourself , R isn't a function
Show by yourself

Accumulative test 3

1
1 b 2 d 3 b 4 d
5 c 6 d 7 d 8 b

2
1 The domain = $\{3, 5, 7\}$
2 $f(X) = 3X$

3 $b = 1$

Accumulative test 4

1
1 b 2 c 3 c 4 b
5 c 6 b 7 b 8 a

2 Graph by yourself.
1 The vertex of the curve is : $(0, 2)$
2 The equation of the axis of symmetry is : $X = 0$
3 The maximum value of the function = 2

3
1 $n(X \times Z) = 6$
2 $(Y \cap X) \times (X - Y) = \{(5, 1), (5, 6)\}$

Accumulative test 5

1
1 b 2 b 3 b 4 c
5 d 6 d 7 a 8 a

2
1 $f(3) + 3g(\sqrt{2}) = 12$
2 Prove by yourself

3 The number is 1

Accumulative test 6

1
1 c 2 b 3 b 4 b
5 c 6 d 7 b 8 d

2 Prove by yourself

3 $\frac{28}{3}$

Accumulative test 7

1
1 d 2 c 3 c 4 d
5 a 6 c 7 d 8 c

2 Prove by yourself

3 Prove by yourself

Accumulative test 8

1
1 c 2 c 3 b 4 d
5 a 6 c 7 d 8 c

2 $y = 3 + \frac{2}{X}$, $\frac{11}{3}$

3 Prove by yourself

Accumulative test 9

1
1 a 2 c 3 c 4 b
5 d 6 c 7 c 8 b

2 The mean = 23 , the standard deviation $\simeq 4.24$

3 Prove by yourself

Answers of model examinations of the school book of algebra & statistics

Model 1

1

1 b 2 c 3 b 4 a 5 c 6 b

2

[a] 1 $Y = \{2 , 5 , 7\}$

2 $Y \times X = \{(2 , 2) , (5 , 2) , (7 , 2)\}$

[b] Let $\frac{a}{b} = \frac{c}{d} = m$, where $m > 0$

$\therefore a = b\,m , c = d\,m$

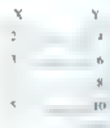
$\therefore$ L.H.S $= \frac{a}{b - a} = \frac{b\,m}{b - b\,m} = \frac{b\,m}{b\,(1 - m)} = \frac{m}{1 - m}$ (1)

, R.H.S $= \frac{c}{d - c} = \frac{d\,m}{d - d\,m} = \frac{d\,m}{d\,(1 - m)} = \frac{m}{1 - m}$ (2)

From (1) , (2) : $\therefore \frac{a}{b - a} = \frac{c}{d - c}$

3

[a] 1 $R = \{(2 , 4) , (3 , 6) , (5 , 10)\}$



2 R is a function because every element of X has only one image in Y

[b] Let the number be X

$\frac{X + 7}{X + 11} = \frac{2}{3}$

$3\,X + 21 = 2\,X + 22$ $\therefore X = 1$

The required number is 1

4

[a] 1 The range $= \{3 , 1 , 5\}$

2 $a + b = 8$

[b] 1 $\because y \propto \frac{1}{X}$ $\therefore X\,y = m$

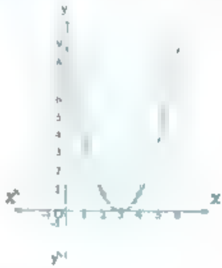
$m = 2 \times 3 = 6$ $\therefore X\,y = 6$

2 At $X = 1.5$ $\therefore y = \frac{6}{1.5} = 4$

5

[a] $f(X) = (X - 3)^2$

X	0	1	2	3	4	5	6
$f(X)$	9	4	1	0	1	4	9



From the graph :

The vertex of the curve is $(3 , 0)$

, the minimum value $= 0$

, the equation of the axis of symmetry is : $X = 3$

[b] Form the table by yourself

, then the arithmetic mean $= 7$

, the standard deviation $\simeq 1.41$

Model 2

1

1 a 2 c 3 d 4 b 5 c 6 a

2

[a] 1 $n(X \times Z) = 2$

2 $(Y \cap X) \times Z = \{2\} \times \{3\} = \{(2 , 3)\}$

[b] $\because$ b is the middle proportional between a and c

$\therefore \frac{a}{b} = \frac{b}{c} = m$ $\therefore b = c\,m , a = c\,m^2$

L.H.S $= \frac{a - b}{a - c} = \frac{c\,m^2 - c\,m}{c\,m^2 - c} = \frac{c\,m\,(m - 1)}{c\,(m^2 - 1)}$

$= \frac{c\,m\,(m - 1)}{c\,(m - 1)\,(m + 1)} = \frac{m}{m + 1}$ (1)

, R.H.S. $= \frac{b}{b + c} = \frac{c\,m}{c\,m + c} = \frac{c\,m}{c\,(m + 1)} = \frac{m}{m + 1}$ (2)

From (1) , (2) : $\therefore \frac{a - b}{a - c} = \frac{b}{b + c}$

3

[a] 1 $R = \{(1 , 6) , (3 , 4) , (4 , 3) , (5 , 2)\}$

2 R is a function because every element of X has only one image in Y

[b] $5a = 3b$ $\qquad \frac{a}{b} = \frac{3}{5}$

$\therefore a = 3m , b = 5m$

$\therefore \frac{7a + 9b}{4a + 2b} = \frac{7 \times 3m + 9 \times 5m}{4 \times 3m + 2 \times 5m} = \frac{66m}{22m} = 3$

4

[a] $\because f(X) = 4X + b , f(3) = 15$

$4 \times 3 + b = 15 \qquad b = 3$

[b] 1 $\because y \propto X \qquad \therefore y = mX$

$6 = m \times 3 \qquad \therefore m = 2$

$\therefore y = 2X$

2 At $X = 5 \qquad \therefore y = 2 \times 5 = 10$

5

[a] $f(X) = 4 - X^2$

X	-3	-2	-1	0	1	2	3
$f(X)$	-5	0	3	4	3	0	-5

From the graph : The vertex of the curve is (0 , 4)

, the maximum value = 4

, the equation of the axis of symmetry is : $X = 0$

[b] Form the tables by yourself

, then the mean = 2.26

, the standard deviation = 1.06

Answers of model for the merge students

1

1 the first	2 the third	3 30
4 X	5 9	6 9

2

1 a	2 a	3 d
4 b	5 c	6 c

3

1 ✔	2 ✘	3 ✘
4 ✔	5 ✔	6 ✔

4

1 1	2 6	3 8
4 10	5 ± 6	6 2

Answers of governorates' examinations of algebra & statistics

1 Cairo

1

1 a 2 a 3 c 4 d 5 b 6 c

2

[a] 1 $X \times Y = \{(2, 3), (2, 4), (2, 5)\}$

2 $n(Y^2) = 9$

3 $X^2 = \{(2, 2)\}$

[b] $\because \frac{a}{b} = \frac{3}{5}$ $\therefore a = 3m, b = 5m$

$\therefore \frac{7a + 9b}{4a + 2b} = \frac{7 \times 3m + 9 \times 5m}{4 \times 3m + 2 \times 5m} = \frac{66m}{22m} = 3$

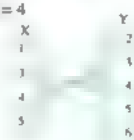
3

[a] 1 $\because y \propto \frac{1}{x}$ $\therefore y = \frac{m}{x}$

$\therefore 3 = \frac{m}{2}$ $\therefore m = 6$ $\therefore y = \frac{6}{x}$

2 at $x = 1.5$ $\therefore y = \frac{6}{1.5} = 4$

[b] $R = \{(1, 6), (3, 4), (4, 3), (5, 2)\}$



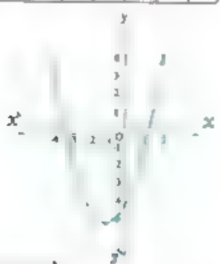
R is a function because every element of X has only one image in Y

4

[a] Form the tables by yourself, then $\sigma \simeq 1.73$

[b] $f(x) = x^2 + 2x - 4$

x	-4	-3	-2	-1	0	1	2
$f(x)$	4	-1	-4	-5	-4	-1	4



From the graph :

1 The vertex of the curve is : $(-1, -5)$

2 The equation of the axis of symmetry is $x = -1$

5

[a] $\because$ b is the middle proportional between a and c

$\therefore b^2 = ac$

$\therefore$ L.H.S $= \frac{a^2 + b^2}{b^2 + c^2} = \frac{a^2 + ac}{ac + c^2} = \frac{a(a + c)}{c(a + c)} = \frac{a}{c} =$ R.H.S.

[b] 1 $\because f(2) = 2^2 - 2 \times 2 = 4 - 4 = 0$

$, g(2) = 2 - 2 = 0$

$\therefore f(2) = g(2)$

2 $\because g(k) = 7$

$\therefore k - 2 = 7$ $\therefore k = 9$

2 Giza

1

1 a 2 c 3 b 4 b 5 b 6 a

2

[a] $\because (x + 3, 9) = (5, y^2)$

$\therefore x + 3 = 5$ $\therefore x = 2$

$, y^2 = 9$ $\therefore y = \pm 3$

[b] 1 $\because y \propto \frac{1}{x}$ $\therefore y = \frac{m}{x}$

$\therefore 4 = \frac{m}{2}$ $\therefore m = 8$ $\therefore y = \frac{8}{x}$

2 at $x = 8$

$\therefore y = \frac{8}{8} = 1$

3

[a] 1 $R = \{(0, 0), (2, 1), (4, 2), (6, 3)\}$

R is not a function because the elements 1, 3 and 5 $\in X$ have no images in X

2 No

3 6 R X $\therefore x = 3$

[b] $\frac{a}{b} = \frac{b}{c} = m$ $\therefore b = cm$, $a = cm^2$

$\therefore \frac{a - b}{a - c} = \frac{cm^2 - cm}{cm^2 - c} = \frac{cm(m - 1)}{c(m^2 - 1)}$

$= \frac{cm(m - 1)}{c(m - 1)(m + 1)} = \frac{m}{m + 1}$ (1)

$, \frac{b}{b + c} = \frac{cm}{cm + c} = \frac{cm}{c(m + 1)} = \frac{m}{m + 1}$ (2)

From (1), (2). $\therefore \frac{a - b}{a - c} = \frac{b}{b + c}$

4

[a] AO = 4 units. $\therefore A(0, 4)$

A (0 , 4) belongs to the curve of the function f

$\therefore$ A satisfies the equation of the curve

$\therefore 4 = m - (0)^2$ $\therefore m = 4$

$\because$ The curve of the function intersects X-axis at the two points B and C

$\therefore 0 = 4 - X^2$ $\therefore X^2 = 4$

$\therefore X = 2$ or -2

$\therefore B = (2, 0)$, $C = (-2, 0)$

$\therefore$ BC = 4 units

The area of $\Delta ABC = \frac{1}{2} \times 4 \times 4 = 8$ square units

[b] 1 $\because f(3) = 9$ $\therefore 2 \times 3 + a = 9$ $\therefore a = 3$

2 At $f(X) = 0$ $\therefore 2X + 3 = 0$

$\therefore 2X = -3$ $\therefore X = \frac{-3}{2}$

$\therefore$ The intersection point with X-axis is $\left(\frac{-3}{2}, 0\right)$

5

[a] $\frac{X}{2} = \frac{y}{3} = \frac{z}{4} = \frac{2X - y + 5z}{3m}$

, multiplying the two terms of the 1st ratio by 2 and the 2nd by − 1 and the 3rd by 5 and adding the antecedents and consequents of the three ratios

$\therefore \frac{2X - y + 5z}{4 - 3 + 20}$ = one of the given ratios.

$\therefore \frac{2X - y + 5z}{21} = \frac{2X - y + 5z}{3m}$

$3m = 21$ $\therefore m = 7$

[b] Form the table by yourself , then $\sigma \simeq 2.83$

3 Alexandria

1

1 c 2 a 3 b 4 d 5 d 6 b

2

[a] 1 $n(X \times Z) = 2$

2 $Y \cap X = \{2\}$

$, (Y \cap X) \times Z = \{2\} \times \{3\} = \{(2, 3)\}$

[b] Let the number be X

$\therefore \frac{5 + X^2}{11 + X^2} = \frac{3}{5}$ $\therefore 25 + 5X^2 = 33 + 3X^2$

$\therefore 2X^2 = 8$ $\therefore X^2 = 4$

$\therefore X = 2$ or $X = -2$ $\therefore$ The number is 2 or − 2

3

[a] $\frac{X}{3} = \frac{y}{4} = \frac{z}{5} = m$

$\therefore X = 3m , y = 4m , z = 5m$

$\therefore \frac{2y - z}{3X - 2y + z} = \frac{8m - 5m}{9m - 8m + 5m} = \frac{3m}{6m} = \frac{1}{2}$

[b] $R = \{(1, 6), (3, 4), (4, 3), (5, 2)\}$

R is a function because every element in X has only one image in Y

4

[a] 1 $\because y \propto \frac{1}{X}$ $\therefore Xy = m$

$\therefore 4 \times 2 = m$ $\therefore m = 8$ $\therefore Xy = 8$

2 at $X = 16$

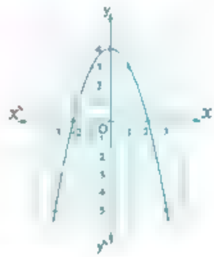
$16y = 8$ $\therefore y = \frac{1}{2}$

[b] Form the tables by yourself , then $\sigma \simeq 4.24$

5

[a] $f(X) = 4 - X^2$

X	−3	−2	−1	0	1	2	3
$f(X)$	−5	0	3	4	3	0	−5



From the graph :

1 The vertex of the curve is (0 , 4)

2 The maximum value = 4

3 The equation of the line of symmetry is : $X = 0$

[b] $f(1) + f(3) = -7$

$\therefore 5 - a + 3 - 2a = -7$

$\therefore -3a = -15$ $\therefore a = 5$

4

1

1 c   2 d   3 a   4 b   5 d   6 c

2

[a] 1 $y \propto \frac{1}{X}$ $\therefore Xy = m$

$\therefore 2 \times 3 = m$ $\therefore m = 6$ $\therefore Xy = 6$

2 at $X = \frac{3}{2}$

$\therefore \frac{3}{2}\, y = 6$ $\therefore y = 4$

[b] $\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a - b + 5c}{3X}$

, multiplying the two terms of the 1st ratio by 2 and the 2nd by − 1 and the 3rd by 5 and adding the antecedents and consequents of the three ratios

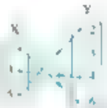
$\therefore \frac{2a - b + 5c}{4 - 3 + 20}$ = one of the given ratios

$\therefore \frac{2a - b + 5c}{21} = \frac{2a - b + 5c}{3X}$

$\therefore 3X = 21$ $\therefore X = 7$

3

[a] 1 $R = \{(1 , 6) , (3 , 4) , (5 , 2)\}$



2 R is a function because every element in X has only one image in Y

[b] $\because \frac{a}{b} = \frac{b}{c} = m$ $\therefore b = cm$, $a = cm^2$

$\therefore \frac{a - c}{a - b} = \frac{cm^2 - c}{cm^2 - cm} = \frac{c(m^2 - 1)}{cm(m - 1)}$

$= \frac{(m - 1)(m + 1)}{m(m - 1)} = \frac{m + 1}{m}$ (1)

, $\frac{b + c}{b} = \frac{cm + c}{cm} = \frac{c(m + 1)}{cm} = \frac{m + 1}{m}$ (2)

From (1) , (2) : $\frac{a - c}{a - b} = \frac{b + c}{b}$

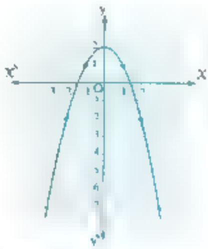
4

[a] 1 $X = \{1\}$, $Y = \{1 , 3 , 5\}$

2 $Y \times X = \{(1 , 1) , (3 , 1) , (5 , 1)\}$

[b] $f(X) = 2 - X^2$

X	−3	−2	−1	0	1	2	3
f (X)	−7	−2	1	2	1	−2	−7



From the graph :

1 The vertex of the curve is : (0 , 2)

2 The maximum value = 2

3 The equation of the line of symmetry is : X = 0

5

[a] 1 The range = {3 , 1 , 5}

2 ∵ R is a function on X

Each element in X appears only once as a first projection in R

$\therefore a = 3 , b = 5$ or $a = 5 , b = 3$

$\therefore a + b = 3 + 5 = 8$

[b] Form the tables by yourself

, then the mean $(\overline{X}) = 6$, $\sigma = 2.32$

5

1

1 b   2 d   3 c   4 c   5 d   6 d

2

[a] 1 $\because Y \cap Z = \{5\}$

$\therefore X \times (Y \cap Z) = \{4 , 3\} \times \{5\}$
$= \{(4 , 5) , (3 , 5)\}$

2 $\because X - Y = \{3\}$

$\therefore (X - Y) \times Z = \{3\} \times \{5 , 6\}$
$= \{(3 , 5) , (3 , 6)\}$

3 $n(Z^2) = 4$

[b] $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = dm$, $b = dm^2$, $a = dm^3$

$\because \frac{ab - cd}{b^2 - c^2} = \frac{d^2 m^5 - d^2 m}{d^2 m^4 - d^2 m^2} = \frac{d^2 m (m^4 - 1)}{d^2 m^2 (m^2 - 1)}$

$= \frac{d^2 m (m^2 - 1)(m^2 + 1)}{d^2 m^2 (m^2 - 1)}$

$= \frac{m^2 + 1}{m}$ (1)

$\because \frac{a + c}{b} = \frac{d m^3 + d m}{d m^2} = \frac{d m (m^2 + 1)}{d m^2} = \frac{m^2 + 1}{m}$ (2)

From (1) , (2) $\therefore \frac{ab - cd}{b^2 - c^2} = \frac{a + c}{b}$

3

[a] 1 $R = \{(-2 , -8) , (-1 , -1) , (1 , 1) , (2 , 8)\}$

2 R is a function because every element of X has only one image in Y , its range $= \{-8 , -1 , 1 , 8\}$

[b] $\because$ (0 , 3) satisfies the function

$\therefore 3 = a \times 0 + b$ $\therefore b = 3$

$, \because f(2) = 7$ $\therefore 7 = 2a + 3$

$\therefore 2a = 4$ $\therefore a = 2$

4

[a] Let the number be x

$\therefore \frac{x^2 + 7}{x^2 + 11} = \frac{4}{5}$ $\therefore 5x^2 + 35 = 4x^2 + 44$

$\therefore x^2 - 9 = 0$ $\therefore (x + 3)(x - 3) = 0$

$\therefore x = -3$ or $x = 3$

$\therefore$ The number is : -3 or 3

[b] 1 $\because y \propto \frac{1}{x^2}$ $\therefore x^2 y = m$

$\therefore 3^2 \times 4 = m$ $\therefore m = 36$ $\therefore x^2 y = 36$

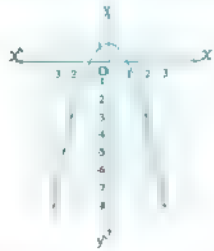
2 at $y = 9$

$\therefore x^2 \times 9 = 36$ $\therefore x^2 = 4$ $\therefore x = \pm 2$

5

[a] $f(x) = 1 - x^2$

x	-3	-2	-1	0	1	2	3
$f(x)$	-8	-3	0	1	0	-3	-8



From the graph :

1 The vertex of the curve is (0 , 1)

2 The equation of the axis of symmetry is $x = 0$

3 The area $= \frac{1}{2} \times 2 \times 1 = 1$ square unit

[b] Form the tables by yourself , then the mean $(\overline{X}) = 2$, $\sigma \simeq 0.96$

El-Gharbia

1

1 c 2 c 3 c 4 a 5 c 6 c

2

[a] $R = \{(2 , \frac{1}{2}) , (3 , \frac{1}{3})\}$

R is not a function because the elements $-1 \in X$, $0 \in X$ have no images in Y

[b] 1 inverse variation

2 $\because y \propto \frac{1}{x}$ $\therefore xy = m$ $\therefore m = 12$

3 at $x = 3$ $\therefore 3y = 12$ $\therefore y = 4$

3

[a] $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = d m$, $b = d m^2$, $a = d m^3$

$\because \frac{a}{b + d} = \frac{d m^3}{d m^2 + d} = \frac{d m^3}{d (m^2 + 1)} = \frac{m^3}{m^2 + 1}$ (1)

$, \frac{c^3}{c^2 d + d^3} = \frac{d^3 m^3}{d^3 m^2 + d^3} = \frac{d^3 m^3}{d^3 (m^2 + 1)} = \frac{m^3}{m^2 + 1}$ (2)

From (1) , (2) $\therefore \frac{a}{b + d} = \frac{c^3}{c^2 d + d^3}$

[b] $\boxed{1}$ $n(X^2) = 1$

$\boxed{2}$ $Z - Y = \{5, 6\}$, $X \cap Z = \{6\}$

$\therefore (Z - Y) \times (X \cap Z) = \{5, 6\} \times \{6\}$

$= \{(5, 6), (6, 6)\}$

4

[a] Let the two numbers be : $2X$, $3X$

$\because \frac{2X + 7}{3X - 12} = \frac{5}{3}$

$\therefore 15X - 60 = 6X + 21$

$\therefore 9X = 81 \qquad \therefore X = 9$

$\therefore$ The two numbers are 18 , 27

[b] $\because$ The straight line passes through $(a, 2a)$

$3a - 6 = 2a \qquad \therefore a = 6$

at $X = 0$

$\because f(0) = 3 \times 0 - 6 = -6$

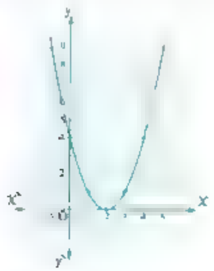
$\therefore$ The straight line intersects the y-axis at $(0, -6)$

5

[a] Form the table by yourself , then $\sigma \simeq 9.3$

[b] $f(X) = (X - 2)^2$

X	-1	0	1	2	3	4	5
$f(X)$	9	4	1	0	1	4	9



From the graph :

$\boxed{1}$ The equation of the axis of symmetry is : $X = 2$

$\boxed{2}$ The minimum value $= 0$

7

1

[a] $\boxed{1}$ a $\qquad$ $\boxed{2}$ d $\qquad$ $\boxed{3}$ d

[b] $\because$ b is the middle proportional between a and c

$\therefore b^2 = ac$

$$\text{L.H.S.} = \frac{2c^2 - 3b^2}{2b^2 - 3a^2} = \frac{2c^2 - 3ac}{2ac - 3a^2} = \frac{c(2c - 3a)}{a(2c - 3a)}$$

$$= \frac{c}{a} = \text{R.H.S.}$$

2

[a] $\boxed{1}$ c $\qquad$ $\boxed{2}$ a $\qquad$ $\boxed{3}$ b

[b] $\because (-1, 2)$ is the vertex of the curve

$-1 = \frac{-b}{2a} \qquad \therefore \frac{6}{2a} = 1$

$2a = 6 \qquad \therefore a = -3$

$\because (-1, 2) \in$ the curve of the function

$2 = -3 + 6 + c \qquad \therefore c = -1$

3

[a] $3a = 4b = 6c$

$a = \frac{4}{3}b$, $c = \frac{2}{3}b$

$a : b : c = \frac{4}{3}b : b : \frac{2}{3}b$ (multiplying by 3)

$a : b : c = 4b : 3b : 2b$ (dividing by b)

$a : b : c = 4 : 3 : 2$

$a = 4m$, $b = 3m$, $c = 2m$

$$\frac{3a + 2b}{a + 4c} = \frac{12m + 6m}{4m + 8m} = \frac{18m}{12m} = \frac{3}{2}$$

[b] $R = \{(-2, 2), (-1, 1), (0, 0), (1, -1), (2, -2)\}$

R is a function because every element of X has only one image in X

4

[a] $\because z \propto \frac{1}{y} \qquad \therefore z = \frac{m}{y}$

$\therefore 2 = \frac{m}{3} \qquad \therefore m = 6$

$\therefore z = \frac{6}{y} \qquad \therefore X = \frac{6}{y} + 8$

, at $X = 3$

$3 = \frac{6}{y} + 8 \qquad \therefore -5 = \frac{6}{y}$

$\therefore -5y = 6 \qquad \therefore y = -\frac{6}{5}$

[b] $\because d(2) = 2 \times 2 + 5 = 9$

$, 3\, r(3) = 3(3 - 6) = -9$

$\therefore d(2) + 3\, r(3) = 9 + (-9) = 0$

5

[a] Form the table by yourself , then $\overline{X} = 7$, $\sigma \simeq 1.41$

[b] $\because (X - 2 , 3^{y-1}) = (3 , 1)$

$\therefore X - 2 = 3 \qquad \therefore X = 5$

$, 3^{y-1} = 1 \qquad \therefore 3^{y-1} = 3^0$

$\therefore y - 1 = 0 \qquad \therefore y = 1$

8

1

1 c 2 c 3 b 4 a 5 d 6 a

2

[a] 1 R = {(2 , 4) , (3 , 6) , (5 , 10)}

2 R is a function because every element in X has only one image in Y

, the range = {4 , 6 , 10}

[b] Let the two numbers be : 3 m , 7 m

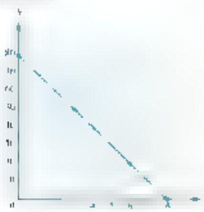
$\therefore \frac{3m + 5}{7m + 5} = \frac{1}{2} \qquad \therefore 6m + 10 = 7m + 5$

$\therefore m = 5$

The two numbers are : 15 , 35

3

[a] 1 y = 80 ... (0 ...



2 8 hours 3 80 pages

[b] $\because \frac{X}{y} = \frac{\ell}{z} = m \qquad \therefore X = y\,m , \ell = z\,m$

$\therefore \frac{y - X}{X} = \frac{y - y\,m}{y\,m} = \frac{y(1 - m)}{y\,m} = \frac{1 - m}{m} \qquad (1)$

$, \frac{z - \ell}{\ell} = \frac{z - z\,m}{z\,m} = \frac{z(1 - m)}{z\,m} = \frac{1 - m}{m} \qquad (2)$

From (1) , (2) : $\frac{y - X}{X} = \frac{z - \ell}{\ell}$

4

[a] $\because y \propto X \qquad \therefore y = m\,X \qquad \therefore 40 = 14\,m$

$\therefore m = \frac{20}{7} \qquad \therefore y = \frac{20}{7}\,X$

at y = 80 $\qquad \therefore 80 = \frac{20}{7}\,X \qquad \therefore X = 28$

[b] 1 $\because X = \{1 , 2\} , Y = \{2 , 3\}$

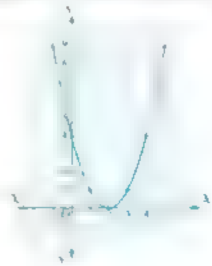
$\therefore X \cup Y = \{1 , 2 , 3\}$

2 n(Y') = 4

5

[a] $f(X) = (X - 2)^2$

X	−1	0	1	2	3	4	5
f (X)	9	4	1	0	1	4	9



From the graph :

1 The vertex of the curve is : (2 , 0)

2 The equation of the line of symmetry is : X = 2

3 The minimum value = 0

[b] Form the table by yourself , then $\sigma \simeq 3.29$

9

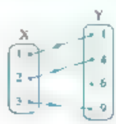
1

1 c 2 d 3 c 4 a 5 b 6 b

2

[a] 1 R = {(1 , 1) , (2 , 4) , (3 , 9)}

2



3] R is a function because every element in X has only one image in Y

[b] ∵ b is the middle proportional between a and c

$b^2 = ac$

$$\text{L.H.S} = \frac{2c^2 - 3b^2}{2b^2 - 3a^2} = \frac{2c^2 - 3ac}{2ac - 3a^2} = \frac{c(2c-3a)}{a(2c-3a)}$$

$$= \frac{c}{a} = \text{R.H.S.}$$

3

[a] ∵ $(2x, 4) = (8, y+1)$

∴ $2x = 8$ ∴ $x = 4$

, $y + 1 = 4$ ∴ $y = 3$

∴ $\sqrt{x^2 + y^2} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$

[b] 1 ∵ $y \propto x$ ∴ $y = mx$

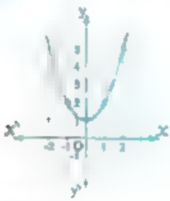
∴ $2 = 8m$ ∴ $m = \frac{1}{4}$ ∴ $y = \frac{1}{4}x$

2 at $x = 12$ ∴ $y = \frac{1}{4} \times 12$ ∴ $y = 3$

4

[a] $f(x) = x^2 + 1$

x	-2	-1	0	1	2
$f(x)$	5	2	1	2	5



From the graph :

1 The vertex of the curve is (0 , 1)

2] The equation of the line of symmetry is $x = 0$

3] The minimum value = 1

[b] ∵ $\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a - b + 5c}{3x}$

, multiplying the two terms of the 1st ratio by 2 and the 2nd by – 1 and the 3rd by 5 and adding the antecedents and consequents of the three ratios.

∴ $\frac{2a - b + 5c}{4 - 3 + 20}$ = one of the given ratios

∴ $\frac{2a - b + 5c}{21} = \frac{2a - b + 5c}{3x}$

∴ $3x = 21$ ∴ $x = 7$

5

[a] 1 The range = $\{3, 1, 5\}$

2 ∵ f is a function on X

∵ Each element in X appears only once as a first projection in R

∴ $a = 3, b = 5$ or $a = 5, b = 3$

∴ $a + b = 3 + 5 = 8$

[b] Form the table by yourself , then $\sigma \simeq 1.41$

10 Past Said

1

1 b 2 c 3 a 4 a 5 d 6 c

2

[a] $R = \{(2, 4), (3, 6), (4, 8)\}$

R is a function because every element of X has only one image in Y

, its range = $\{4, 6, 8\}$

[b] ∵ $f(3) = 15$ ∴ $4 \times 3 + b = 15$

∴ $12 + b = 15$ ∴ $b = 3$

3

[a] 1 $f(\sqrt{2}) + 3g(\sqrt{2})$

$= (\sqrt{2})^2 - 3 \times \sqrt{2} + 3(\sqrt{2} - 3)$

$= 2 - 3\sqrt{2} + 3\sqrt{2} - 9 = -7$

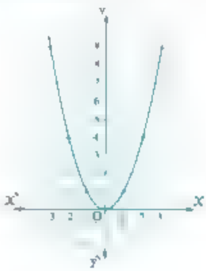
2 ∵ $f(3) = (3)^2 - 3 \times 3 = 9 - 9 = 0$

, $g(3) = 3 - 3 = 0$

∴ $f(3) = g(3) = 0$

[b] $f(x) = x^2$

x	-3	-2	-1	0	1	2	3
$f(x)$	9	4	1	0	1	4	9



From the graph :

The vertex of the curve is $(0 , 0)$

, the minimum value $= 0$

, the equation of the axis of symmetry is : $X = 0$

4

[a] ∵ b is the middle proportional between a and b

∴ $b^2 = ac$

L.H.S $= \frac{a^2 + b^2}{b^2 + c^2} = \frac{a^2 + ac}{ac + c^2} = \frac{a(a+c)}{c(a+c)} = \frac{a}{c} =$ R.H.S

[b] 1 ∵ $y \propto X$ ∴ $y = m X$

∵ $14 = 42\ m$ ∴ $m = \frac{1}{3}$ ∴ $y = \frac{1}{3} X$

2 at $X = 60$

∴ $y = \frac{1}{3} \times 60$ ∴ $y = 20$

5

[a] Form the table by yourself , then $\sigma \simeq 9.32$

[b] ∵ $h \propto \frac{1}{r^2}$ ∴ $\frac{h_1}{h_2} = \frac{r_2^2}{r_1^2}$

∴ $\frac{22.5}{r_2^2} = \frac{15.75}{[illegible]}$

∴ $r_2 = \frac{22.5 \times [illegible]}{248.0625} = 12$ cm

11 [illegible]

1

1 a 2 c 3 a 4 d 5 d 6 c

2

[a] $X - Y = \{1 , 9\}$

$(X - Y) \times Z = \{1 , 9\} \times \{4\} = \{(1 , 4) , (9 , 4)\}$

[b] ∵ b is the middle proportional between a and c

∴ $b^2 = ac$

L.H.S. $= \frac{a^2 + b^2}{b^2 + c^2} = \frac{a^2 + ac}{ac + c^2} = \frac{a(a+c)}{c(a+c)} = \frac{a}{c} =$ R.H.S

3

[a] 1 ∵ $y \propto \frac{1}{X}$ ∴ $y = \frac{m}{X}$ ∴ $3 = \frac{m}{2}$

∴ $m = 6$ ∴ $y = \frac{6}{X}$

2 at $X = 1.5$ ∴ $y = \frac{6}{1.5} = 4$

[b] 1 ∵ $\frac{y}{X - z} = \frac{X}{y} = \frac{X + y}{z}$

, adding the antecedents and consequents of the three ratios

$\frac{y + X + X + y}{X - z + y + z} = \frac{2(X + y)}{(X + y)} = 2$

= one of the given ratios.

∴ Each ratio = 2 unless $X + y = 0$

2 ∵ $\frac{X}{y} = 2$ ∴ $X = 2y$

∵ $\frac{X + y}{z} = 2$ ∴ $X + y = 2z$

∴ $2y + y = 2z$ ∴ $3y = 2z$

4

[a] ∵ $(X^3 , y + 1) = (8 , 3)$

∴ $X^3 = 8$ ∴ $X = \sqrt[3]{8} = 2$

, $y + 1 = 3$ ∴ $y = 2$

∴ $\sqrt[3]{X + 3y} = \sqrt[3]{2 + 3 \times 2} = \sqrt[3]{8} = 2$

[b] 1 $R = \{(-1 , 1) , (0 , 0) , (2 , 4) , (3 , 9)\}$

2 R is a function because every element of X has only one image in Y

, its range $= \{0 , 1 , 4 , 9\}$

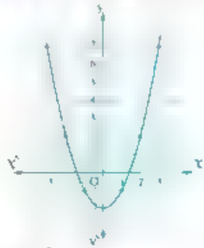
5

[a] Form the table by yourself

, then the mean $(\bar{X}) = 63$, $\sigma \simeq 7.07$

[b] $f(X) = X^2 - 2$

X	-3	-2	-1	0	1	2	3
$f(X)$	7	2	-1	-2	-1	2	7



From the graph :

1 The vertex of the curve is : $(0 , -2)$

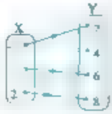
2 The equation of the axis of symmetry is : $X = 0$

12 Kafr El-Sheikh

1

1 b 2 a 3 a 4 c 5 a 6 c

[a] $R = \{(-1, 2), (1, 6), (2, 8)\}$



R is a function because every element in X has only one image in Y

, the range = $\{2, 6, 8\}$

[b] $\because \frac{21X - y}{7X - z} = \frac{y}{z}$

$\therefore 7Xy - zy = 21Xz - zy$

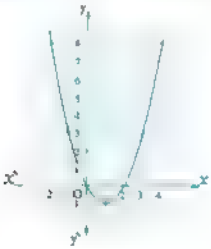
$\therefore 7Xy = 21Xz$

$\therefore y = \frac{21Xz}{7X} = 3z \qquad \therefore y \propto z$

3

[a] $f(X) = X^2 - 2X$

X	-2	-1	0	1	2	3	4
$f(X)$	8	3	0	-1	0	3	8



From the graph :

1 The equation of the line of symmetry is : $X = 1$

2 The minimum value $= -1$

[b] $\because \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = dm \quad , \quad b = dm^2 \quad , \quad a = dm^3$

$\therefore \frac{a}{b+d} = \frac{dm^3}{dm^2 + d} = \frac{dm^3}{d(m^2+1)} = \frac{m^3}{m^2+1} \qquad (1)$

$\therefore \frac{c^3}{c^2 d + d^3} = \frac{d^3 m^3}{d^3 m^2 + d^3} = \frac{d^3 m^3}{d^3(m^2+1)} = \frac{m^3}{m^2+1} \qquad (2)$

From (1) , (2) $\therefore \frac{a}{b+d} = \frac{c^3}{c^2 d + d^3}$

4

[a] $\because \frac{X+y}{3} = \frac{y+z}{8} = \frac{z+X}{6}$

, adding the antecedents and consequents of the three ratios

$\therefore \frac{X+y+y+z+z+X}{3+8+6} = \frac{2X+2y+2z}{17}$

$= \frac{2(X+y+z)}{17}$ = one of the given ratios $\qquad (1)$

, multiplying the terms of 2^{nd} ratio by 2 and adding the antecedents and consequents of the three ratios

$\therefore \frac{X+y+2y+2z+z+X}{3+16+6} = \frac{2X+3y+3z}{25}$

= one of the given ratios. $\qquad (2)$

From (1) and (2) : $\therefore \frac{2(X+y+z)}{17} = \frac{2X+3y+3z}{25}$

$\therefore \frac{X+y+z}{2X+3y+3z} = \frac{17}{50}$

[b] $(a, 4)$ satisfies the function

$\therefore 2a + b = 4$ (multiplying by 3)

$\therefore 6a + 3b = 12$

5

[a] Form the tables by yourself

, then the mean $(\bar{X}) = 40.75$, $\sigma \simeq 13.4$

[b] $\because$ The straight line intersects from the positive part of y-axis a part of length 3 units

$\therefore b = 3$

, $\because$ the straight line passes through $(1, 5)$

$\therefore a + 3 = 5 \qquad \therefore a = 2$

13 El-Beheira

1

1 b   2 d   3 b   4 d   5 a   6 c

2

[a] 1 $R = \{(2, 4), (3, 6), (5, 10)\}$

2 R is a function because every element of X has only one image in Y

[b] b is the middle proportional between a and c

$\therefore b^2 = ac$

$L.H.S = \frac{2c^2 - 3b^2}{2b^2 - 3a^2} = \frac{2c^2 - 3ac}{2ac - 3a^2} = \frac{c(2c - 3a)}{a(2c - 3a)}$

$= \frac{c}{a} = R.H.S$

3

[a] 1 $y \propto \frac{1}{X} \qquad \therefore y = \frac{m}{X} \qquad \therefore 9 = \frac{m}{2}$

$\therefore m = 18 \qquad \therefore y = \frac{18}{X}$

2 at $X = 3 \qquad \therefore y = \frac{18}{3} \qquad \therefore y = 6$

[b] $\because f(2) = 12 \qquad \therefore 12 = 5 \times 2 + a$

$\therefore 10 + a = 12 \qquad \therefore a = 2$

4

[a] 1 $X - Y = \{3\}$

$\therefore (X - Y) \times Z = \{3\} \times \{6, 5\}$

$= \{(3, 6), (3, 5)\}$

2 $n(X \times Y) = 4$

[b] Let the number be x

$\because \frac{49 - 3x}{69 - 3x} = \frac{2}{3}$ $\therefore 147 - 9x = 138 - 6x$

$\therefore 3x = 9$ $\therefore x = 3$

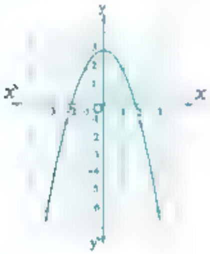
5

[a] Form the table by yourself

, then the mean $(\overline{X}) = 16$, $\sigma \simeq 4.43$

[b] $f(x) = 3 - x^2$

x	-3	-2	-1	0	1	2	3
$f(x)$	-6	-1	2	3	2	-1	-6



From the graph :

1 The equation of the axis of symmetry is : $x = 0$

2 The maximum value $= 3$

14 El-Fayoum

1

1 b 2 b 3 d 4 a 5 b 6 a

2

[a] 1 $n(X \times Y) = 4$

2 $Y \cap X = \{2\}$

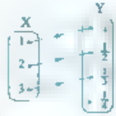
, $(Y \cap X) \times Z = \{2\} \times \{3\} = \{(2, 3)\}$

[b] $\because a = 2b$

$\therefore \frac{8a + 5b}{7a - 2b} = \frac{16b + 5b}{14b - 2b} = \frac{21b}{12b} = \frac{7}{4}$

3

[a] 1 $R = \left\{(1, 1), \left(2, \frac{1}{2}\right), \left(3, \frac{1}{3}\right)\right\}$



2 Yes, its range $= \left\{1, \frac{1}{2}, \frac{1}{3}\right\}$

[b] $f\left(\frac{1}{4}\right) = 12$ $\therefore 12 = 4 \times \frac{1}{4} + a$

$\therefore 1 + a = 12$ $\therefore a = 11$

4

[a] $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\because c = dm$, $b = dm^2$, $a = dm^3$

$\therefore \frac{a}{b + d} = \frac{dm^3}{dm^2 + d} = \frac{dm^3}{d(m^2 + 1)} = \frac{m^3}{m^2 + 1}$ (1)

$\therefore \frac{c^3}{c^2 d + d^3} = \frac{d^3 m^3}{d^3 m^2 + d^3} = \frac{d^3 m^3}{d^3(m^2 + 1)} = \frac{m^3}{m^2 + 1}$ (2)

From (1), (2) $\therefore \frac{a}{b + d} = \frac{c^3}{c^2 d + d^3}$

[b] 1 $\because y \propto \frac{1}{x}$ $\therefore y = \frac{m}{x}$

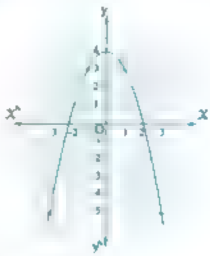
$\because 3 = \frac{m}{2}$ $\therefore m = 6$ $\therefore y = \frac{6}{x}$

2 at $x = 3$ $\therefore y = \frac{6}{3}$ $\therefore y = 2$

5

[a] $f(x) = 4 - x^2$

x	-3	-2	-1	0	1	2	3
$f(x)$	-5	0	3	4	3	0	-5



From the graph :

1 The vertex of the curve is $(0, 4)$

2 The equation of the axis of symmetry is : $x = 0$

[b] Form the table by yourself

, then the mean $(\overline{X}) = 8$, $\sigma = 4$

15 Beni Suef

1

1 c 2 a 3 b 4 d 5 b 6 d

2

[a] 1 $X \times Z = \{(2, 3), (5, 3)\}$

2 $Y^2 = \{(3, 3), (3, 2), (2, 3), (2, 2)\}$

3 $X \cap Y = \{2\}$

$, (X \cap Y) \times Z = \{2\} \times \{3\} = \{(2, 3)\}$

[b] Let the number be X

$\because \frac{5 + X^2}{11 + X^2} = \frac{3}{5}$ $\therefore 25 + 5X^2 = 33 + 3X^2$

$\therefore 2X^2 = 8$ $\therefore X^2 = 4$

$\therefore X = 2$ or $X = -2$ (refused)

$\therefore$ The number is 2

3

[a] 1 $f(3) + 3g\left(\sqrt{2}\right) = 3^2 - \sqrt{2} \times 3 + 3\left(\sqrt{2} + 1\right)$

$= 9 - 3\sqrt{2} + 3\sqrt{2} + 3 = 12$

2 $f\left(\sqrt{2}\right) = \left(\sqrt{2}\right)^2 - \sqrt{2} \times \sqrt{2} = 2 - 2 = 0$

$, g(-1) = -1 + 1 = 0$

$\therefore f\left(\sqrt{2}\right) = g(-1)$

[b] 1 $\because y \propto \frac{1}{X}$ $\therefore y = \frac{m}{X}$

$3 = \frac{m}{2}$ $\therefore m = 6$ $\therefore y = \frac{6}{X}$

2 at $X = 1.5$ $\therefore y = \frac{6}{1.5}$ $\therefore y = 4$

4

[a] $R = \{(1, 7), (2, 6)\}$

R is not a function because the element $3 \in X$ has no image in Y

[b] $\frac{X}{3} = \frac{y}{4} = \frac{z}{5} = m$

$\therefore X = 3m$, $y = 4m$, $z = 5m$

$\text{L.H.S} = \frac{2y - z}{3X - 2y + z} = \frac{8m - 5m}{9m - 8m + 5m}$

$= \frac{3m}{6m} = \frac{1}{2} = \text{R.H.S}$

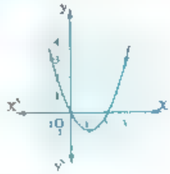
5

[a] Form the table by yourself

, then the mean $(\overline{X}) = 10$, $\sigma = 4$

[b] $f(X) = X^2 - 2X$

X	-1	0	1	2	3
$f(X)$	3	0	-1	0	3



From the graph :

- The equation of the axis of symmetry is $X = 1$
- The minimum value $= -1$

16 El-Menia

1

1 a 2 c 3 b 4 d 5 a 6 b

2

[a] Form the table by yourself , then $\sigma \simeq 2.83$

[b] 1 $n(X \times Z) = 2$

2 $Y \cap X = \{2\}$

$, (Y \cap X) \times Z = \{2\} \times \{3\} = \{(2, 3)\}$

3

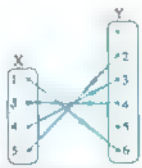
[a] 1 $\because y \propto \frac{1}{X}$ $\therefore Xy = m$

$\therefore 2 \times 3 = m$ $\therefore m = 6$ $\therefore Xy = 6$

2 at $X = 1.5$ $\therefore 1.5y = 6$ $\therefore y = 4$

[b] $R = \{(1, 6), (3, 4), (4, 3), (5, 2)\}$

R is a function because every element in X has only one image in Y



4

[a] $\because \frac{X}{y} = \frac{2}{3} = m$ $\therefore X = 2m , y = 3m$

$\therefore \frac{3X + 2y}{6y - X} = \frac{6m + 6m}{18m - 2m} = \frac{12m}{16m} = \frac{3}{4}$

[b] ∵ The straight line intersects the X-axis at (2 , b)

∴ b = 0

, ∵ (2 , 0) belongs to the straight line

∴ $4 \times 2 - a = 0$ ∴ 8 − a = 0 ∴ a = 8

5

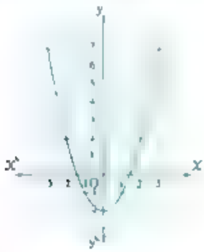
[a] ∵ b is the middle proportional between a and c

∴ $b^2 = ac$

∵ $L.H.S = \frac{a^2 + b^2}{b^2 + c^2} = \frac{a^2 + ac}{ac + c^2} = \frac{a(a+c)}{c(a+c)} = \frac{a}{c} = R.H.S$

[b] $f(X) = X^2 - 2$

X	−3	−2	−1	0	1	2	3
$f(X)$	7	2	−1	−2	−1	2	7



From the graph :

* The vertex of the curve is (0 , −2)
* The equation of the axis of symmetry is : $X = 0$
* The minimum value = −2

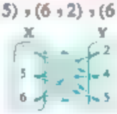
17 Assiut

1

1 d 2 b 3 c 4 b 5 a 6 d

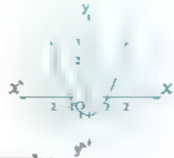
2

[a] $X \times Y$ = {(1 , 2) , (1 , 4) , (1 , 5) , (5 , 2) , (5 , 4) , (5 , 5) , (6 , 2) , (6 , 4) , (6 , 5)}



[b] $f(X) = X^2 - 1$

X	−2	−1	0	1	2
$f(X)$	3	0	−1	0	3



From the graph :

1 The equation of the axis of symmetry is : $X = 0$

2 The minimum value = −1

3

[a] ∵ $f(3) = 15$ ∴ $15 = 4 \times 3 + m$

∴ $12 + m = 15$ ∴ $m = 3$

[b] ∵ $\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a - b + 5c}{3X}$

, multiplying the two terms of the 1st ratio by 2 , the 2nd ratio by (−1) and the 3rd ratio by 5 and adding the antecedents and consequents of the three ratios

∴ $\frac{2a - b + 5c}{4 - 3 + 20}$ = one of the given ratios

∴ $\frac{2a - b + 5c}{21} = \frac{2a - b + 5c}{3X}$

∴ $3X = 21$ ∴ $X = 7$

4

[a] 1 $y \propto X$ ∴ $y = mX$

∴ $3 = 2m$ ∴ $m = \frac{3}{2}$ ∴ $y = \frac{3}{2}X$

2 at $X = \frac{1}{3}$ ∴ $y = \frac{3}{2} \times \frac{1}{3}$ ∴ $y = \frac{1}{2}$

[b] ∵ b is the middle proportional between a and c

∴ $b^2 = ac$

∵ $L.H.S = \frac{a^2 + b^2}{b^2 + c^2} = \frac{a^2 + ac}{ac + c^2} = \frac{a(a+c)}{c(a+c)} = \frac{a}{c} = R.H.S$

5

[a] 1 R = {(1 , 6) , (3 , 4) , (5 , 2)}

2 R is a function because every element in X has only one image in Y

, the range = {2 , 4 , 6}

[b] Form the table by yourself

, then the mean $(\overline{X}) = 7$, $\sigma \simeq 1.41$

18 Souhag

1

1 c 2 d 3 c 4 c 5 c 6 d

2

[a] $\because \frac{x}{y} = \frac{3}{4} = m \quad \therefore x = 3m , y = 4m$

$\therefore \frac{3x+y}{x+5y} = \frac{9m+4m}{3m+20m} = \frac{13m}{23m} = \frac{13}{23}$

[b] $R = \left\{(1 , 1) , \left(2 , \frac{1}{2}\right) , \left(3 , \frac{1}{3}\right)\right\}$

R is a function because every element in X has only one image in Y

3

[a] 1 $\because$ R is a function on X

$\therefore$ Each element in X has to appear only once as a first projection in R

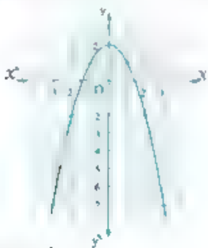
$\therefore a = 5 , b = 7$ or $a = 7 , b = 5$

$\therefore a + b = 5 + 7 = 12$

2 The range $= \{5 , 7\}$

[b] $f(x) = 2 - x^2$

x	-3	-2	-1	0	1	2	3
$f(x)$	-7	-2	1	2	1	-2	-7



From the graph :

1 The vertex of the curve is : $(0 , 2)$

2 The equation of the axis of symmetry is : $x = 0$

3 The maximum value $= 2$

4

[a] $\because$ b is the middle proportional between a and c

$\therefore b^2 = ac$

$\therefore \text{L.H.S.} = \frac{a^2+b^2}{b^2+c^2} = \frac{a^2+ac}{ac+c^2} = \frac{a(a+c)}{c(a+c)} = \frac{a}{c} = \text{R.H.S.}$

[b] 1 The variation is inverse

2 $y \propto \frac{1}{x} \quad \therefore xy = m \quad \therefore m = 12$

3 at $x = 2\frac{2}{5} \quad \therefore \left(2\frac{2}{5}\right) y = 12$

$\therefore y = 12 \times \frac{5}{12} = 5$

5

[a] $\because (a , 3)$ lies on the straight line which represents the function

$\therefore f(a) = 3 \quad \therefore 4a - 5 = 3$

$\therefore 4a = 8 \quad \therefore a = 2$

[b] Form the table by yourself , then $\sigma \simeq 3.22$

10 [illegible]

1

1 a 2 c 3 b 4 b 5 a 6 c

2

[a] $R = \{(1 , 2) , (2 , 3) , (3 , 4)\}$

R is a function because every element in X has only one image in Y

, the range $= \{2 , 3 , 4\}$

[b] $\frac{a}{4} = \frac{b}{5} = \frac{c}{3} = m$

$\therefore a = 4m , b = 5m , c = 3m$

$\therefore \text{L.H.S} = \frac{a - b + c}{a + b - c} = \frac{4m - 5m + 3m}{4m + 5m - 3m}$

$= \frac{2m}{6m} = \frac{1}{3} = \text{R.H.S}$

3

[a] $\because y \propto x \quad \therefore y = mx \quad \therefore \frac{5}{6} = \frac{1}{6} m$

$\therefore m = 5 \quad \therefore y = 5x$

, at $y = 15 \quad \therefore 15 = 5x \quad \therefore x = 3$

[b] $\because (a , -a)$ lies on the straight line that represents the function

$\therefore f(a) = -a \quad \therefore a - 6 = -a$

$\therefore 2a = 6 \quad \therefore a = 3$

4

[a] $\because \frac{x}{y} = \frac{y}{z} = m \quad \therefore y = zm , x = zm^2$

$\therefore \frac{xz}{y(y+z)} = \frac{z^2m^2}{zm(zm+z)} = \frac{z^2m^2}{z^2m(m+1)}$

$= \frac{m}{m+1} \quad (1)$

$, \frac{x}{x+y} = \frac{zm^2}{zm^2+zm} = \frac{zm^2}{zm(m+1)} = \frac{m}{m+1} \quad (2)$

From (1) , (2) $\therefore \frac{xz}{y(y+z)} = \frac{x}{x+y}$

[b] $\boxed{1}$ $X - Y = \{2 , 3\}$

$, (X - Y) \times Z = \{2 , 3\} \times \{4 , 5\}$
$= \{(2 , 4) , (2 , 5) , (3 , 4) , (3 , 5)\}$

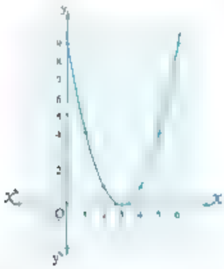
$\boxed{2}$ $Y \cap Z = \{5\}$

$, X \times (Y \cap Z) = \{2 , 3\} \times \{5\}$
$= \{(2 , 5) , (3 , 5)\}$

5

[a] $f(X) = (X - 3)^2$

X	0	1	2	3	4	5	6
$f(X)$	9	4	1	0	1	4	9



From the graph :

$\boxed{1}$ The vertex of the curve is $(3 , 0)$

$\boxed{2}$ The minimum value = 0

[b] Form the table by yourself

, then the mean $(\overline{X}) = 64$, $\sigma \simeq 7.07$

20 Answer

1

$\boxed{1}$ c $\boxed{2}$ b $\boxed{3}$ d $\boxed{4}$ c $\boxed{5}$ b $\boxed{6}$ d

2

[a] $\boxed{1}$ $X \times Y = \{(2 , 4) , (2 , 0) , (1 , 4) , (1 , 0)\}$

$\boxed{2}$ $Y \cap Z = \{4\}$

$(Y \cap Z) \times X = \{4\} \times \{2 , 1\}$
$= \{(4 , 2) , (4 , 1)\}$

$\boxed{3}$ $n(Y^2) = 4$

[b] Let the number be : X

$\because \frac{15 - X}{13 + X} = \frac{3}{4}$ $\therefore 39 + 3X = 60 - 4X$

$\therefore 7X = 21$ $\therefore X = 3$ $\therefore$ The number is 3

3

[a] $\because f(2) + g(-4) = 30$

$\therefore 2 \times 2 + a + (-4)^2 + a = 30$

$\therefore 4 + a + 16 + a = 30$ $\therefore 2a + 20 = 30$

$\therefore 2a = 10$ $\therefore a = 5$

[b] $\because \frac{a}{b} = \frac{c}{d} = m$ $\therefore a = bm , c = dm$

$\therefore \frac{a + c}{b + d} = \frac{bm + dm}{b + d} = \frac{m(b + d)}{b + d} = m$ (1)

$, \frac{a^2 + c^2}{ab + cd} = \frac{b^2 m^2 + d^2 m^2}{b^2 m + d^2 m} = \frac{m^2 (b^2 + d^2)}{m (b^2 + d^2)} = m$ (2)

From (1) , (2) $\frac{a + c}{b + d} = \frac{a^2 + c^2}{ab + cd}$

4

[a] $R = \left\{(1 , 1) , \left(2 , \frac{1}{2}\right) , \left(\frac{1}{2} , 2\right)\right\}$

R is not a function

[b] $\boxed{1}$ $y \propto X^3$ $y = m X^3$ $, 64 = 8m$

$m = 8$ $\therefore y = 8 X^3$

$\boxed{2}$ at $X = \frac{1}{2}$ $y = 8 \times \left(\frac{1}{2}\right)^3$

$\therefore y = 8 \times \frac{1}{8} = 1$

5

[a] Form the table by yourself

, then the mean $(\overline{X}) = 20$, $\sigma \simeq 1.26$

[b] $f(X) = X^2 - 4X + 5$

X	0	1	2	3	4
$f(X)$	5	2	1	2	5

From the graph :

$\boxed{1}$ The equation of the axis of symmetry is : $X = 2$

$\boxed{2}$ The minimum value = 1

21 Aswan

1

[1] a [2] c [3] c [4] d [5] b [6] b

2

[a] [1] $R = \{(1, 6), (3, 4), (4, 3), (5, 2)\}$

[2] R is a function because every element in X has only one image in Y

[b] [1] $\because y \propto X \quad \therefore y = m X \quad \therefore 6 = 3 m$

$\therefore m = 2 \quad \therefore y = 2 X$

[2] at $X = 5 \quad \therefore y = 2 \times 5 \quad \therefore y = 10$

3

[a] $f(X) = 4 - X^2$

X	-3	-2	-1	0	1	2	3
$f(X)$	-5	0	3	4	3	0	-5

From the graph :

* The vertex of the curve is . (0 , 4)
* The maximum value = 4
* The equation of the line of symmetry is : $X = 0$

[b] Let the number be X

$\therefore \frac{29 + X^2}{46 - X^2} = \frac{3}{2}$

$\therefore 58 + 2 X^2 = 138 - 3 X^2$

$\therefore 5 X^2 = 80 \quad \therefore X^2 = 16$

$\therefore X = 4$ or $X = -4$ (refused)

$\therefore$ The number is . 4

4

[a] $\because$ The straight line intersects the y-axis at (b , 2)

$\therefore b = 0$

, $\because$ (0 , 2) satisfies the function

$\therefore 2 = 6 \times 0 - a \quad \therefore a = -2$

[b] Form the tables by yourself , then $\sigma \simeq 1.73$

5

[a] [1] The range = $\{3, 1, 5\}$

[2] $\because$ R is a function on X

$\therefore$ Each element in X has to appear only once as a first projection in R

$\therefore a = 3, b = 5$ or $a = 5, b = 3$

$\therefore a + b = 3 + 5 = 8$

[b] $\because \frac{a}{b} = \frac{c}{d} = m \qquad \therefore a = bm , c = dm$

$\frac{a}{b - a} = \frac{bm}{b - bm} = \frac{bm}{b(1 - m)} = \frac{m}{1 - m} \qquad (1)$

$, \frac{c}{d - c} = \frac{dm}{d - dm} = \frac{dm}{d(1 - m)} = \frac{m}{1 - m} \qquad (2)$

From (1) , (2) $\therefore \frac{a}{b - a} = \frac{c}{d - c}$

22 New Valley

1

[1] b [2] b [3] c [4] a [5] c [6] a

2

[a] [1] $R = \{(2, 4), (3, 6), (5, 10)\}$

[2] R is a function because every element in X has only one image in Y

[b] $\because \frac{X}{y} = \frac{2}{3} = m \qquad \therefore X = 2 m , y = 3 m$

$\therefore \frac{3 X + 2 y}{6 y - X} = \frac{6 m + 6 m}{18 m - 2 m} = \frac{12 m}{16 m} = \frac{3}{4}$

3

[a] [1] $X = \{1\} , Y = \{1, 3, 5\}$

[2] $Y \times X = \{(1, 1), (3, 1), (5, 1)\}$

[3] $Y^2 = \{(1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5)\}$

[b] $\frac{21 X - y}{7 X - z} = \frac{y}{z}$

$\therefore 7 X y - y z = 21 X z - y z$

$7\,x\,y = 21\,x\,z$ $\qquad y = \frac{21\,x\,z}{7\,x} = 3\,z$

$y \propto z$

4

[a] $\frac{1}{3} f(3) = 5$ $\qquad \therefore f(3) = 15$

$4 \times 3 + b = 15$ $\qquad 12 + b = 15$ $\qquad b = 3$

[b] $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = d\,m$, $b = d\,m^2$, $a = d\,m^3$

$\therefore \frac{a^2 - 3c^2}{b^2 - 3d^2} = \frac{d^2 m^6 - 3d^2 m^2}{d^2 m^4 - 3d^2} = \frac{d^2 m^2 (m^4 - 3)}{d^2 (m^4 - 3)}$

$= m^2$ (1)

$, \frac{b}{d} = \frac{d\,m^2}{d} = m^2$ (2)

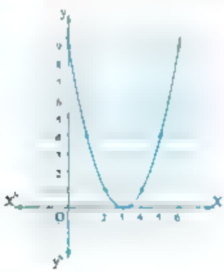
From (1) , (2) : $\therefore \frac{a^2 - 3c^2}{b^2 - 3d^2} = \frac{b}{d}$

5

[a] Form the table by yourself , then $\sigma \simeq 3.29$

[b] $f(x) = (x - 3)^2$

x	0	1	2	3	4	5	6
$f(x)$	9	4	1	0	1	4	9



From the graph :

- The vertex of the curve is : (3 , 0)
- The minimum value = 0
- The equation of the axis of symmetry is : $x = 3$

23 South Sinai

1

1 b $\quad$ 2 a $\quad$ 3 d $\quad$ 4 c $\quad$ 5 a $\quad$ 6 c

2

[a] 1 $Y \cap Z = \{2\}$

$, X \times (Y \cap Z) = \{1\} \times \{2\} = \{(1 , 2)\}$

2 $n(X \times Y) = 2$

3 $Z - Y = \{5 , 6\}$

[b] $f(x) = x^2 - 4$

x	-3	-2	-1	0	1	2	3
$f(x)$	5	0	-3	-4	-3	0	5

From the graph :

- The vertex of the curve is : (0 , − 4)
- The minimum value = − 4

3

$R = \{(1 , 6) , (3 , 4) , (4 , 3) , (5 , 2)\}$

R is a function because every element in X has only one image in Y

4

[a] $y \propto x$ $\qquad \therefore \frac{y_1}{y_2} = \frac{x_1}{x_2}$

$\therefore \frac{6}{y_2} = \frac{3}{5}$ $\qquad y_2 = \frac{6 \times 5}{3} = 10$

[b] Let the number be : x

$\therefore \frac{5 + x^2}{11 + x^2} = \frac{3}{5}$ $\quad \therefore 25 + 5x^2 = 33 + 3x^2$

$\therefore 2x^2 = 8$ $\quad \therefore x^2 = 4$

$\therefore x = 2$ or $x = -2$ (refused)

$\therefore$ The number is : 2

5

[a] $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$\therefore c = d\,m$, $b = d\,m^2$, $a = d\,m^3$

$\frac{c^2 - d^2}{a - c} = \frac{d^2 m^2 - d^2}{d\,m^3 - d\,m} = \frac{d^2 (m^2 - 1)}{d\,m (m^2 - 1)} = \frac{d}{m}$ (1)

$\therefore \frac{b\,d}{a} = \frac{d^2 m^2}{d\,m^3} = \frac{d}{m}$ (2)

From (1) , (2) $\therefore \frac{c^2 - d^2}{a - c} = \frac{b\,d}{a}$

[b] Form the tables by yourself , then $\sigma \simeq 1.73$

24 North Sinai

1

1 c 2 b 3 b 4 a 5 d 6 c

2

[a] 1 $R = \{(2 , 4) , (3 , 6) , (4 , 8)\}$

2 R is function because every element in X has only one image in Y

, its range $= \{4 , 6 , 8\}$

[b] $\because y \propto x$ $\quad \therefore \frac{y_1}{y_2} = \frac{x_1}{x_2}$

$\therefore \frac{2}{y_2} = \frac{8}{12}$ $\quad \therefore y_2 = \frac{12 \times 2}{8} = 3$

3

[a] $\because f(3) = 15$ $\quad \therefore 4 \times 3 + b = 15$

$\therefore 12 + b = 15$ $\quad \therefore b = 3$

[b] $\because \frac{x}{y} = \frac{2}{3} = m$ $\quad \therefore x = 2m$, $y = 3m$

$\therefore \frac{3x + 2y}{6y - x} = \frac{6m + 6m}{18m - 2m} = \frac{12m}{16m} = \frac{3}{4}$

4

[a] $(6 , b - 3) = (2 - a , -1)$

$\therefore 2 - a = 6$ $\quad \therefore -a = 4$ $\quad \therefore a = -4$

, $b - 3 = -1$ $\quad \therefore b = 2$

$\therefore a + b = -4 + 2 = -2$

[b] $\because$ b is the middle proportional between a and c

$\therefore b^2 = a\,c$

$\therefore$ L.H.S. $= \frac{a^2 + b^2}{b^2 + c^2} = \frac{a^2 + a\,c}{a\,c + c^2} = \frac{a(a + c)}{c(a + c)} = \frac{a}{c} =$ R.H.S.

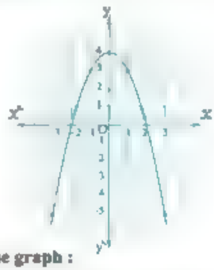
6

[a] Form the table by yourself

, then the mean $(\bar{X}) = 16$, $\sigma \simeq 3.9$

[b] $f(x) = 4 - x^2$

x	-3	-2	-1	0	1	2	3
$f(x)$	-5	0	3	4	3	0	-5



From the graph :

1 The vertex of the curve is (0 , 4)

2 The equation of the axis of symmetry is $x = 0$

3 The maximum value = 4

25 Red Sea

1

1 c 2 b 3 d 4 a 5 b 6 c

2

[a] 1 $Y = \{2 , 5 , 7\}$

2 $X = \{2\}$, $X^2 = \{2\} \times \{2\} = \{(2 , 2)\}$

[b] $\because 5a = 3b$ $\quad \therefore \frac{a}{b} = \frac{3}{5} = m$

$\therefore a = 3m$, $b = 5m$

$\therefore \frac{7a + 9b}{4a + 2b} = \frac{21m + 45m}{12m + 10m} = \frac{66m}{22m} = 3$

3

[a] 1 $\because y \propto \frac{1}{x}$ $\quad \therefore x\,y = m$

$\therefore 2 \times 3 = m$ $\quad \therefore m = 6$ $\quad \therefore x\,y = 6$

2 at $x = 1.5$ $\quad \therefore (1.5)\,y = 6$ $\quad \therefore y = 4$

[b] $\boxed{1}$ The vertex of the curve is : $(0 , -2)$

$\boxed{2}$ The equation of the line of symmetry is $X = 0$

$\boxed{3}$ The minimum value $= -2$

4

[a] $R = \{(-2 , 2) , (-1 , 1) , (0 , 0) , (1 , -1) , (2 , -2)\}$

[b] $\because \frac{a}{b} = \frac{b}{c} = m$

$\therefore b = cm , a = cm^2$

$\therefore \frac{a - b}{a - c} = \frac{cm^2 - cm}{cm^2 - c} = \frac{cm (m - 1)}{c (m^2 - 1)} = \frac{m (m - 1)}{(m - 1) (m + 1)}$

$= \frac{m}{m + 1}$ (1)

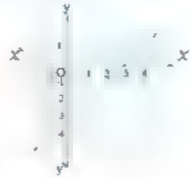
$, \frac{b}{b + c} = \frac{cm}{cm + c} = \frac{cm}{c (m + 1)} = \frac{m}{m + 1}$ (2)

From (1) , (2) $\therefore \frac{a - b}{a - c} = \frac{b}{b + c}$

5

[a] $f (X) = X - 3$

X	0	1	3
$f(X)$	-3	-2	0



From the graph :

- The intersection point with X-axis is $(3 , 0)$
- The intersection point with y-axis is : $(0 , -3)$

[b] Form the table by yourself , then $\sigma \simeq 1.41$

Answers of accumulative tests on trigonometry & geometry

Accumulative test 1

1

1 b	2 a	3 b	4 c
5 b	6 a	7 c	8 d

2 1 $\sin B = \frac{4}{5}$ 2 $48\ cm^2$

3 1 $\tan(\angle ACB) + \tan(\angle ACD) = \frac{56}{15}$

2 $\sin(\angle B)\cos(\angle CAD) + \cos(\angle B)\sin(\angle CAD) = \frac{56}{65}$

Accumulative test 2

1

1 b	2 b	3 c	4 b
5 c	6 b	7 a	8 a

2 1 Prove by yourself

2 $22^\circ\ 37'$

3 $X = \frac{1}{16}$

Accumulative test 3

1

1 d	2 a	3 c	4 b
5 d	6 c	7 c	8 c

2 Prove by yourself

3 $m(\angle X) = 45^\circ$

Accumulative test 4

1

1 c	2 d	3 d	4 b
5 a	6 d	7 a	8 c

2 $DC = 5$ cm. , $\cos(\angle BCD) = \frac{4}{5}$

3 The perimeter of the triangle AOB

= 24 length units

Accumulative test 5

1

1 c	2 d	3 c	4 d
5 c	6 a	7 c	8 b

2 Prove by yourself

3 1 The point of intersection of its diagonals is $(4 , 1)$

2 $D = (6 , 4)$

Accumulative test 6

1

1 d	2 c	3 b	4 b
5 b	6 b	7 d	8 c

2 $y = -X + 3$

3 1 $\cos A \cos B - \sin A \sin B = 0$

2 $m(\angle B) = 36^\circ\ 52'\ 12''$

Answers of model examinations of the school book of trigonometry & geometry

Model 1

1

1 a	2 c	3 b
4 a	5 b	6 a

2

[a] ∵ $\sin 60° = \frac{\sqrt{3}}{2}$ (1)

, $2 \sin 30° \cos 30° = 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$ (2)

From (1) , (2) : ∴ $\sin 60° = 2 \sin 30° \cos 30°$

[b] ∵ The slope of $\overline{AB} = \frac{5+1}{6+3} = \frac{2}{3}$

, the slope of $\overline{BC} = \frac{3-5}{3-6} = \frac{2}{3}$

The slope of $\overline{AB}$ = the slope of $\overline{BC}$

$\overleftrightarrow{AB} \parallel \overleftrightarrow{BC}$

, ∵ B is a common point between the two straight lines.

The points A , B and C are collinear

3

[a] ∵ $4 \cos 60° \sin 30° = 4 \times \frac{1}{2} \times \frac{1}{2} = 1$

$\tan X = 1$ $X = 45°$

[b] Let B (X , y)

∴ $(6 , -4) = \left(\frac{X+5}{2} , \frac{y+3}{2}\right)$

$\frac{X+5}{2} = 6$ ∴ $X + 5 = 12$ ∴ $X = 7$

, $\frac{y+3}{2} = -4$ $y + 3 = -8$ $y = -5$

∴ B $(7 , -5)$

4

[a] $m_1 = \frac{k-1}{2-3} = 1-k$

, $m_2 = \tan 45° = 1$

, $L_1 \parallel L_2$ $m_1 = m_2$

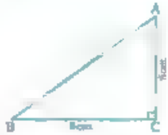
$1 - k = 1$ $k = 0$

[b] ∵ m (∠ C) = 90°

$(AB)^2 = (6)^2 + (8)^2$

$= 100$

∴ AB = 10 cm.



(1) $\cos A \cos B - \sin A \sin B$

$= \frac{6}{10} \times \frac{8}{10} - \frac{8}{10} \times \frac{6}{10} = 0$

(2) ∵ $\cos B = \frac{8}{10}$ ∴ m (∠ B) ≃ 36° 52′ 12″

5

[a] The slope of the straight line = 2

∴ The equation of the straight line is

$y = 2X + c$

, ∵ (1 , 0) satisfies the equation

$0 = 2 \times 1 + c$ ∴ $c = -2$

∴ The equation of the straight line is : $y = 2X - 2$

[b] ∵ $MA = \sqrt{(-1-3)^2 + (2+1)^2} = \sqrt{16+9} = \sqrt{25}$

= 5 length units

, $MB = \sqrt{(-1+4)^2 + (2-6)^2} = \sqrt{9+16}$

$= \sqrt{25} = 5$ length units

, $MC = \sqrt{(-1-2)^2 + (2+2)^2} = \sqrt{9+16}$

$= \sqrt{25} = 5$ length units

∴ MA = MB = MC

∴ A , B and C lie on the circle M

, the circumference $= 2\pi r = 2 \times \pi \times 5$

$= 10\pi$ length units

Model 2

1

1 a	2 d	3 b
4 c	5 b	6 b

2

[a] $\cos E \tan 30° = \cos^2 45°$

$\cos E \times \frac{1}{\sqrt{3}} = \left(\frac{1}{\sqrt{2}}\right)^2$

$\cos E = \frac{\sqrt{3}}{2}$ m (∠ E) = 30°

[b] $\because AB = \sqrt{(3-1)^2 + (3-5)^2} = \sqrt{4+4}$

$= 2\sqrt{2}$ length units

, $BC = \sqrt{(1-1)^2 + (5-3)^2} = \sqrt{4} = 2$ length units

, $AC = \sqrt{(3-1)^2 + (3-3)^2} = \sqrt{4} = 2$ length units

$\therefore BC = AC$ $\therefore \Delta ABC$ is isosceles

3

[a] $\because$ The slope of the straight line $= \frac{-3-3}{-1-1} = 3$

$\therefore$ The equation of the straight line is : $y = 3X + c$

, $\because (1, 3)$ satisfies the equation

$\therefore 3 = 3 \times 1 + c$ $\therefore c = 0$

$\therefore$ The equation of the straight line is : $y = 3X$

, $\because c = 0$

$\therefore$ The straight line passes through the origin point.

[b] $\because (3, 1) = \left(\frac{1+X}{2}, \frac{y+3}{2}\right)$

$\therefore \frac{1+X}{2} = 3$ $\therefore 1 + X = 6$

$\therefore X = 5$, $\frac{y+3}{2} = 1$

$\therefore y + 3 = 2$ $\therefore y = -1$

$\therefore (X, y) = (5, -1)$

4

[a] $\because$ The straight line passes through the two points $(1, 0)$ and $(0, 4)$

$\therefore$ The slope $= \frac{4-0}{0-1} = -4$

$\therefore$ The equation of the straight line is : $y = -4X + c$

, $\because$ the intercepted part from y-axis $= 4$

$\therefore$ The equation of the straight line is $y = -4X + 4$

[b] $\because m(\angle B) = 90°$

$\therefore (AB)^2 = (10)^2 - (8)^2 = 36$

$\therefore AB = 6$ cm.

$\therefore \sin^2 A + 1 = \left(\frac{8}{10}\right)^2 + 1 = \frac{41}{25}$ (1)

, $2\cos^2 C + \cos^2 A = 2 \times \left(\frac{8}{10}\right)^2 + \left(\frac{6}{10}\right)^2 = \frac{41}{25}$ (2)

From (1) , (2)

$\therefore \sin^2 A + 1 = 2\cos^2 C + \cos^2 A$

5

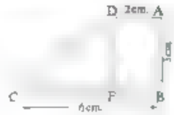
[a] $\because m_1 = \frac{4-3}{2+1} = \frac{1}{3}$

, $m_2 = \frac{1}{3}$ $\therefore m_1 = m_2$ $\therefore L_1 // L_2$

[b] Const : Draw $\overline{DF} \perp \overline{BC}$

Proof : $\because \overline{AD} // \overline{BC}$, $\overline{AB} \perp \overline{BC}$

, $\overline{DF} \perp \overline{BC}$



$\therefore$ ABFD is a rectangle

$\therefore BF = AD = 2$ cm

, $AB = DF = 3$ cm

$\therefore FC = 6 - 2 = 4$ cm.

From Δ DFC which is right-angled at F

$\therefore (DC)^2 = (3)^2 + (4)^2 = 25$

$\therefore DC = 5$ cm

$\therefore \cos(\angle BCD) = \frac{4}{5}$

Answers of model for the merge students

1

1 ✔	2 ✔	3 ✗
4 ✗	5 ✗	6 ✔

2

1 b	2 c	3 d
4 c	5 a	6 c

3

1 0	2 1	3 10
4 2	5 -3	6 $\frac{\sqrt{3}}{2}$

4

1 $\frac{1}{2}$	2 $\frac{3}{5}$	3 3
4 2	5 5 length units	6 $(-5, 2)$

Answers of governorates' examinations of trigonometry & geometry

1 Cairo

1

1 a　　2 b　　3 a　　4 d　　5 d　　6 c

2

[a] $4 \sin 45° \cos 45° = 4 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 2$

[b] $\because$ The slope of the given straight line = 3

$\therefore$ The slope of the required straight line = 3

$\therefore$ The equation of the required straight line is :

$y = 3X + c$

, $\because$ (1 , 2) satisfies the equation

$\therefore 2 = 3 \times 1 + c$　　　$\therefore c = -1$

$\therefore$ The equation is : $y = 3X - 1$

3

[a] $\because X \sin 30° = \sin 30° \cos 60° + \cos 30° \sin 60°$

$\therefore \frac{1}{2} X = \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

$\frac{1}{2} X = 1$　　　$X = 2$

[b] $\because m_1 = \frac{2-5}{3-0} = -1$, $m_2 = \tan 45° = 1$

, $m_1 \times m_2 = -1 \times 1 = -1$

$\therefore$ The two straight lines are perpendicular

4

[a] $\because$ In the parallelogram , the two diagonals bisect each other

$M = \left(\frac{3+1}{2} , \frac{-1+7}{2}\right) = (2 , 3)$

[b] $\because AB = \sqrt{(2+1)^2 + (8-4)^2} = \sqrt{9+16} = \sqrt{25}$

$= 5$ length units.

, $BC = \sqrt{(-1-3)^2 + (4-1)^2} = \sqrt{16+9} = \sqrt{25}$

$= 5$ length units

, $AC = \sqrt{(2-3)^2 + (8-1)^2} = \sqrt{1+49} = \sqrt{50}$

$= 5\sqrt{2}$ length units.

, $\because (AC)^2 = (AB)^2 + (BC)^2$

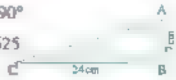
$\therefore \triangle ABC$ is a right-angled triangle at B

, $\because AB = BC$

$\therefore \triangle ABC$ is an isosceles triangle

5

[a] In $\triangle ABC$: $\because m(\angle B) = 90°$



$(AC)^2 = (7)^2 + (24)^2 = 625$

$AC = 25$ cm.

1 $3 \tan A \times \tan C = 3 \times \frac{24}{7} \times \frac{7}{24} = 3$

2 $\sin^2 A + \sin^2 C = \left(\frac{24}{25}\right)^2 + \left(\frac{7}{25}\right)^2$

$= \frac{576}{625} + \frac{49}{625} = 1$

[b] Let A (0 , 1) , B (a , 3) , C (2 , 5)

, $\because$ The points are collinear

$\therefore$ The slope of $\overline{AB}$ = the slope of $\overline{AC}$

$\frac{3-1}{a-0} = \frac{5-1}{2-0}$　　　$\therefore \frac{2}{a} = 2$　　　$\therefore a = 1$

2 Giza

1

1 b　　2 b　　3 c　　4 b　　5 c　　6 c

2

[a] Draw $\overline{DF} \perp \overline{BC}$

$\because \overline{AD} \parallel \overline{BC}$, $\overline{AB} \perp \overline{BC}$

, $\overline{DF} \perp \overline{BC}$

$\therefore$ ABFD is a rectangle

$BF = AD = 6$ cm.

$\therefore FC = 4$ cm. , $DF = AB = 3$ cm

$\therefore$ From $\triangle$ DFC which is right-angled at F

$(DC)^2 = 3^2 + 4^2 = 25$　　　$\therefore DC = 5$ cm.

$\therefore \cos(\angle DCB) - \tan(\angle ACB) = \frac{4}{5} - \frac{3}{10} = \frac{1}{2}$

[b] $\because m_1 = \frac{k-1}{2-3} = 1 - k$, $m_2 = \tan 45° = 1$

, $\because L_1 \parallel L_2$　　　$\therefore m_1 = m_2$

$1 - k = 1$　　　$\therefore k = 0$

3

[a] In $\triangle ABC$: $\because m(\angle A) = 90°$

$\therefore (BC)^2 = (20)^2 + (15)^2 = 625$

$\therefore BC = 25$ cm

$\therefore \cos C \cos B - \sin C \sin B$

$= \frac{15}{25} \times \frac{20}{25} - \frac{20}{25} \times \frac{15}{25} = 0$

[b] $\because$ The two diagonals of the parallelogram bisect each other

$\therefore M = \left(\frac{3+1}{2}, \frac{-1+7}{2}\right) = (2, 3)$

Let $D(X, y)$

$\therefore (2, 3) = \left(\frac{6+X}{2}, \frac{2+y}{2}\right)$

$\therefore \frac{6+X}{2} = 2$ $\quad 6 + X = 4$ $\quad \therefore X = -2$

, $\frac{2+y}{2} = 3$ $\quad 2 + y = 6$ $\quad \therefore y = 4$

$D(-2, 4)$

4

[a] $\tan X = 4 \sin 30 \cos 60$

$\tan X = 4 \times \frac{1}{2} \times \frac{1}{2} = 1$

$X = 45°$

[b] The slope of the given straight line $= \frac{-5}{-2} = \frac{5}{2}$

The slope of the required straight line $= \frac{-2}{5}$

The equation of the required straight line is

$y = \frac{-2}{5} X + c$

, $(3, 4)$ satisfies the equation

$4 = \frac{-2}{5} \times 3 + c$ $\quad \therefore c = \frac{26}{5}$

$\therefore$ The equation is : $y = \frac{-2}{5} X + \frac{26}{5}$

5

[a] $\because \sqrt{(a-0)^2 + (7-3)^2} = 5$ (squaring both sides)

$a^2 + (4)^2 = 25$ $\quad \therefore a^2 + 16 = 25$

$\therefore a^2 = 9$ $\quad \therefore a = \pm\sqrt{9}$

$\therefore a = 3$ or $a = -3$

[b] $\because \Delta ABO$ is equilateral

, C is the midpoint of AB

$\therefore \overline{OC} \perp \overline{AB}$ $\quad \therefore (\angle BOC) = 30°$

$\therefore \tan(\angle BOC) = \tan 30° = \frac{1}{\sqrt{3}}$

$\therefore$ The equation of $\overleftrightarrow{OC}$ is : $y = \frac{1}{\sqrt{3}} X + c$

, $\because O \in \overleftrightarrow{OC}$ $\quad \therefore c = 0$

$\therefore$ The equation of $\overleftrightarrow{OC}$ is : $y = \frac{1}{\sqrt{3}} X$

3 Alexandria

1

1 a	2 c	3 b	4 d	5 a	6 c

2

[a] $\because m(\angle C) = 90°$

$(AB)^2 = \ell^2 + \ell^2 = 2\ell^2$ $\quad \therefore AB = \sqrt{2}\,\ell$

1 $AC : BC : AB = \ell : \ell : \sqrt{2}\,\ell = 1 : 1 : \sqrt{2}$

2 $\tan B = \frac{\ell}{\ell} = 1$, $\sin A = \frac{\ell}{\sqrt{2}\,\ell} = \frac{1}{\sqrt{2}}$

[b] $\sqrt{(X-6)^2 + (5-1)^2} = 2\sqrt{5}$ (squaring both sides)

$\therefore (X-6)^2 + (4)^2 = 20$

$\therefore X^2 - 12X + 36 + 16 - 20 = 0$

$X^2 - 12X + 32 = 0$ $\quad (X-8)(X-4) = 0$

$X = 8$ or $X = 4$

3

[a] 1 Let E be the point of intersection of the two diagonals

$\therefore E = \left(\frac{3-1}{2}, \frac{2-2}{2}\right) = (1, 0)$

2 $AC = \sqrt{(-1-3)^2 + (-2-2)^2} = \sqrt{16+16} = \sqrt{32}$

$= 4\sqrt{2}$ length units

$BD = \sqrt{(-2-4)^2 + (3+3)^2} = \sqrt{36+36}$

$= \sqrt{72} = 6\sqrt{2}$ length unit

The area of the rhombus $= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$

$= 24$ square unit.

[b] $\because 2 \sin X = \sin 30° \cos 60° + \cos 30° \sin 60°$

$\therefore 2 \sin X = \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = 1$

$\sin X = \frac{1}{2}$ $\quad X = 30°$

4

[a] The slope of the given straight line $= \frac{-4+3}{5-2} = \frac{-1}{3}$

$\therefore$ The slope of the required straight line $= 3$

$\therefore$ The equation of the required straight line is

$y = 3X + c$

, $\because (1, 2)$ satisfies the equation

$\therefore 2 = 3 \times 1 + c$ $\quad \therefore c = -1$

$\therefore$ The equation is : $y = 3X - 1$

[b] $\tan 60° = \sqrt{3}$ (1)

, $\frac{2 \tan 30°}{1 - \tan^2 30°} = \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}} = \sqrt{3}$ (2)

From (1) , (2) $\tan 60° = \frac{2 \tan 30°}{1 - \tan^2 30°}$

5

[a] $m_1 = \frac{k - 1}{2 - 3} = 1 - k$, $m_2 = \tan 45° = 1$

, $L_1 \parallel L_2$ $m_1 = m_2$

, $1 - k = 1$ $k = 0$

[b] $\because$ The slope of $\overleftrightarrow{AB} = \frac{3 - 5}{3 + 2} = \frac{-2}{5}$

, The slope of $\overleftrightarrow{BC} = \frac{2 - 3}{4 - 3} = \frac{1}{7}$

, $\because$ The slope of $\overleftrightarrow{AB} \neq$ the slope of $\overleftrightarrow{BC}$

A , B and C are not collinear

4 El-Kalyoubia

1

1 c 2 d 3 a 4 b 5 d 6 b

2

[a] $\because \frac{x}{2} + \frac{y}{3} = 1$ (multiplying by 3)

$\frac{3x}{2} + y = 3$ $y = \frac{-3}{2}x + 3$

The slope $= \frac{-3}{2}$ and the intercepted part of the y axis = 3 units

[b] $\sin x = \tan 30° \sin 60°$

$\sin x = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{2} = \frac{1}{2}$ $x = 30°$

$4 \cos 30° \sin 30° = 4 \times \frac{\sqrt{3}}{2} \times \frac{1}{2} = \sqrt{3}$

3

[a] The slope of the given straight line $= \frac{7 - 1}{2 + 2} = \frac{3}{2}$

The slope of the required straight line $= \frac{3}{2}$

$\therefore$ The equation of the required straight line is :

$y = \frac{3}{2}x + c$

, $\because$ (2 , −5) satisfies the equation

$-5 = \frac{3}{2} \times 2 + c$ $\therefore c = -8$

The equation is : $y = \frac{3}{2}x - 8$

[b] $\because 2 AB = \sqrt{3} AC$

$\therefore \frac{AB}{AC} = \frac{\sqrt{3}}{2}$

Let $AB = \sqrt{3}$ length units.

, AC = 2 length units.

BC = 1 length units

1 $\sin C = \frac{\sqrt{3}}{2}$ $m(\angle C) = 60°$

2 $\sin^2 A - \cos^2 C = \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = \frac{1}{4} - \frac{1}{4} = 0$

4

[a] $m_1 = \frac{-3}{-4} = \frac{3}{4}$, $m_2 = \frac{-4}{a}$

, $L_1 \perp L_2$ $m_1 \times m_2 = -1$

$\frac{3}{4} \times \frac{-4}{a} = -1$ $\therefore \frac{-3}{a} = -1$ $\therefore a = 3$

[b] $\because AC = \sqrt{(-1-3)^2 + (-2-2)^2}$

$= \sqrt{16 + 16} = \sqrt{32} = 4\sqrt{2}$ length units.

, $BD = \sqrt{(-2-4)^2 + (3+3)^2}$

$= \sqrt{36 + 36} = \sqrt{72} = 6\sqrt{2}$ length units.

$\therefore$ The area $= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2} = 24$ square units.

5

[a] $\cos^2 60° = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$ (1)

, $\cos^2 30° \tan^2 30° \tan 45° = \left(\frac{\sqrt{3}}{2}\right)^2 \times \left(\frac{1}{\sqrt{3}}\right)^2 \times 1$

$= \frac{1}{4}$ (2)

From (1) , (2) : $\therefore \cos^2 60° = \cos^2 30° \tan^2 30° \tan 45°$

[b] $\because$ C is the midpoint of $\overline{AB}$

$\therefore (3 , 4) = \left(\frac{x + 0}{2} , \frac{0 + y}{2}\right)$

$\frac{x}{2} = 3$ $\therefore x = 6$ $\therefore$ A (6 , 0)

, $\frac{y}{2} = 4$ $\therefore y = 8$ $\therefore$ B (0 , 8)

$\therefore AB = \sqrt{(0-6)^2 + (8-0)^2} = \sqrt{36 + 64}$

$= \sqrt{100} = 10$ length units.

, $\because$ OA = 6 length units , OB = 8 length units

$\therefore$ The perimeter of Δ AOB = 6 + 8 + 10

= 24 length units

5 El-Sharkia

1

1 c 2 a 3 d 4 d 5 b 6 c

2

[a] $(4 , y) = \left(\frac{X+6}{2} , \frac{3+5}{2}\right)$

$\therefore \frac{X+6}{2} = 4$ $\therefore X + 6 = 8$ $\therefore X = 2$

$, y = \frac{3+5}{2}$ $\therefore y = 4$

$\therefore X + y = 2 + 4 = 6$

[b] $\because AB = \sqrt{(3-5)^2 + (-2-3)^2} = \sqrt{4+25}$

$= \sqrt{29}$ length units.

$, BC = \sqrt{(-2-3)^2 + (-4+2)^2} = \sqrt{25+4}$

$= \sqrt{29}$ length units.

$, AC = \sqrt{(-2-5)^2 + (-4-3)^2} = \sqrt{49+49}$

$= 7\sqrt{2}$ length units

$, \because AC \neq AB + BC$

$\therefore$ A , B and C are non collinear

$\therefore$ A , B and C are vertices of a triangle

$, \quad (AC)^2 > (AB)^2 + (BC)^2$

$\therefore \Delta ABC$ is an obtuse-angled trianlge

3

[a] 1 In ΔABC : $\because m(\angle B) = 90°$

$(AC)^2 = (5)^2 + (12)^2 = 169$

$\therefore AC = 13$ cm.

2 In ΔADC : $5 \tan(\angle ACD) - 13 \sin(\angle DAC)$

$= 5 \times \frac{12}{5} - 13 \times \frac{5}{13} = 7$

[b] $\because$ The slope of $\overleftrightarrow{AB} = \frac{3+1}{5-3} = 2$

$\therefore$ The slope of the axis of symmetry of $\overline{AB} = -\frac{1}{2}$

$\therefore$ The equation of the axis of symmetry of $\overline{AB}$ is :

$y = \frac{-1}{2} X + c$

$, \because$ The midpoint of $\overline{AB} = \left(\frac{3+5}{2} , \frac{-1+3}{2}\right) = (4 , 1)$

$\therefore (4 , 1)$ satisfies the equation

$\therefore 1 = -\frac{1}{2} \times 4 + c$ $\therefore c = 3$

$\therefore$ The equation is $y = \frac{-1}{2} X + 3$

4

[a] $\frac{\cos^2 60° + \cos^2 30°}{\sin 60° \tan 60°} = \frac{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}{\frac{\sqrt{3}}{2} \times \sqrt{3}} = \frac{\frac{1}{4} + \frac{3}{4}}{\frac{3}{2}} = \frac{2}{3}$

[b] 1 $\because L_1 \parallel L_2$ $\therefore m_1 = m_2$

$\therefore \frac{-6}{k} = \frac{2}{3}$ $k = \frac{-6 \times 3}{2} = -9$

2 $\because L_1 \perp L_2$ $\therefore m_1 \times m_2 = -1$

$\therefore \frac{-6}{k} \times \frac{2}{3} = -1$ $\therefore \frac{-4}{k} = -1$ $\therefore k = 4$

5

[a] $\because$ The slope of the given straight line $= \frac{-1}{2}$

The slope of the required straight line $= \frac{-1}{2}$

The equation of the required straight line is :

$y = -\frac{1}{2} X + c$

$, \because (1 , 4)$ satisfies the equation.

$\therefore 4 = -\frac{1}{2} \times 1 + c$ $\therefore c = \frac{9}{2}$

$\therefore$ The equation is : $y = -\frac{1}{2} X + \frac{9}{2}$

[b] 1 $\because$ The two diagonals of the square bisect each other

$\therefore M = \left(\frac{2-7}{2} , \frac{4+5}{2}\right) = \left(\frac{-5}{2} , \frac{9}{2}\right)$

Let $D(X , y)$

$\left(\frac{-5}{2} , \frac{9}{2}\right) = \left(\frac{X-3}{2} , \frac{y+0}{2}\right)$

$\frac{X-3}{2} = \frac{-5}{2}$ $\therefore X - 3 = -5$ $\therefore X = -2$

$, \frac{y}{2} = \frac{9}{2}$ $\therefore y = 9$ $\therefore D(-2 , 9)$

2 $AB = \sqrt{(-3-2)^2 + (0-4)^2}$

$= \sqrt{25+16} = \sqrt{41}$ length units.

The area of the square ABCD

$= \left(\sqrt{41}\right)^2 = 41$ square units.

6 El-Menofia

1

1 c 2 a 3 c 4 c 5 b 6 a

2

[a] ∵ $AB=\sqrt{(1-3)^2+(4-0)^2}=\sqrt{4+16}$
$=2\sqrt{5}$ length units.

, $BC=\sqrt{(-1-1)^2+(2-4)^2}=\sqrt{4+4}$
$=2\sqrt{2}$ length units.

, $AC=\sqrt{(-1-3)^2+(2-0)^2}=\sqrt{16+4}$
$=2\sqrt{5}$ length units.

∴ AB = AC

∴ Δ ABC is an isosceles triangle

[b] $\frac{\tan 45^\circ+\tan 30^\circ}{1-\tan 45^\circ \tan 30^\circ}=\frac{1+\frac{1}{\sqrt{3}}}{1-1\times\frac{1}{\sqrt{3}}}=\frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}=2+\sqrt{3}$

3

[a] The slope of $\overleftrightarrow{AB}=\frac{0-4}{-3-2}=\frac{4}{5}$

, The slope of $\overleftrightarrow{CD}=\frac{9-5}{-2+7}=\frac{4}{5}$

$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ (1)

, ∵ The slope of $\overleftrightarrow{BC}=\frac{5-0}{-7+3}=\frac{-5}{4}$

, The slope of $\overleftrightarrow{AD}=\frac{9-4}{-2-2}=\frac{-5}{4}$

, $\overleftrightarrow{BC} \parallel \overleftrightarrow{AD}$ (2)

From (1) , (2) : ∴ ABCD is a parallelogram

, ∵ The slope of $\overleftrightarrow{AB}$ × the slope of $\overleftrightarrow{BC}$
$=\frac{4}{5}\times\frac{-5}{4}=-1$

∴ $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$ ∴ ABCD is a rectangle.

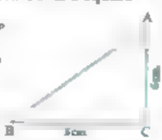
, ∵ The slope of $\overleftrightarrow{AC}=\frac{5-4}{-7-2}=\frac{-1}{9}$

, The slope of $\overleftrightarrow{BD}=\frac{9-0}{-2+3}=9$

, The slope of $\overleftrightarrow{AC}$ × the slope of $\overleftrightarrow{BD}=\frac{-1}{9}\times 9$
$=-1$

∴ $\overleftrightarrow{AC} \perp \overleftrightarrow{BD}$ ∴ ABCD is a square

[b] In Δ ABC ∵ ∵ m (∠ C) = 90°



∴ $(AB)^2=(6)^2+(8)^2=100$

∴ AB = 10 cm.

cos A cos B − sin A sin B
$=\frac{6}{10}\times\frac{8}{10}-\frac{8}{10}\times\frac{6}{10}=0$

4

[a] ∵ $m_1=\frac{5+2}{4+3}=1$, $m_2=\tan 45^\circ=1$

∴ $m_1=m_2$

∴ The two straight lines are parallel.

[b] ∵ $\sqrt{3}\sin X\tan 30^\circ=\tan 45^\circ\cos 2X$

∴ $\sqrt{3}\times\sin X\times\frac{1}{\sqrt{3}}=1\times\cos 2X$

∴ $\sin X=\cos 2X$

∴ $X+2X=90^\circ$ ∴ $X=30^\circ$

5

[a] ∵ The slope of the given straight line $=\frac{-3}{-4}=\frac{3}{4}$

∴ The slope of the required straight line $=\frac{-4}{3}$

and it intercepts from the positive part of y-axis 4 units

∴ The equation is : $y=\frac{-4}{3}X+4$

[b] $\boxed{1}$ In Δ ABC :

∵ m (∠ B) = 90°

∵ sin (∠ ACB) $=\frac{3}{5}$

∴ m (∠ ACB) ≈ 36° 52′ 12″

$\boxed{2}$ ∵ $(BC)^2=(5)^2-(3)^2=16$

∵ BC = 4 cm

∴ The area of the rectangle ABCD = 3 × 4
$=12\ cm^2$

El-Gharbia

1

$\boxed{1}$ a $\boxed{2}$ c $\boxed{3}$ c $\boxed{4}$ b $\boxed{5}$ c $\boxed{6}$ b

2

[a] ∵ m (∠ Y) = 90°

∴ $(YZ)^2=(13)^2-(5)^2=144$

∴ YZ = 12 cm.

∵ cos X cos Z − sin X sin Z
$=\frac{5}{13}\times\frac{12}{13}-\frac{12}{13}\times\frac{5}{13}=0$

[b] Let the positive measure of the angle with the positive direction of the X-axis be θ

, ∵ The slope of $\overleftrightarrow{AB}=\frac{1+2}{6-3}=1$

$\therefore \tan\theta = 1$ $\qquad \therefore \theta = 45^\circ$

The measure of the positive angle that $\overline{AB}$ makes with the negative direction of the X-axis $= 180^\circ - 45^\circ = 135^\circ$

3

[a] $\because \cos(3X + 6^\circ) = \frac{1}{2}$

$\therefore 3X + 6^\circ = 60^\circ$

$\therefore 3X = 54^\circ$ $\qquad \therefore X = 18^\circ$

[b] $\because \frac{y-1}{X} = \frac{1}{3}$

$\therefore 3y - 3 = X$ $\qquad \therefore 3y - X - 3 = 0$

$\therefore$ The slope of the given straight line $= \frac{1}{3}$

$\therefore$ The slope of the required straight line $= \frac{1}{3}$

and intercepts from the negative part of y-axis 3 units.

$\therefore$ The equation is $y = \frac{1}{3}X - 3$

4

[a] $\because X - \sin 30^\circ \cos^2 45^\circ = \sin^2 60^\circ$

$X - \frac{1}{2} \times \left(\frac{1}{\sqrt{2}}\right)^2 = \left(\frac{\sqrt{3}}{2}\right)^2$

$\therefore X - \frac{1}{4} = \frac{3}{4}$ $\qquad \therefore X = 1$

[b] Let D be the midpoint of $\overline{BC}$

$\therefore D = \left(\frac{3+1}{2}, \frac{4-6}{2}\right) = (2, -1)$

$\because AB = AC$ $\qquad \therefore \overline{AD} \perp \overline{BC}$

$\therefore AD = \sqrt{(2+3)^2 + (-1-0)^2} = \sqrt{25+1}$

$= \sqrt{26}$ length unit.

5

[a] $\because MA = \sqrt{(3+1)^2 + (-1-2)^2} = \sqrt{16+9}$

$= 5$ length unit.

$\therefore$ The circumference of the circle

$= 2 \times \frac{22}{7} \times 5 = 31\frac{3}{7}$ length units.

[b] $\because$ The slope of $\overline{AB} = \frac{-4+3}{5-2} = \frac{-1}{3}$

$\therefore$ The slope of the required straight line $= 3$

$\therefore$ The equation of the required straight line is :

$y = 3X + c$

$\because (1, 2)$ satisfies the equation.

$\therefore 2 = 3 \times 1 + c$ $\qquad \therefore c = -1$

$\therefore$ The equation is : $y = 3X - 1$

El-Dakahlia

1

[a] 1 c $\qquad$ 2 c $\qquad$ 3 d

[b] 1 Let $A(X, 0)$, $B(0, y)$

$\therefore (4, 3) = \left(\frac{X+0}{2}, \frac{0+y}{2}\right)$

$\therefore \frac{X}{2} = 4$ $\qquad \therefore X = 8$ $\qquad \therefore A = (8, 0)$

$, \frac{y}{2} = 3$ $\qquad \therefore y = 6$ $\qquad \therefore B = (0, 6)$

2 $\therefore OA = 8$ units , $OB = 6$ units

$\therefore$ Area of $\Delta AOB = \frac{1}{2} \times 8 \times 6 = 24$ square units.

2

[a] 1 a $\qquad$ 2 d $\qquad$ 3 c

[b] $\because 2\sin X = \tan^2 60^\circ - 2\tan 45^\circ$

$\therefore 2\sin X = \left(\sqrt{3}\right)^2 - 2 \times 1$

$\therefore 2\sin X = 1$ $\qquad \therefore \sin X = \frac{1}{2}$

$\therefore X = 30^\circ$

3

[a] $\because$ The straight line passes through $(2, 0)$, $(0, 3)$

$\therefore$ The slope $= \frac{3-0}{0-2} = \frac{-3}{2}$

and it intercepts from the positive part of y-axis 3 units.

$\therefore$ The equation is $y = \frac{-3}{2}X + 3$

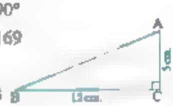
[b] In ΔABC : $\because m(\angle C) = 90^\circ$

$\therefore (AB)^2 = (12)^2 + (5)^2 = 169$

$\therefore AB = 13$ cm

$, \cos A \cos B - \sin A \sin B$

$= \frac{5}{13} \times \frac{12}{13} - \frac{12}{13} \times \frac{5}{13} = 0$



4

[a] $\because$ The two diagonals of the parallelogram bisect each other

$\therefore$ The midpoint of $\overline{AC} = \left(\frac{3+0}{2}, \frac{2-3}{2}\right)$

$= \left(\frac{3}{2}, \frac{-1}{2}\right)$

Let $D(X, y)$

$\therefore \left(\frac{3}{2}, \frac{-1}{2}\right) = \left(\frac{4+X}{2}, \frac{5+y}{2}\right)$

$\therefore \frac{4+X}{2} = \frac{3}{2}$ $\qquad \therefore 4 + X = 3$

$\therefore x = -1$, $\frac{5+y}{2} = \frac{-1}{2}$

$\therefore 5 + y = -1$ $\therefore y = 4$ D $(-1 , 4)$

[b] $\because 2 \sin 30^\circ + 4 \cos 60^\circ = 2 \times \frac{1}{2} + 4 \times \frac{1}{2} = 3$ (1)

, $\tan^2 60^\circ = \left(\sqrt{3}\right)^2 = 3$ (2)

From (1) and (2) :

$\therefore 2 \sin 30^\circ + 4 \cos 60^\circ = \tan^2 60^\circ$

5

[a] $\because$ The slope of $\overline{AB} = \frac{-7-1}{-1-5} \ldots 4$

, The slope of $\overline{BC} = \frac{3+7}{1-3} = -5$

The slope of $\overline{AB} \neq$ The slope of $\overline{BC}$

$\therefore$ The points A , B and C are not collinear

[b] $\because$ The slope of $\overline{AB} = \frac{5-1}{4-2} = 2$

The slope of the required straight line $= \frac{-1}{2}$

$\therefore$ The equation of the required straight line is :

$y = \frac{-1}{2} x + c$

The midpoint of $\overline{AB} = \left(\frac{2+4}{2} , \frac{1+5}{2}\right) = (3 , 3)$

$\because$ (3 , 3) satisfies the equation

$\therefore 3 = \frac{-1}{2} \times 3 + c$ $\therefore c = \frac{9}{2}$

$\therefore$ The equation is : $y = \frac{-1}{2} x + \frac{9}{2}$

6 Ismailia

1

1 c 2 b 3 d 4 b 5 a 6 d

2

[a] In Δ ABC : $\because m(\angle B) = 90^\circ$

$(AC)^2 = (BC)^2 + (AB)^2$

$\therefore \sin^2 A + \sin^2 C = \left(\frac{BC}{AC}\right)^2 + \left(\frac{AB}{AC}\right)^2$

$= \frac{(BC)^2}{(AC)^2} + \frac{(AB)^2}{(AC)^2} = \frac{(BC)^2 + (AB)^2}{(AC)^2}$

$= \frac{(AC)^2}{(AC)^2} = 1$

[b] $\because m_1 = \frac{3-4}{-1-2} = \frac{1}{3}$, $m_2 = \frac{1}{3}$

$\therefore m_1 = m_2$

$\therefore$ The two straight lines are parallel.

3

[a] In Δ ABC : $\because m(\angle B) = 90^\circ$

$\therefore \sin(\angle ACB) = \frac{15}{25}$

$\therefore m(\angle ACB) \simeq 36^\circ\ 52'\ 12''$

, $(BC)^2 = (25)^2 - (15)^2 = 400$

$\therefore$ BC = 20 cm

$\therefore$ The area of rectangle ABCD $= 15 \times 20$

$= 300 \text{ cm}^2$

[b] 1 $\because$ The slope of the straight line $= \frac{3-1}{2-1} = 2$

$\therefore$ The equation of the straight line is $y = 2x + c$

, $\because$ (1 , 1) satisfies the equation

$\therefore 1 = 2 \times 1 + c$ $\therefore c = -1$

$\therefore$ The equation is : $y = 2x - 1$

2 The length of the intercepted part of y-axis is 1 unit

4

[a] $\because$ The midpoint of $\overline{AC} = \left(\frac{-1+7}{2} , \frac{3+4}{2}\right)$

$= \left(3 , \frac{7}{2}\right)$

, the midpoint of $\overline{BD} = \left(\frac{5+1}{2} , \frac{1+6}{2}\right) = \left(3 , \frac{7}{2}\right)$

The midpoint of $\overline{AC}$ = The midpoint of $\overline{BD}$

ABCD is a parallelogram

[b] $\because$ The straight line passes through (3 , 0) , (0 , 4)

The slope of straight line $= \frac{4-0}{0-3} = \frac{-4}{3}$

and it intersects from the positive part of y-axis 4 units.

$\therefore$ The equation is : $y = \frac{-4}{3} x + 4$

5

[a] $\sin 45^\circ \cos 45^\circ + \sin 30^\circ \cos 60^\circ - \cos^2 30^\circ$

$= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{2} - \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{2} + \frac{1}{4} - \frac{3}{4} = 0$

[b] 1 $BC = \sqrt{(12-6)^2 + (8-0)^2} = \sqrt{36 + 64}$

= 10 units.

, $AC = \sqrt{(12-2)^2 + (8-3)^2} = \sqrt{100 + 25}$

$= 5\sqrt{5}$ units

$\therefore$ Saeid's house is nearer to the school

[3] ∵ The slope of $\overleftrightarrow{AB} = \frac{0-3}{6-2} = \frac{-3}{4}$

, The slope of $\overleftrightarrow{BC} = \frac{8-0}{12-6} = \frac{4}{3}$

, ∵ The slope of $\overleftrightarrow{AB}$ × the slope of $\overleftrightarrow{BC}$

$= \frac{-3}{4} \times \frac{4}{3} = -1$

∴ $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$

10

1

[1] c [2] b [3] a [4] c [5] a [6] b

2

[a] ∵ The slope of the straight line = 2 and it intersects from the positive part of y-axis 7 units.

∴ Its equation is : $y = 2x + 7$

[b] ∵ $4x = \cos^2 30° \tan^2 30° \tan^2 45°$

∴ $4x = \left(\frac{\sqrt{3}}{2}\right)^2 \times \left(\frac{1}{\sqrt{3}}\right)^2 \times (1)^2$

∴ $4x = \frac{3}{4} \times \frac{1}{3} \times 1$ ∴ $4x = \frac{1}{4}$

∴ $x = \frac{1}{16}$

3

[a] ∵ The diagonals of the parallelogram bisect each other

∴ $E = \left(\frac{4-2}{2}, \frac{3-3}{2}\right) = (1, 0)$

Let $D(x, y)$

∴ $(1, 0) = \left(\frac{0+x}{2}, \frac{2+y}{2}\right)$

∴ $\frac{x}{2} = 1$ ∴ $x = 2$

, $\frac{2+y}{2} = 0$ ∴ $2 + y = 0$ ∴ $y = -2$

∴ $D(2, -2)$

[b] ∵ $\tan^2 60° - \tan^2 45° = (\sqrt{3})^2 - (1)^2 = 3 - 1 = 2$ (1)

, $\sin^2 60° + \cos^2 60° + 2 \sin 30°$

$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + 2 \times \frac{1}{2} = \frac{3}{4} + \frac{1}{4} + 1 = 2$ (2)

From (1) , (2) :

∴ $\tan^2 60° - \tan^2 45°$

$= \sin^2 60° + \cos^2 60° + 2 \sin 30°$

4

[a] $m_1 = \frac{3+1}{6-2} = 1$, $m_2 = \tan 45° = 1$

∴ $m_1 = m_2$

∴ The two straight lines are parallel

[b] ∵ $2AB = \sqrt{3}\,AC$

∴ $\frac{AB}{AC} = \frac{\sqrt{3}}{2}$

Let $AB = \sqrt{3}$ length units

, AC = 2 length units.

∴ BC = 1 length unit

$\sin C = \frac{\sqrt{3}}{2}$, $\tan A = \frac{1}{\sqrt{3}}$

5

[a] ∵ $AB = \sqrt{(3+3)^2 + (4-0)^2} = \sqrt{36+16}$

$= 2\sqrt{13}$ length units

, $BC = \sqrt{(1-3)^2 + (-6-4)^2} = \sqrt{4+100}$

$= 2\sqrt{26}$ length units

, $AC = \sqrt{(1+3)^2 + (-6-0)^2} = \sqrt{16+36}$

$= 2\sqrt{13}$ length units

∴ AB = AC

∴ Δ ABC is an isosceles triangle

[b] ∵ The slope of the given straight line = $\frac{-1}{2}$

∴ The slope of the required straight line = 2

∴ The equation of the required straight line is :

$y = 2x + c$

∵ (3 , 5) satisfies the equation

∴ $5 = 2 \times 3 + c$ ∴ $c = -1$

∴ The equation is : $y = 2x - 1$

11

1

[1] b [2] c [3] d [4] a [5] b [6] d

2

[a] [1] $\sin A \cos B + \cos A \sin B = \frac{12}{13} \times \frac{12}{13} + \frac{5}{13} \times \frac{5}{13}$

$= 1$

[2] $1 + \tan^2 A = 1 + \left(\frac{12}{5}\right)^2 = \frac{169}{25}$

[b] $\sin 45° \cos 45° + \sin 30° \cos 60° - \cos^2 30°$

$= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{2} - \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{2} + \frac{1}{4} - \frac{3}{4} = 0$

3

[a] $\because \sin E = \sin 60° \cos 30° - \cos 60° \sin 30°$

$\therefore \sin E = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$

$\therefore m(\angle E) = 30°$

[b] $\because m_1 = \frac{5+2}{4+3} = 1$, $m_2 = \tan 45° = 1$

$\therefore m_1 = m_2$

$\therefore$ The two straight lines are parallel

4

[a] $\because$ The slope of the given straight line

$= \frac{-4+3}{5-2} = \frac{-1}{3}$

$\therefore$ The slope of the required straight line $= 3$

The equation of the required straight line is

$y = 3X + c$

, $\because (1 , 2)$ satisfies the equation.

$2 = 3 \times 1 + c$ $\therefore c = -1$

The equation is : $y = 3X - 1$

[b] $\because MA = \sqrt{(3+1)^2 + (-1-2)^2} = \sqrt{16+9}$

$= 5$ length units

, $MB = \sqrt{(-4+1)^2 + (6-2)^2} = \sqrt{9+16}$

$= 5$ length units

, $MC = \sqrt{(2+1)^2 + (-2-2)^2} = \sqrt{9+16}$

$= 5$ length units

$\therefore MA = MB = MC$

$\therefore$ The points A , B and C are located on the circle M

5

[a] $\because$ The diagonals of the parallelogram bisect each other

Let E be the point of intersection of the diagonals

$\therefore E = \left(\frac{3+0}{2} , \frac{2-3}{2}\right) = \left(\frac{3}{2} , \frac{-1}{2}\right)$

Let $D(X , y)$

$\therefore \left(\frac{3}{2} , \frac{-1}{2}\right) = \left(\frac{4+X}{2} , \frac{-5+y}{2}\right)$

$\therefore \frac{4+X}{2} = \frac{3}{2}$ $\therefore 4 + X = 3$ $\therefore X = -1$

, $\frac{-5+y}{2} = \frac{-1}{2}$ $\therefore -5 + y = -1$ $\therefore y = 4$

$\therefore D(-1 , 4)$

[b] 1 $c = 3$ units from the positive part of y-axis

2 6 units from the negative part of X-axis

3 The straight line passes through $(-6 , 0) , (0 , 3)$

$\therefore$ The slope $= \frac{3-0}{0+6} = \frac{1}{2}$

12 Damietta

1

1 c	2 b	3 b	4 d	5 c	6 b

2

[a] In $\triangle ABC$ $\because m(\angle C) = 90°$

$\therefore (AB)^2 = (6)^2 + (8)^2 = 100$

$\therefore AB = 10$ cm

$\therefore \cos A \cos B - \sin A \sin B$

$= \frac{6}{10} \times \frac{8}{10} - \frac{8}{10} \times \frac{6}{10} = 0$

[b] The straight line passes through $(3 , 0) , (0 , 2)$

The slope $= \frac{2-0}{0-3} = \frac{-2}{3}$

and intercepts from the positive part of y-axis 2 units

$\therefore$ The equation is : $y = \frac{-2}{3} X + 2$

3

[a] $\sqrt{(6-X)^2 + (1-5)^2} = 2\sqrt{5}$ (squaring both sides)

$\therefore (6-X)^2 + (-4)^2 = 20$

$\therefore X^2 - 12X + 36 + 16 = 20$

$\therefore X^2 - 12X + 32 = 0$

$\therefore (X-8)(X-4) = 0$ $\therefore X = 8$ or $X = 4$

[b] The slope $= \frac{1+1}{1-2} = -2$

$\therefore$ The equation is : $y = -2X + c$

, $\because (1 , 1)$ satisfies the equation

$\therefore 1 = -2 \times 1 + c$ $\therefore c = 3$

$\therefore$ The equation is : $y = -2X + 3$

, $\because (0 , k)$ satisfies the equation

$\therefore k = -2 \times 0 + 3$ $\therefore k = 3$

4

[a] $\because 4X = \cos^2 30° \tan^2 30° \tan^2 45°$

$\therefore 4X = \left(\frac{\sqrt{3}}{2}\right)^2 \times \left(\frac{1}{\sqrt{3}}\right)^2 \times (1)^2$

$\therefore 4\,X = \frac{3}{4} \times \frac{1}{3} \times 1 \qquad 4\,X = \frac{1}{4}$

$\therefore X = \frac{1}{16}$

[b] $m_1 = \frac{3-0}{0-a} = \frac{-3}{a}$, $m_2 = \tan 30° = \frac{1}{\sqrt{3}}$

, $\because$ the two straight lines are perpendicular

$m_1 \times m_2 = -1 \quad \therefore \frac{-3}{a} \times \frac{1}{\sqrt{3}} = -1 \quad \therefore a = \sqrt{3}$

5

[a] $\sin 45° \cos 45° + \sin 30° \cos 60° - \cos^2 30°$

$= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{2} - \left(\frac{\sqrt{3}}{2}\right)^2$

$= \frac{1}{2} + \frac{1}{4} - \frac{3}{4} = 0$

[b] $\because$ The slope of $\overleftrightarrow{AB} = \frac{5-3}{3-1} = 1$

The slope of the required straight line $= -1$

$\therefore$ The equation of the required straight line is :

$y = -X + c$

, $C = \left(\frac{1+3}{2}, \frac{3+5}{2}\right) = (2, 4)$

$(2, 4)$ satisfies the equation

$\therefore 4 = -2 + c \qquad \therefore c = 6$

The equation is : $y = -X + 6$

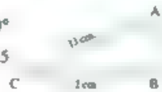
13 Kafr El-Sheikh

1

1 b 2 d 3 c 4 b 5 c 6 b

2

[a] In ΔABC $m(\angle B) = 90°$



$\therefore (AB)^2 = (13)^2 - (12)^2 = 25$

$\therefore AB = 5$ cm

$\therefore \sin^2 C + \sin^2 A = \left(\frac{5}{13}\right)^2 + \left(\frac{12}{13}\right)^2 = 1$

[b] 1 $\because MA = \sqrt{(5-1)^2 + (2+1)^2} = \sqrt{16+9}$

$= 5$ length units

$\therefore$ The area $= \pi \times (5)^2 = 25\,\pi$ square units.

2 $\because$ The slope of $\overline{AM} = \frac{-1-2}{1-5} = \frac{3}{4}$

$\therefore$ The equation is : $y = \frac{3}{4} X + c$

, $\because (1, -1)$ satisfies the equation.

$\therefore -1 = \frac{3}{4} \times 1 + c \qquad \therefore c = -\frac{7}{4}$

The equation is $y = \frac{3}{4} X - \frac{7}{4}$

3

[a] The slope of $\overleftrightarrow{AB} = \frac{7-5}{-1+3} = 1$

The slope of the axis of symmetry of $\overline{AB} = -1$

The equation of the axis of symmetry of $\overline{AB}$ is : $y = -X + c$

, $\because$ The midpoint of $\overline{AB} = \left(\frac{-3-1}{2}, \frac{5+7}{2}\right)$

$= (-2, 6)$

$(-2, 6)$ satisfies the equation .

$\therefore 6 = 2 + c \qquad \therefore c = 4$

$\therefore$ The equation is : $y = -X + 4$

[b] $\because \tan^2 60° - \tan^2 45° = \left(\sqrt{3}\right)^2 - (1)^2$

$= 3 - 1 = 2 \qquad (1)$

, $\sin^2 60° + \cos^2 60° + 2 \sin 30°$

$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + 2 \times \frac{1}{2} = \frac{3}{4} + \frac{1}{4} + 1 = 2 \quad (2)$

From (1) , (2)

$\tan^2 60° - \tan^2 45°$

$= \sin^2 60° + \cos^2 60° + 2 \sin 30°$

4

[a] $\because$ The midpoint of $\overline{AC} = \left(\frac{-1+7}{2}, \frac{3+4}{2}\right)$

$= \left(3, \frac{7}{2}\right)$

, The midpoint of $\overline{BD} = \left(\frac{5+1}{2}, \frac{1+6}{2}\right)$

$= \left(3, \frac{7}{2}\right)$

The midpoint of $\overline{AC}$ = The midpoint of $\overline{BD}$

$\therefore$ A , B , C and D are vertices of a parallelogram

[b] Draw $\overline{AF} \perp \overline{BC}$

, $\overline{DE} \perp \overline{BC}$

$\overline{AD} \parallel \overline{BC}$

$\therefore$ AFED is a rectangle

, FE = 4 cm. $\qquad \therefore$ BF + CE = 8 cm.

BF = CE = 4 cm. ($\Delta ABF \equiv \Delta DCE$)

From ΔABF which is right at F

$(AF)^2 = (5)^2 - (4)^2 = 9 \quad \therefore AF = 3$ cm.

$\therefore$ DE = AF = 3 cm.

$\therefore \frac{\tan B \cos C}{\cos^2 C + \sin^2 C} = \frac{\frac{3}{4} \times \frac{4}{5}}{\left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2} = \frac{3}{5}$

5

[a] $m_1 = \frac{k-1}{2-3} = 1-k$, $m_2 = \tan 45^\circ = 1$

1) $L_1 \parallel L_2$ ∴ $m_1 = m_2$

$1 - k = 1$ ∴ $k = 0$

2) $L_1 \perp L_2$ ∴ $m_1 \times m_2 = -1$

$(1-k) \times 1 = -1$ ∴ $1 - k = -1$

$k = 2$

[b] 1 Let A $(x , 0)$, B $(0 , y)$

$(3 , 4) = \left(\frac{x+0}{2} , \frac{0+y}{2}\right)$

$\frac{x}{2} = 3$ ∴ $x = 6$ ∴ A $(6 , 0)$

, $\frac{y}{2} = 4$ ∴ $y = 8$ ∴ B $(0 , 8)$

2 ∵ The slope of $\overleftrightarrow{AB} = \frac{8-0}{0-6} = \frac{-4}{3}$

and it intercepts from the positive part of y-axis 8 units

∴ The equation of $\overleftrightarrow{AB}$ is : $y = \frac{-4}{3}x + 8$

14 El-Beheira

1

1 b 2 d 3 b 4 c 5 d 6 c

2

[a] ∵ $m(\angle C) = 90^\circ$

$(AB)^2 = (8)^2 + (6)^2 = 100$

∴ AB = 10 cm

1 $\cos A \cos B - \sin A \sin B = \frac{6}{10} \times \frac{8}{10} - \frac{8}{10} \times \frac{6}{10}$
$= 0$

2 ∵ $\cos B = \frac{8}{10}$

∴ $m(\angle B) \simeq 36^\circ\ 52'\ 12''$

[b] $AB = \sqrt{(3+2)^2 + (-1-4)^2} = \sqrt{25+25}$
$= 5\sqrt{2}$ length units.

, $BC = \sqrt{(4-3)^2 + (5+1)^2} = \sqrt{1+36}$
$= \sqrt{37}$ length unit.

, $AC = \sqrt{(4+2)^2 + (5-4)^2} = \sqrt{36+1}$
$= \sqrt{37}$ length units

∴ BC = AC

∴ Δ ABC is an isosceles triangle

3

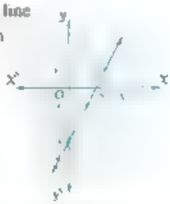
[a] $\tan^2 60^\circ - \tan^2 45^\circ = \left(\sqrt{3}\right)^2 - (1)^2 = 3 - 1$
$= 2$ (1)

, $\cos^2 30^\circ + \cos^2 60^\circ + 2 \sin 30^\circ$
$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + 2 \times \frac{1}{2} = \frac{3}{4} + \frac{1}{4} + 1 = 2$ (2)

From (1) , (2)

∴ $\tan^2 60^\circ - \tan^2 45^\circ$
$= \cos^2 30^\circ + \cos^2 60^\circ + 2 \sin 30^\circ$

[b] ∵ The slope of the straight line = 2 and it intersects from the negative part of y-axis 3 units



∴ The equation is :
$y = 2x - 3$

4

[a] ∵ $x \sin 30^\circ \cos^2 45^\circ = \sin^2 60^\circ$

$x \times \frac{1}{2} \times \left(\frac{1}{\sqrt{2}}\right)^2 = \left(\frac{\sqrt{3}}{2}\right)^2$

$\frac{1}{4}x = \frac{3}{4}$ ∴ $x = 3$

[b] $m_1 = \frac{k-1}{2-3} = 1-k$, $m_2 = \tan 45^\circ = 1$

, $L_1 \parallel L_2$ ∴ $m_1 = m_2$

$1 - k = 1$ ∴ $k = 0$

5

[a] $(3 , 1) = \left(\frac{1+x}{2} , \frac{y+3}{2}\right)$

$\frac{1+x}{2} = 3$ ∴ $1 + x = 6$ ∴ $x = 5$

, $\frac{y+3}{2} = 1$ ∴ $y + 3 = 2$ ∴ $y = -1$

∴ The point $(x , y) = (5 , -1)$

[b] ∵ The slope of the given straight line $= \frac{-1}{2}$

∴ The slope of the required straight line = 2

∴ The equation of the required straight line is

$y = 2x + c$

, ∵ $(3 , -5)$ satisfies the equation

∴ $-5 = 2 \times 3 + c$ ∴ $c = -11$

∴ The equation is : $y = 2x - 11$

15 El-Fayoum

1

1 c 2 b 3 b 4 d 5 a 6 c

2

[a] 1 In $\triangle ABC$: $\because m(\angle B) = 90°$

$\sin(\angle ACB) = \frac{15}{25}$

$\therefore m(\angle ACB) \simeq 36° \, 52' \, 12''$

2 $\because (BC)^2 = (25)^2 - (15)^2 = 400$

$\therefore BC = 20$ cm.

$\therefore$ The area of the rectangle ABCD $= 15 \times 20$

$= 300 \text{ cm}^2$

[b] $\because \sqrt{(-2-a)^2 + (3-7)^2} = 5$ (squaring both sides)

$\therefore (-2-a)^2 + (-4)^2 = 25$

$\therefore 4 + 4a + a^2 + 16 - 25 = 0$

$\therefore a^2 + 4a - 5 = 0$

$(a-1)(a+5) = 0 \quad \therefore a = 1$ or $a = -5$

3

[a] $\because 2 \sin X = \sin 30° \cos 60° + \cos 30° \sin 60°$

$\therefore 2 \sin X = \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

$\therefore 2 \sin X = \frac{1}{4} + \frac{3}{4}$

$\therefore 2 \sin X = 1 \quad \therefore \sin X = \frac{1}{2} \quad \therefore X = 30°$

[b] $m_1 = \frac{4-3}{2+1} = \frac{1}{3}$, $m_2 = \frac{1}{3}$

$\because m_1 = m_2$

$\therefore$ The two straight lines are parallel

4

[a] $AB = \sqrt{(6-5)^2 + (-2-3)^2} = \sqrt{1+25}$

$= \sqrt{26}$ length units.

$, BC = \sqrt{(1-6)^2 + (-1+2)^2} = \sqrt{25+1}$

$= \sqrt{26}$ length units.

$, CD = \sqrt{(0-1)^2 + (4+1)^2} = \sqrt{1+25}$

$= \sqrt{26}$ length units

$, AD = \sqrt{(0-5)^2 + (4-3)^2} = \sqrt{25+1}$

$= \sqrt{26}$ length units.

$\therefore AB = BC = CD = AD$

$\therefore$ ABCD is a rhombus.

[b] Let D be the midpoint of $\overline{BC}$

$\therefore D = \left(\frac{3+1}{2}, \frac{7-3}{2}\right) = (2, 2)$

$\therefore$ The slope of $\overleftrightarrow{AD} = \frac{-6-2}{5-2} = \frac{-8}{3}$

The equation of $\overleftrightarrow{AD}$ is $y = \frac{-8}{3} X + c$

$\therefore D \in \overleftrightarrow{AD}$

$\therefore (2, 2)$ satisfies its equation

$\therefore 2 = \frac{-8}{3} \times 2 + c \quad \therefore c = \frac{22}{3}$

$\therefore$ The equation of $\overleftrightarrow{AD}$ is $y = \frac{-8}{3} X + \frac{22}{3}$

5

[a] L.H.S $= \frac{\cos^2 60° + \cos^2 30° + \tan^2 45°}{\sin 60° \tan 60° - \sin 30°}$

$= \frac{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + (1)^2}{\frac{\sqrt{3}}{2} \times \sqrt{3} - \frac{1}{2}} = \frac{\frac{1}{4} + \frac{3}{4} + 1}{\frac{3}{2} - \frac{1}{2}} = 2 =$ R.H.S

[b] $m_1 = \frac{y-1}{2-3} = 1 - y$, $m_2 = \tan 45° = 1$

$\because L_1 \perp L_2 \quad \therefore m_1 \times m_2 = -1$

$\therefore (1-y) \times 1 = -1 \quad \therefore 1 - y = -1$

$\therefore y = 2$

16 Beni Suef

1

1 c 2 a 3 d 4 b 5 b 6 c

2

[a] $AB = \sqrt{(5+1)^2 + (1-3)^2} = \sqrt{36+4}$

$= 2\sqrt{10}$ length units.

$, BC = \sqrt{(6-5)^2 + (4-1)^2} = \sqrt{1+9}$

$= \sqrt{10}$ length units.

$\therefore$ The area of rectangle ABCD $= 2\sqrt{10} \times \sqrt{10}$

$= 20$ square units.

[b] $\because X \cos 60° = \sin 30° + \tan 45°$

$\therefore X \times \frac{1}{2} = \frac{1}{2} + 1 \quad \therefore \frac{1}{2} X = \frac{3}{2}$

$\therefore X = 3$

3

[a] $\because m_1 = \frac{4-0}{3+1} = 1$, $m_2 = \tan 45° = 1$

$m_1 = m_2$

The two straight lines are parallel

[b] In Δ ABC $m(\angle A) = 90°$

$(BC)^2 = (20)^2 + (15)^2 = 625$

$BC = 25$ cm

$\cos C \cos B - \sin C \sin B$

$= \frac{15}{25} \times \frac{20}{25} - \frac{20}{25} \times \frac{15}{25} = 0$

4

[a] $(X , -3) = \left(\frac{3+9}{2} , \frac{y+11}{2}\right)$

$X = \frac{3+9}{2}$ $\qquad X = 3$

, $\frac{y+11}{2} = -3$ $\qquad y + 11 = -6$

$y = -17$ $\qquad X + y = 3 - 17 = -14$

[b] $\sin 45° \cos 45° + 3 \sin 30° \cos 60° - \cos^2 30°$

$= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + 3 \times \frac{1}{2} \times \frac{1}{2} - \left(\frac{\sqrt{3}}{2}\right)^2$

$= \frac{1}{2} + \frac{3}{4} - \frac{3}{4} = \frac{1}{2}$

5

[a] $\because$ The slope of the given straight line = 2

$\therefore$ The slope of the required straight line = $\frac{-1}{2}$

The equation of the required straight line is :

$y = \frac{-1}{2} X + c$

, $\because (2 , -5)$ satisfies the equation

$-5 = \frac{-1}{2} \times 2 + c$ $\qquad \therefore c = -4$

$\therefore$ The equation is : $y = -\frac{1}{2} X - 4$

[b] $\because$ The slope of $\overleftrightarrow{AD} = \frac{1-3}{2-6} = \frac{1}{2}$

, The slope of $\overleftrightarrow{BC} = \frac{-1-2}{0-6} = \frac{1}{2}$

The slope of $\overleftrightarrow{AD}$ = the slope of $\overleftrightarrow{BC}$

$\therefore \overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$ (1)

, $\because$ The slope of $\overleftrightarrow{AB} = \frac{2-3}{6-2} = \frac{-1}{4}$

, the slope of $\overleftrightarrow{CD} = \frac{1+1}{-2-0} = -1$

$\therefore$ The slope of $\overleftrightarrow{AB} \neq$ The slope of $\overleftrightarrow{CD}$

$\therefore \overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are not parallel (2)

From (1) , (2) : $\therefore$ ABCD is a trapezoid

17

1

1 c 2 d 3 a 4 b 5 c 6 c

2

[a] $\because$ The slope of $\overleftrightarrow{AB} = m_1 = \frac{-4-0}{2-6} = 1$

, The slope of $\overleftrightarrow{BC} = m_2 = \frac{2+4}{-4-2} = -1$

, $\because m_1 \times m_2 = 1 \times -1 = -1$ $\qquad \overleftrightarrow{AB} \perp \overleftrightarrow{BC}$

Δ ABC is right-angled at B

[b] $\tan X \tan Y = \frac{YZ}{XZ} \times \frac{XZ}{YZ} = 1$

3

[a] $4X = \cos^2 30° \tan^2 30° \tan^2 45°$

$4X = \left(\frac{\sqrt{3}}{2}\right)^2 \times \left(\frac{1}{\sqrt{3}}\right)^2 \times (1)^2$

$4X = \frac{3}{4} \times \frac{1}{3} \times 1$ $\qquad 4X = \frac{1}{4}$

$\therefore X = \frac{1}{16}$

[b] $\because$ The slope of the given straight line = $\frac{1}{2}$

$\therefore$ The slope of the required straight line = $\frac{-1}{2}$

$\therefore$ The equation of the required straight line is :

$y = \frac{-1}{2} X + c$

, $\because (3 , -5)$ satisfies the equation.

$\therefore -5 = \frac{-1}{2} \times 3 + c$ $\qquad \therefore c = \frac{-7}{2}$

$\therefore$ The equation is : $y = -\frac{1}{2} X - \frac{7}{2}$

4

[a] $\because$ The diagonals of the parallelogram bisect each other

Let M be the point of intersection of the two diagonals.

$M = \left(\frac{-2-4}{2} , \frac{5+2}{2}\right) = \left(-3 , \frac{7}{2}\right)$

Let D (X , y)

$\therefore \left(-3 , \frac{7}{2}\right) = \left(\frac{3+X}{2} , \frac{3+y}{2}\right)$

$\therefore \frac{3+X}{2} = -3$ $\qquad \therefore 3 + X = -6$ $\qquad X = -9$

, $\frac{3+y}{2} = \frac{7}{2}$ $\qquad \therefore 3 + y = 7$ $\qquad \therefore y = 4$

$\therefore$ D $(-9 , 4)$

[b] $\because \sin^2 30° = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$ (1)

, $5\cos^2 60° - \tan^2 45° = 5 \times \left(\frac{1}{2}\right)^2 - (1)^2$

$= \frac{5}{4} - 1 = \frac{1}{4}$ (2)

From (1) , (2) $\therefore \sin^2 30° = 5\cos^2 60° - \tan^2 45°$

5

[a] $m_1 = \frac{k-1}{2-3} = 1 - k$, $m_2 = \tan 45° = 1$

, $\because L_1 \perp L_2$ $\therefore m_1 \times m_2 = -1$

$\therefore (1-k) \times 1 = -1$ $\therefore 1 - k = -1$

$\therefore k = 2$

[b] $\because$ The straight line passes through (2 , 0) , (0 , 3)

$\therefore$ The slope of the straight line $= \frac{3-0}{0-2} = \frac{-3}{2}$

and intersects from the positive part of y-axis 3 units

$\therefore$ The equation is : $y = \frac{-3}{2}x + 3$

10 Assiut

1

1 b 2 a 3 b 4 c 5 b 6 c

2

[a] $\because$ The slope of $\overleftrightarrow{AB} = m_1 = \frac{5+1}{6+3} = \frac{2}{3}$

, the slope of $\overleftrightarrow{BC} = m_2 = \frac{3-5}{3-6} = \frac{2}{3}$

$\therefore m_1 = m_2$ $\therefore \overleftrightarrow{AB} \parallel \overleftrightarrow{BC}$

, $\because$ B is a common point

$\therefore$ A , B and C are collinear

[b] $\because x \sin 30° \cos^2 45° = \sin^2 60°$

$\therefore x \times \frac{1}{2} \times \left(\frac{1}{\sqrt{2}}\right)^2 = \left(\frac{\sqrt{3}}{2}\right)^2$

$\therefore \frac{1}{4}x = \frac{3}{4}$ $\therefore x = 3$

3

[a] $\because \Delta XYZ$ is right-angled at Y

$\therefore \overline{XY} \perp \overline{YZ}$, the slope of $\overleftrightarrow{XY} = \frac{5-2}{3-4} = -3$

$\therefore$ The slope of $\overleftrightarrow{YZ} = \frac{1}{3}$

, $\because$ the slope of $\overleftrightarrow{YZ} = \frac{a-2}{-5-4} = \frac{a-2}{-9} = \frac{1}{3}$

$\therefore a - 2 = -3$ $\therefore a = -1$

[b] $\because$ The slope of the straight line = 2 and it intersects from the positive part of y-axis 7 units.

Its equation is : $y = 2x + 7$

4

[a] 1 In ΔABC : $m(\angle B) = 90°$

$\therefore \sin(\angle ACB) = \frac{15}{25}$

$\therefore m(\angle ACB) \simeq 36° \, 52' \, 12''$

2 $\because (BC)^2 = (25)^2 - (15)^2 = 400$

$\therefore BC = 20$ cm

The area of the rectangle ABCD = 15×20

$= 300 \text{ cm}^2$

[b] $m_1 = \frac{0-3}{0-2} = \frac{3}{2}$, $m_2 = \frac{7-4}{1+1} = \frac{3}{2}$

$\therefore m_1 = m_2$

$\therefore$ The two straight lines are parallel

5

[a] $\because AB = \sqrt{(6-5)^2 + (-2-3)^2} = \sqrt{1+25}$

$= \sqrt{26}$ length units.

, $BC = \sqrt{(1-6)^2 + (-1+2)^2} = \sqrt{25+1}$

$= \sqrt{26}$ length units.

, $CD = \sqrt{(0-1)^2 + (4+1)^2} = \sqrt{1+25}$

$= \sqrt{26}$ length units.

, $AD = \sqrt{(0-5)^2 + (4-3)^2} = \sqrt{25+1}$

$= \sqrt{26}$ length units.

$\therefore AB = BC = CD = AD$

$\therefore$ ABCD is a rhombus

[b] $\because 2x - 3y - 6 = 0$ $\therefore 3y = 2x - 6$

$\therefore y = \frac{2}{3}x - 2$

The slope $= \frac{2}{3}$ and the intercepted part = 2 units from the negative part of y-axis

10 Sohag

1

1 b 2 c 3 a 4 a 5 a 6 b

2

[a] Let B (x , y)

$\therefore (2 , 3) = \left(\frac{x-1}{2} , \frac{y+3}{2}\right)$

$\therefore \frac{x-1}{2} = 2$ $\therefore x - 1 = 4$ $\therefore x = 5$

, $\frac{y+3}{2} = 3$ $\therefore y + 3 = 6$ $\therefore y = 3$

$\therefore B(5 , 3)$

[b] [1] ∵ $\cos X = \sin 30° \cos 60°$

∴ $\cos X = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

∴ $X \simeq 75° \ 31' \ 21''$

[2] $\tan 75° \ 31' \ 21'' \simeq 3.873$

3

[a] $m_1 = -\frac{a}{2}$, $m_2 = \tan 45° = 1$

∵ the two straight lines are parallel

∴ $m_1 = m_2$ ∴ $-\frac{a}{2} = 1$

∴ $a = -2$

[b] ∵ $\tan^2 60° - \tan^2 45° = \left(\sqrt{3}\right)^2 - (1)^2$

$= 3 - 1 = 2$ (1)

, $4 \sin 30° = 4 \times \frac{1}{2} = 2$ (2)

From (1) , (2) ∴ $\tan^2 60° - \tan^2 45° = 4 \sin 30°$

4

[a] [1] In ΔABC ∵ $m(\angle B) = 90°$

∴ $\sin(\angle ACB) = \frac{6}{10}$

∴ $m(\angle ACB) \simeq 36° \ 52' \ 12''$

[2] ∵ $(BC)^2 = (10)^2 - (6)^2 = 64$

∴ $BC = 8$ cm

∴ The area of the rectangle ABCD $= 6 \times 8$

$= 48 \text{ cm}^2$

[b] ∵ The slope of the given straight line $= \frac{-1}{3}$

∴ The slope of the required straight line $= 3$

The equation of the required straight line is :

$y = 3X + c$

∵ (1 , 2) satisfies the equation.

∴ $2 = 3 \times 1 + c$ ∴ $c = -1$

∴ The equation is : $y = 3X - 1$

5

[a] ∵ $MA = \sqrt{(3+1)^2 + (-1-2)^2} = \sqrt{16+9}$

$= 5$ length units

, $MB = \sqrt{(-4+1)^2 + (6-2)^2} = \sqrt{9+16}$

$= 5$ length units.

, $MC = \sqrt{(2+1)^2 + (-2-2)^2} = \sqrt{9+16}$

$= 5$ length units.

∵ $MA = MB = MC$

∴ A , B and C belong to the circle M

, its area $= \pi \times (5)^2 = 25\pi$ square units

[b] ∵ $4X + 5y - 10 = 0$ ∴ $5y = -4X + 10$

∴ $y = -\frac{4}{5}X + 2$

∴ The slope $= -\frac{4}{5}$ and the intercepted part = 2 units

from the positive part of y-axis 2

20 Quiz

1

1 c 2 d 3 b 4 a 5 a 6 c

2

[a] $\cos 60° \sin 30° - \sin 60° \cos 30°$

$= \frac{1}{2} \times \frac{1}{2} - \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{1}{4} - \frac{3}{4} = -\frac{1}{2}$

[b] The slope of the straight line $= \tan 135° = -1$

and it intercepts from the positive part of y-axis 5 units

∴ Its equation is : $y = -X + 5$

3

[a] ∵ $AB = \sqrt{(-1-1)^2 + (-2-4)^2} = \sqrt{4+36}$

$= 2\sqrt{10}$ length units

, $BC = \sqrt{(2+1)^2 + (-3+2)^2} = \sqrt{9+1}$

$= \sqrt{10}$ length units.

, $AC = \sqrt{(2-1)^2 + (-3-4)^2} = \sqrt{1+49}$

$= 5\sqrt{2}$ length units.

∵ $(AC)^2 = (AB)^2 + (BC)^2$

∴ ΔABC is right-angled triangle at B

, its area $= \frac{1}{2} \times AB \times BC = \frac{1}{2} \times 2\sqrt{10} \times \sqrt{10}$

$= 10$ square units

[b] In ΔABC : ∵ $m(\angle C) = 90°$

∴ $\sin B = \frac{AC}{AB}$ ∴ $\sin 60° = \frac{AC}{6}$

∴ $AC = 6 \sin 60° = 6 \times \frac{\sqrt{3}}{2} = 3\sqrt{3}$ cm

4

[a] The slope $= \frac{-2}{-6} = \frac{1}{3}$

Put $X = 0$ ∴ $2 \times 0 - 6y = 12$

∴ $-6y = 12$ ∴ $y = -2$

∴ The intersection point with y-axis is : $(0 , -2)$

Put $y = 0$

∴ $2X - 6 \times 0 = 12$ ∴ $2X = 12$ ∴ $X = 6$

∴ The intersection point with X-axis is $(6 , 0)$

[b] $\tan X = 4 \cos 60° \sin 30°$

$\tan X = 4 \times \frac{1}{2} \times \frac{1}{2}$ $\therefore \tan X = 1$

$\therefore X = 45°$

5

[a] $\because m_1 = \frac{4-3}{2-1} = 1$, $m_2 = \frac{1}{1} = 1$

$\therefore m_1 = m_2$

$\therefore$ The two straight lines are parallel

[b] $\because$ The midpoint of $\overline{AC} = \left(\frac{7+1}{2}, \frac{0+8}{2}\right) = (4, 4)$

, the midpoint of $\overline{BD} = \left(\frac{-1+9}{2}, \frac{4+4}{2}\right) = (4, 4)$

$\therefore$ The midpoint of $\overline{AC}$ = the midpoint of $\overline{BD}$

$\therefore$ The two diagonals bisect each other

$\therefore$ ABCD is a parallelogram.

, $\because$ the slope of $\overleftrightarrow{AB} = \frac{4-0}{-1-1} = -2$

, the slope of $\overleftrightarrow{BC} = \frac{8-4}{7+1} = \frac{1}{2}$

$\because$ slope of $\overleftrightarrow{AB}$ × the slope of $\overleftrightarrow{BC} = -2 \times \frac{1}{2} = -1$

$\therefore \overleftrightarrow{AB} \perp \overleftrightarrow{BC}$ $\therefore$ ABCD is a rectangle

21 Answers

1

[1] c [2] d [3] c [4] a [5] b [6] d

2

[a] $\because \tan 2X = 4 \sin 30° \cos 30°$

$\therefore \tan 2X = 4 \times \frac{1}{2} \times \frac{\sqrt{3}}{2}$ $\therefore \tan 2X = \sqrt{3}$

$\therefore 2X = 60°$ $\therefore X = 30°$

[b] $\because$ The slope of the given straight line $= \frac{-2}{-3} = \frac{2}{3}$

$\therefore$ The slope of the required straight line $= \frac{2}{3}$

$\therefore$ The equation of the required straight line is :

$y = \frac{2}{3} X + c$

, $\because$ (3 , 5) satisfies the equation

$\therefore 5 = \frac{2}{3} \times 3 + c$ $\therefore c = 3$

$\therefore$ The equation is : $y = \frac{2}{3} X + 3$

3

[a] $\because m_1 = \frac{-1+3}{5-7} = -1$, $m_2 = \tan 45° = 1$

, $m_1 \times m_2 = -1 \times 1 = -1$

$\therefore$ The two straight lines are perpendicular

[b] $\because 2 \sin 30° + 4 \cos 60° = 2 \times \frac{1}{2} + 4 \times \frac{1}{2}$

$= 1 + 2 = 3$ (1)

, $\tan^2 60° = \left(\sqrt{3}\right)^2 = 3$ (2)

From (1) , (2) : $\therefore 2 \sin 30° + 4 \cos 60° = \tan^2 60°$

4

[a] $\sqrt{(0-a)^2 + (1-0)^2} = \sqrt{2}$ (squaring both sides)

$\therefore a^2 + 1 = 2$ $\therefore a^2 = 1$

$\therefore a = 1$ or $a = -1$

[b] $M = \left(\frac{4-2}{2}, \frac{-1+7}{2}\right) = (1, 3)$

, $MA = \sqrt{(4-1)^2 + (-1-3)^2} = \sqrt{9+16}$

$= 5$ length units.

5

[a] $\because$ The slope of $\overleftrightarrow{AB} = m_1 = \frac{0+4}{1+1} = 2$

, the slope of $\overleftrightarrow{BC} = m_2 = \frac{2-0}{2-1} = 2$

$\because m_1 = m_2$ $\therefore \overleftrightarrow{AB} \parallel \overleftrightarrow{BC}$

, $\because$ B is a common point

$\therefore$ A , B and C are collinear

[b] In △ ADC : $\because m(\angle D) = 90°$

$\therefore (CD)^2 = (5)^2 - (4)^2 = 9$ $\therefore CD = 3$ cm

In △ CAB : $\because m(\angle CAB) = 90°$

$\therefore (AB)^2 = (13)^2 - (5)^2 = 144$ $\therefore AB = 12$ cm

$\therefore \tan(\angle DAC) \sin(\angle ACB)$

$- \sin(\angle B) \cos(\angle CAD)$

$= \frac{3}{4} \times \frac{12}{13} - \frac{5}{13} \times \frac{4}{5} = \frac{5}{13}$

22 Answers

1

[1] c [2] a [3] c [4] b [5] c [6] d

2

[a] $\because$ The slope of the straight line $= \frac{-3-3}{-1-1} = 3$

$\therefore$ The equation of the straight line is : $y = 3X + c$

, ∵ (1 , 3) satisfies the equation ,

∴ $3 = 3 \times 1 + c$ ∴ $c = 0$

∴ The equation is : $y = 3x$

[b] $MA = \sqrt{(3+1)^2 + (-1-2)^2} = \sqrt{16+9}$

$= 5$ length units.

, $MB = \sqrt{(-4+1)^2 + (6-2)^2} = \sqrt{9+16}$

$= 5$ length units

, $MC = \sqrt{(2+1)^2 + (-2-2)^2} = \sqrt{9+16}$

$= 5$ length units

, ∵ MA = MB = MC

∴ A , B and C lie on the circle M

, its circumference $= 2 \times \pi \times 5$

$= 10\pi$ length units

3

[a] ∵ $2 \sin E = \sin 30° \cos 60° + \cos 30° \sin 60°$

∴ $2 \sin E = \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

∴ $2 \sin E = 1$ ∴ $\sin E = \frac{1}{2}$ ∴ $E = 30°$

[b] ∵ $(4 , 6) = \left(\frac{x+6}{2} , \frac{3+y}{2}\right)$

∴ $\frac{x+6}{2} = 4$ ∴ $x + 6 = 8$ ∴ $x = 2$

, $\frac{3+y}{2} = 6$ ∴ $3 + y = 12$ ∴ $y = 9$

4

[a] In Δ ABC m (∠ C) = 90°

∴ $(AB)^2 = (6)^2 + (8)^2 = 100$

∴ AB = 10 cm

1 $\cos A \cos B - \sin A \sin B$

$= \frac{6}{10} \times \frac{8}{10} - \frac{8}{10} \times \frac{6}{10} = 0$

2 $\sin B = \frac{6}{10}$

∴ m (∠ B) ≃ 36° 52′ 12″

[b] $m_1 = \frac{k-1}{2-3} = 1 - k$, $m_2 = \tan 45° = 1$

1 ∵ $L_1 /\!/ L_2$ ∴ $m_1 = m_2$

$1 - k = 1$ ∴ $k = 0$

2 ∵ $L_1 \perp L_2$ ∴ $m_1 \times m_2 = -1$

$(1 - k) \times 1 = -1$ ∴ $1 - k = -1$

∴ $k = 2$

5

[a] ∵ The slope of the given straight line $= \frac{-1}{2}$

∴ The slope of the required straight line $= \frac{-1}{2}$

∴ The equation of the required straight line is

$y = \frac{-1}{2} x + c$

, ∵ (3 , − 5) satisfies the equation

∴ $-5 = \frac{-1}{2} \times 3 + c$ ∴ $c = \frac{-7}{2}$

∴ The equation is : $y = \frac{-1}{2} x - \frac{7}{2}$

[b] ∵ $x \sin 30° \cos^2 45° = \sin^2 60°$

∴ $x \times \frac{1}{2} \times \left(\frac{1}{\sqrt{2}}\right)^2 = \left(\frac{\sqrt{3}}{2}\right)^2$

∴ $\frac{1}{4} x = \frac{3}{4}$ ∴ $x = 3$

23 North Sinai

1

1 b 2 b 3 a 4 c 5 c 6 a

2

[a] ∵ $\sin 60° = \frac{\sqrt{3}}{2}$ (1)

, $2 \sin 30° \cos 30° = 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$ (2)

From (1) , (2)

∴ $\sin 60° = 2 \sin 30° \cos 30°$

[b] ∵ $AB = \sqrt{(-3-2)^2 + (0-4)^2} = \sqrt{25+16}$

$= \sqrt{41}$ length units.

, $BC = \sqrt{(-7+3)^2 + (5-0)^2} = \sqrt{16+25}$

$= \sqrt{41}$ length units

, $CD = \sqrt{(-2+7)^2 + (9-5)^2} = \sqrt{25+16}$

$= \sqrt{41}$ length units.

, $AD = \sqrt{(-2-2)^2 + (9-4)^2} = \sqrt{16+25}$

$= \sqrt{41}$ length units.

, ∵ AB = BC = CD = AD

∴ ABCD is a rhombus

, ∵ $AC = \sqrt{(-7-2)^2 + (5-4)^2} = \sqrt{81+1}$

$= \sqrt{82}$ length units.

$, BD = \sqrt{(-2+3)^2 + (9-0)^2} = \sqrt{1+81}$

$= \sqrt{82}$ length units

$\therefore AC = BD$

$\therefore ABCD$ is a square

3

[a] The slope of the straight line = 3

$\therefore$ The equation of the straight line is $y = 3X + c$

$, \because (5, 0)$ satisfies the equation.

$\therefore 0 = 3 \times 5 + c \qquad \therefore c = -15$

$\therefore$ The equation is : $y = 3X - 15$

[b] In ΔXYZ : $\because m(\angle Z) = 90°$

$\therefore (YZ)^2 = (25)^2 - (7)^2 = 576$

$\therefore YZ = 24$ cm.

$\boxed{1}$ $\tan X \tan Y = \frac{24}{7} \times \frac{7}{24} = 1$

$\boxed{2}$ $\sin^2 X + \sin^2 Y = \left(\frac{24}{25}\right)^2 + \left(\frac{7}{25}\right)^2 = 1$

4

[a] $\because 2 \sin X = \tan^2 60° - 2 \tan 45°$

$\therefore 2 \sin X = \left(\sqrt{3}\right)^2 - 2 \times 1$

$\therefore 2 \sin X = 3 - 2 \qquad \therefore 2 \sin X = 1$

$\sin X = \frac{1}{2} \qquad \therefore X = 30°$

[b] $\because$ The slope of $\overline{AB} = m_1 = \frac{0+4}{1+1} = 2$

$,$ the slope of $\overline{BC} = m_2 = \frac{2-0}{2-1} = 2$

$\therefore m_1 = m_2 \qquad \therefore \overleftrightarrow{AB} \parallel \overleftrightarrow{BC}$

$,$ B is a common point

$\therefore A, B$ and C are collinear

5

[a] $m_1 = \frac{5+2}{4+3} = 1 \quad , \quad m_2 = \tan 45° = 1$

$m_1 = m_2$

$\therefore$ The two straight lines are parallel

[b] $\because m_1 = \frac{k-3}{1+2} = \frac{k-3}{3} \quad , \quad m_2 = -3$

$, \because$ The two straight lines are perpendicular

$\therefore m_1 \times m_2 = -1 \qquad \therefore \frac{k-3}{3} \times -3 = -1$

$\therefore 3 - k = -1 \qquad \therefore k = 4$

24 Red Sea

1

$\boxed{1}$ c $\boxed{2}$ d $\boxed{3}$ c $\boxed{4}$ b $\boxed{5}$ d $\boxed{6}$ a

2

[a] $\cos 60° \sin 30° - \sin 60° \tan 60° + \cos^2 30°$

$= \frac{1}{2} \times \frac{1}{2} - \frac{\sqrt{3}}{2} \times \sqrt{3} + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} - \frac{3}{2} + \frac{3}{4} = -\frac{1}{2}$

[b] $\because m_1 = \frac{5+2}{4+3} = 1 \quad , \quad m_2 = \tan 45° = 1$

$\therefore m_1 = m_2$

The two straight lines are parallel

3

[a] $\because 3X + 4y - 5 = 0 \qquad \therefore 4y = -3X + 5$

$\therefore y = \frac{-3}{4} X + \frac{5}{4} \qquad \therefore$ The slope $= \frac{-3}{4}$

and the intercepted part $= \frac{5}{4}$ from the positive part of y-axis

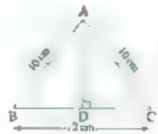
[b] $\because X \sin 30° \cos^2 45° = \sin^2 60°$

$X \times \frac{1}{2} \times \left(\frac{1}{\sqrt{2}}\right)^2 = \left(\frac{\sqrt{3}}{2}\right)^2$

$\therefore \frac{1}{4} X = \frac{3}{4} \qquad X = 3$

4

[a] Draw $\overline{AD} \perp \overline{BC}$



$AB = AC , \overline{AD} \perp \overline{BC}$

$BD = CD = 6$ cm

In ΔABD

$\because m(\angle ADB) = 90°$

$(AD)^2 = (10)^2 - (6)^2 = 64 \qquad \therefore AD = 8$ cm

$\boxed{1}$ $\cos B = \frac{6}{10} \qquad \therefore m(\angle B) = 53° \, 7' \, 48''$

$\boxed{2}$ $\sin^2 B + \cos^2 B = \left(\frac{8}{10}\right)^2 + \left(\frac{6}{10}\right)^2 = 1$

[b] $\because AB = \sqrt{(-1-1)^2 + (-2-4)^2} = \sqrt{4+36}$

$= 2\sqrt{10}$ length units

$, BC = \sqrt{(2+1)^2 + (-3+2)^2} = \sqrt{9+1}$

$= \sqrt{10}$ length units.

$, AC = \sqrt{(2-1)^2 + (-3-4)^2} = \sqrt{1+49}$

$= 5\sqrt{2}$ length units

$, \because (AC)^2 = (AB)^2 + (BC)^2$

$\therefore \Delta ABC$ is right-angled at B

$,$ its area $= \frac{1}{2} \times AB \times BC = \frac{1}{2} \times 2\sqrt{10} \times \sqrt{10}$

$= 10$ square units

5

[a] Let D be the midpoint of $\overline{BC} = \left(\frac{3+1}{2}, \frac{7-3}{2}\right)$

$= (2, 2)$

$\therefore$ The slope of $\overleftrightarrow{AD} = \frac{2-6}{2-4} = 2$

$\therefore$ The equation of $\overleftrightarrow{AD}$ is : $y = 2X + c$

$, \because A \in \overleftrightarrow{AD}$

$\therefore (4, 6)$ satisfies the equation.

$\therefore 6 = 2 \times 4 + c \qquad \therefore c = -2$

The equation of $\overleftrightarrow{AD}$ is : $y = 2X - 2$

[b] 1 $\because$ The diagonals of the parallelogram bisect each other

$\therefore M = \left(\frac{3+5}{2}, \frac{3-1}{2}\right) = (4, 1)$

2 Let D (X, y)

$\therefore (4, 1) = \left(\frac{2+X}{2}, \frac{-2+y}{2}\right)$

$\therefore \frac{2+X}{2} = 4 \qquad \therefore 2 + X = 8 \qquad \therefore X = 6$

$, \frac{-2+y}{2} = 1 \qquad \therefore -2 + y = 2 \qquad \therefore y = 4$

$\therefore D (6, 4)$

26 Matrouh

1

1 c 2 c 3 b 4 a 5 b 6 c

2

[a] $\because 4X = \cos^2 30° \tan^2 30° \tan^2 45°$

$\therefore 4X = \left(\frac{\sqrt{3}}{2}\right)^2 \times \left(\frac{1}{\sqrt{3}}\right)^2 \times (1)^2$

$\therefore 4X = \frac{3}{4} \times \frac{1}{3} \times 1 \qquad \therefore 4X = \frac{1}{4}$

$\therefore X = \frac{1}{16}$

[b] 1 Let A (X, y)

$\therefore (5, 7) = \left(\frac{X+8}{2}, \frac{y+11}{2}\right)$

$\therefore \frac{X+8}{2} = 5 \qquad \therefore X + 8 = 10 \qquad \therefore X = 2$

$, \frac{y+11}{2} = 7 \qquad \therefore y + 11 = 14 \qquad \therefore y = 3$

$\therefore A (2, 3)$

2 $MB = \sqrt{(8-5)^2 + (11-7)^2} = \sqrt{9+16}$

$= 5$ length units.

3

[a] $\because$ The slope of $\overleftrightarrow{AB} = m_1 = \frac{3-5}{3+2} = \frac{-2}{5}$

$,$ the slope of $\overleftrightarrow{BC} = m_2 = \frac{2-3}{-4-3} = \frac{1}{7}$

$, \because m_1 \neq m_2 \quad \therefore A, B$ and C are not collinear

$, \because$ The slope of $\overleftrightarrow{CD} = m_3 = \frac{4-2}{-9+4} = \frac{-2}{5}$

$,$ the slope of $\overleftrightarrow{AD} = m_4 = \frac{4-5}{-9+2} = \frac{1}{7}$

$, \because m_1 = m_3 \qquad \therefore \overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ (1)

$, \because m_2 = m_4 \qquad \therefore \overleftrightarrow{BC} \parallel \overleftrightarrow{AD}$ (2)

From (1) , (2) : $\therefore$ ABCD is a parallelogram

[b] $\frac{\cos^2 60° + \cos^2 30° - \tan^2 45°}{\sin 60° \tan 60° - \sin 30°}$

$= \frac{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - (1)^2}{\frac{\sqrt{3}}{2} \times \sqrt{3} - \frac{1}{2}} = \frac{\frac{1}{4} + \frac{3}{4} - 1}{\frac{3}{2} - \frac{1}{2}} = 0$

4

[a] $\because$ The slope of the given straight line $= \frac{-5}{-2} = \frac{5}{2}$

$\therefore$ The slope of the required straight line $= \frac{-2}{5}$

$\therefore$ The equation of the required straight line is

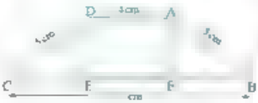
$y = \frac{-2}{5} X + c$

$, \because (3, 4)$ satisfies the equation

$\therefore 4 = \frac{-2}{5} \times 3 + c \qquad \therefore c = \frac{26}{5}$

$\therefore$ The equation is $y = \frac{-2}{5} X + \frac{26}{5}$

[b] Draw $AF \perp \overline{BC}$



, $DE \perp BC$

, $\because \overline{AD} \parallel \overline{BC}$

, $\overline{AF} \perp \overline{BC}$

, $\overline{DE} \perp \overline{BC}$

AFED is a rectangle

$FE = AD = 4$ cm

$BF + CE = 8$ cm

, $BF = CE = 4$ cm. ($\Delta AFB \equiv \Delta DEC$)

In ΔAFB $\quad m(\angle AFB) = 90°$

$(AF)^2 = (5)^2 \quad (4)^2 = 9 \qquad \therefore AF = 3$ cm

$DE = AF = 3$ cm. (ABCD is a rectangle)

$$\frac{5 \tan B \cos C}{\sin^2 C + \cos^2 C} = \frac{5 \times \frac{3}{4} \times \frac{4}{5}}{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2} = 3$$

6

[a] $m_1 \quad \frac{k}{2} \quad \frac{1}{3} = 1 \quad k$, $m_2 = \tan 45° \quad 1$

1 $\quad L \quad L_2 \qquad m \quad m_2$

$1 \quad k = 1 \qquad k \quad 0$

2 $\quad L \perp L_2 \qquad m_1 \times m_2 =$

$(1 \quad k) \times 1 \quad 1 \qquad 1 \quad k = -1 \qquad k = 2$

[b] $2x \quad 3y + 6 \qquad 3y = 2x - 6$

$y = \frac{2}{3}x \quad 2 \qquad$ The slope $= \frac{2}{3}$

and the intercepted part = 2 units from the negative part of y-axis

Answers of multidisciplinary exams

1 Cairo

1 c	2 b	3 c	4 d
5 a	6 b	7 a	8 d

2 Giza

1 d	2 b	3 c	4 b
5 c	6 d	7 c	8 b

3 Alexandria

1 c	2 a	3 c	4 d
5 a	6 b	7 a	8 b

4 El-Kalyoubia

1 c	2 a	3 c	4 b
5 a	6 b	7 b	8 d

5 El-Sharkia

1 c	2 b	3 d	4 a
5 b	6 d	7 a	8 c

6 El-Gharbia

1 d	2 c	3 d	4 c
5 c	6 b	7 a	8 c

7 Ismailia

1 b	2 c	3 d	4 a
5 c	6 b	7 a	8 c

8 Kafr El-Sheikh

1 b	2 d	3 d	4 d
5 b	6 a	7 c	8 a

9 Assiut

1 b	2 a	3 c	4 b
5 c	6 a	7 d	8 b

10 Qena

1 d	2 b	3 c	4 d
5 a	6 b	7 d	8 c

2020

Governorates' Examinations on Algebra and Statistics

1 Cairo Governorate

Answer the following questions : *(Calculator is allowed)*

1 Choose the correct answer from those given :

1 The simplest dispersion measure is ....

(a) the arithmetic mean. (b) the median.

(c) the range. (d) the mode.

2 $2x^2 \times 3x =$ ....

(a) $6x^3$ (b) $5x^3$ (c) $6x^2$ (d) $5x^2$

3 If $X = \{3\}$, $n(Y) = 5$, then $n(X \times Y) =$ ....

(a) 1 (b) 5 (c) 8 (d) 15

4 The simplest form of the expression : $3x - 4y + 5x + 7y$ is ....

(a) $7x + 12y$ (b) $11xy$ (c) $10x + 9y$ (d) $8x + 3y$

5 The relation which represents an inverse variation between the two variables y and x is ....

(a) $xy = 5$ (b) $y = x + 3$ (c) $\frac{x}{5} = \frac{y}{2}$ (d) $y = 2x$

6 If $\sqrt{x} = 4$, then $x =$ .... where $x \in \mathbb{Z}^+$

(a) 2 (b) 4 (c) 8 (d) 16

2 **[a]** Graph the curve of the function $f : f(x) = x^2$ where $x \in [-3, 3]$

, from the graph find :

1 The maximum or the minimum value of the function.

2 The equation of the axis of symmetry.

[b] Find the standard deviation to the set of the values : 15 , 19 , 20 , 21 , 25

3 **[a]** If $X = \{3, 4\}$, $Y = \{4, 5\}$, $Z = \{5, 6\}$, **find :**

1 $X \times Y$ 2 $(X - Y) \times Z$

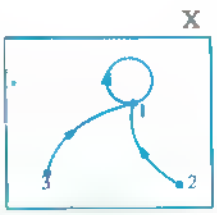
[b] If x , y , z and ℓ are proportional quantities , **prove that :** $\frac{y - x}{x} = \frac{\ell - z}{z}$

4 **[a]** Find the number which if added to both of terms of the ratio 3 : 5 , then it becomes 1 : 2

[b] In the opposite figure :

The arrow diagram represents

the relation R on the set X



1 Write R 2 Is R a function ? If it's , find its range.

5 **[a]** If $y \propto x$ and $y = 20$ as $x = 4$, **find :**

1 The constant of variation between y and x

2 The value of x when $y = 40$

[b] If $f(x) = 2x + k$, $f(5) = 13$, **find the value of :** k

2 Giza Governorate

Answer the following questions :

1 **Choose the correct answer :**

1 Double the number 2^8 is ....

(a) 2^{10} (b) 2^{16} (c) 4^8 (d) 2^9

2 If $x y = 3$, then $y \propto$ ....

(a) x (b) $3x$ (c) $\frac{1}{x}$ (d) $\frac{1}{3}x$

3 If $x^2 + y^2 = 25$, $(x + y)^2 = 49$, then $x y =$ ....

(a) 6 (b) 10 (c) 12 (d) 24

4 If $f(x) = 3$, then $f(3) + f(-3) =$ ....

(a) 0 (b) 1 (c) -6 (d) 6

5 $]-2 , 5[\cup \{-2 , 5\} =$ ....

(a) $[-2 , 5]$ (b) $[-2 , 5[$ (c) $]-2 , 5]$ (d) $]-2 , 5[$

6 The range of the set of the values : 5 , 14 , 4 , 23 , 15 is ....

(a) 12 (b) 14 (c) 19 (d) 23

2 **[a]** If $X = \{2 , 5\}$, $Y = \{1 , 2\}$, $Z = \{3\}$, **then find :**

1 $n(X \times Z)$ 2 $(Y \cap X) \times Z$

[b] If $f(x) = 4x + b$, $f(2) = 10$, **then find the value of :** b

3 [a] If $X = \{2, 3, 5\}$, $Y = \{4, 6, 8, 10\}$ and R is a relation from X to Y where "a R b" means "$a = \frac{b}{2}$" for each $a \in X$, $b \in Y$, write R and represent it by an arrow diagram. Is R a function ? and why ?

[b] Find the number which if added to the two terms of the ratio 7 : 11 , it will be 2 : 3

4 [a] If $2a = 3b = 3c$, **then find the numerical value of :** $\frac{6a + b + c}{4a + 6b + 6c}$

[b] **Calculate the standard deviation for the following values :** 55 , 53 , 57 , 56 , 54

5 [a] If $y \propto X$ and $y = 6$ when $X = 3$, **find :**

1 The relation between X , y

2 The value of y when $X = 4$

[b] Represent graphically the curve of the function $f : f(X) = 4 - X^2$ where $X \in [-3, 3]$ and from the graph deduce the vertex of the curve and the equation of the symmetry axis.

3 Alexandria Governorate

Answer the following questions : *(Calculators are allowed)*

1 **Choose the correct answer from those given :**

1 If $n(X) = 5$, $n(X \times Y) = 10$, then $n(Y) =$ ....

(a) 4 (b) 3 (c) 2 (d) 1

2 If $X = \frac{1}{\sqrt{3} + \sqrt{2}}$, $y = \sqrt{3} + \sqrt{2}$, then $(X + y)^2 =$ ....

(a) 12 (b) $2\sqrt{3}$ (c) $2\sqrt{2}$ (d) zero

3 The arithmetic mean of the set of values : 8 , 9 , 7 , 6 and 5 equals ....

(a) 25 (b) 7 (c) 35 (d) 5

4 For any set Y , then $\varnothing$ .... Y

(a) $\in$ (b) $\notin$ (c) $\subset$ (d) $\not\subset$

5 The relation representing the direct variation between the two variables X and y is ....

(a) $X y = 5$ (b) $y = X + 3$ (c) $\frac{X}{3} = \frac{4}{y}$ (d) $\frac{X}{5} = \frac{y}{2}$

6 $2^{100} = 2^{99} +$ ....

(a) 2 (b) 1 (c) 2^{99} (d) 99

2 [a] If $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = 3X$, mention the degree of f **, then find :** $f(-2)$, $f(\sqrt{3})$

[b] If $5a = 3b$, **then find the value of :** $\frac{7a + 9b}{4a + 2b}$

3 **[a]** If $X = \{-1 , 1 , 2\}$, $Y = \{2 , 4 , 6 , 8\}$ and R is a relation from X to Y where "a R b" means "$b = 2a + 4$" for all $a \in X$, $b \in Y$, write R and represent it by an arrow diagram. Is R a function ? Why ?

[b] If $x^4 y^2 - 14 x^2 y + 49 = 0$, **prove that :** $y \propto \frac{1}{x^2}$

4 **[a]** If $(x - 2 , 3) = (5 , y + 1)$, **find the value of each of :** x , y

[b] **The following frequency distribution shows the number of children of some families in a new city :**

Number of children	0	1	2	3	4
Number of families	8	16	50	20	6

Calculate the mean and standard deviation to the number of children.

5 **[a]** If a , b , c and d are in continued proportion , **then prove that :** $\frac{a}{b + d} = \frac{c^3}{c^2 d + d^3}$

[b] Represent graphically the function f where $f(x) = x^2 + 2x + 1$, taking $x \in [-4 , 2]$ **and from the drawing deduce :**

1. The coordinates of the vertex of the curve.
2. The equation of the symmetry axis.
3. The minimum or the maximum value of the function.

4 El-Kalyoubia Governorate

Answer the following questions :

1 **Choose the correct answer :**

1. $\sqrt[3]{x^6} = \sqrt{\ldots\ldots}$

   (a) x^3 (b) x^2 (c) x (d) x^4

2. If $(x + 5 , 8) = (1 , 6y + x)$, then $y = \ldots\ldots$

   (a) 5 (b) 6 (c) 2 (d) 12

3. The solution set of the equation : $x^2 + 4 = 0$ in $\mathbb{R}$ is $\ldots\ldots$

   (a) $\{4\}$ (b) $\{-2 , 2\}$ (c) $\{-2\}$ (d) $\varnothing$

4. If $xy = 7$, then $y \propto \ldots\ldots$

   (a) $\frac{1}{x}$ (b) $x - 7$ (c) x (d) $x + 7$

5. If $x^2 - y^2 = 16$ and $x + y = 8$, then $x - y = \ldots\ldots$

   (a) 2 (b) 1 (c) 128 (d) 64

6. If $\sum (x - \bar{x})^2 = 36$ to the set of 9 values , then $\sigma = \ldots\ldots$

   (a) 2 (b) 4 (c) 18 (d) 27

2 [a] Represent graphically the function f where $f(X) = (X-2)^2$, $X \in [0, 4]$

From the graph , deduce :

1 The equation of the symmetry axis.

2 The maximum (minimum) value of the function.

[b] If $y \propto \frac{1}{X}$ and $X = 2\frac{4}{5}$ when $y = \frac{4}{7}$, find the value of y when $X = 3\frac{1}{5}$

3 [a] If $X = \{2, 3, 5\}$, $Y = \{4, 6, 8, 10\}$ and R is a relation from X to Y , where "a R b" means "2 a = b" for each $a \in X$, $b \in Y$

1 Write R and represent it by an arrow diagram.

2 Is R a function ?

[b] If a , b , c and d are proportional , **prove that :** $\sqrt[3]{\frac{5a^3 - 3c^3}{5b^3 - 3d^3}} = \frac{a+c}{b+d}$

4 [a] If $X = \{2, 4\}$, $Y = \{4, 0\}$, $Z = \{4, 5, -2\}$, **find :**

1 $(Z - Y) \times (X \cap Y)$ 2 $n(X^2)$

[b] If $f(X) = 4X + b$, $f(3) = 15$, **find the value of :** b

5 [a] If $\frac{a}{2X+y} = \frac{b}{3y-X} = \frac{c}{4X+5y}$, **prove that :** $\frac{a+2b}{7} = \frac{4b+c}{17}$

[b] **Find the standard deviation for this distribution :**

X	zero	1	2	3	4	5	Total
K	3	16	17	25	20	19	100

5 El-Sharkia Governorate

Answer the following questions : ***(Calculators are allowed)***

1 **Choose the correct answer from those given :**

1 If the arithmetic mean of the quantities $2X$, 3 , 4 , 5 equals 4 , then $X =$ ......

(a) 1 (b) 2 (c) 3 (d) 4

2 If $X \times Y = \{(1, 2), (3, 4)\}$, then $X \cap Y =$ ....

(a) $\{1, 2\}$ (b) $\{(3, 4)\}$ (c) $\varnothing$ (d) $\{1, 4\}$

3 If $y = mX$ where m is a constant $\neq$ zero , which of the following statements is false ?

(a) $y \propto X$ (b) $X \propto y$ (c) $X = \frac{1}{m}y$ (d) $X \propto \frac{1}{y}$

4 If a , b , c , d are proportional quantities , then $\frac{a\,d - b\,c}{a^2 + b^2 + c^2} = \ldots\ldots\ldots$

(a) zero (b) 1 (c) 2 (d) 3

5 $f : f(X) = (2\,a - 2)\,X^3 + 3\,X^2 + X + 2$ is a polynomial function from the second degree when a = ……

(a) zero (b) 2 (c) 3 (d) 1

6 If the point (a – 5 , 5 – a) lies in the fourth quadrant , then ……………

(a) $a \geq 5$ (b) $a \leq 5$ (c) $a > 5$ (d) $a < 5$

2 [a] If $X = \{1 , 2 , 3\}$, $Y = \{3 , 4\}$, **find :**

1 $X - Y$ 2 $(Y \cap X) \times Y$ 3 $n(Y^2)$

[b] If a , b , c and d are in continued proportional , **prove that :** $\frac{b + d}{c^2\,d + d^3} = \frac{a}{c^3}$

3 [a] If $X = \left\{\frac{1}{2} , 1 , \text{zero} , -\frac{1}{2} , -1\right\}$, $Y = \{1 , 2 , \text{zero} , -1 , -2\}$

and R is a relation from X to Y where " a R b " means "a is the multiplicative inverse of b" for each $a \in X$ and $b \in Y$

Write R and represent it by an arrow diagram. Is R a function ? and why ?

[b] If y varies inversely as X^2 where y = 9 at $X = \frac{2}{3}$

, **find :** 1 The relation between y and X

2 The value of y when $X = \frac{1}{2}$

4 [a] Represent graphically the quadratic function f where $f(X) = (X - 3)^2 + 1$ taking $X \in [0 , 6]$ **From the graph deduce :**

1 The coordinates of the vertex of the curve.

2 The minimum value of the function.

3 The equation of the axis of symmetry of the curve.

[b] If $\frac{X}{3} = \frac{y}{2} = \frac{z}{5}$, **find the value of :** $\frac{X\,y + y\,z}{X^2 + y^2}$

5 [a] **Calculate the standard deviation for the values :** 12 , 13 , 16 , 18 , 21

[b] If $f(X) = a\,X + b$ and $f(a) = b$

Find the value of the expression : $a\,b^2 + 5$

6 El-Monofia Governorate

Answer the following questions : (Using calculator is permitted)

1 Choose the correct answer from those given :

1. The number 3 belongs to the solution set of the inequality ....

(a) $x > 3$ (b) $x < 3$ (c) $-x \geq -3$ (d) $-x \geq 3$

2. $\left(\frac{-3}{4}\right)^{zero}$ .... $\left(\frac{-3}{4}\right)^2$

(a) $<$ (b) $>$ (c) $=$ (d) $\leq$

3. The number lying between 0.02 and 0.03 is ....

(a) 0.00025 (b) 0.0025 (c) 0.025 (d) 0.25

4. If $a < 5$, then the point $(2 , a - 5)$ lies in the .... quadrant.

(a) first (b) second (c) third (d) fourth

5. If $\frac{a}{3} = \frac{b}{5}$, then $5a - 3b + 4 =$ ....

(a) 3 (b) 4 (c) 5 (d) 6

6. If $\sum (x - \overline{x})^2 = 48$ of a set of values and the number of these values is 12 , then $\sigma =$ ....

(a) 2 (b) -2 (c) -4 (d) 4

2

[a] If $X = \{-1 , 1 , 2\}$, $Y = \{2 , 4 , 6 , 8\}$, R is a relation from X to Y where "a R b" means "$b = 2a + 4$" for each $a \in X$ and $b \in Y$

1. Write R and represent it by an arrow diagram.
2. Show that R is a function and write its range.

[b] If the straight line which represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = 6x - a$ cuts y-axis at the point $(b , 3)$, **find the value of :** $2a - 5b$

3

[a] If $X = \{1\}$, $Y = \{2 , 3\}$, $Z = \{3 , 4 , 5\}$, **find each of the following :**

1. $X \times Y$ 2. $X \times (Y - Z)$ 3. $n(Z^2)$

[b] If b is the middle proportional between a and c , **prove that :** $\frac{a^2 + b^2}{b^2} = \frac{b^2 + c^2}{c^2}$

4

[a] If $a : b : c = 2 : 3 : 5$ and $a + b + c = 35$, **then find the value of each of :** a , b and c

[b] If $y = a + 7$, $a \propto \frac{1}{x^2}$ and $a = 3$ when $x = 2$, **then find :**

1. The relation between x and y 2. The value of y when $x = \sqrt{3}$

5 [a] Draw the curve of the function $f : f(X) = X^2 - 4X$, taking $X \in [-1, 5]$

and from the graph find :

1. The coordinates of the vertex of the curve.
2. The equation of the line of symmetry.
3. The maximum or the minimum value of the function.

[b] **Find the standard deviation for the following set of values :** 20 , 27 , 5 , 16 , 32

7 El-Gharbia Governorate

Answer the following questions : *(Calculators are allowed)*

1 **Choose the correct answer :**

1. The following functions are polynomial functions except the function f where $f(X) = \dots\dots$

(a) $X + 3$ (b) $\sqrt{2}\,X + 1$ (c) $X\left(X + \frac{1}{X}\right)$ (d) $X^2(X + 4)$

2. The solution set of the equation : $(X - 5)^{zero} = 1$ in $\mathbb{R}$ is $\dots\dots$

(a) $\{5\}$ (b) $\{5, -5\}$ (c) $\mathbb{R}$ (d) $\mathbb{R} - \{5\}$

3. If $(a - 7, 26) = (-3, b^3 - 1)$, then $\sqrt{a^2 + b^2} = \dots\dots$

(a) 5 (b) -5 (c) ± 5 (d) ± 7

4. The second proportional to the numbers 2 , ... , 8 is $\dots\dots$

(a) 4 (b) 6 (c) ± 4 (d) ± 6

5. The range of the set of the values : 7 , 3 , 6 , 9 , 5 is $\dots\dots$

(a) 3 (b) 4 (c) 6 (d) 12

6. If $y \propto X$ and $y = 2$ when $X = 8$, then $y = 3$ when $X = \dots\dots$

(a) 16 (b) 12 (c) 24 (d) 6

2 [a] If $X = \{-2, -3, 2\}$, $Y = \left\{\frac{1}{8}, \frac{1}{27}, 8\right\}$ and R is a relation from X to Y where "a R b" means "$a^3 = b$" for all $a \in X$, $b \in Y$, write R , and represent it by an arrow diagram. Is R a function or not with a reason ?

[b] If $X^4 y^2 - 14X^2 y + 49 = 0$, **then prove that :** $y \propto \frac{1}{X^2}$

3 [a] If a , b , c and d are proportional quantities, **then prove that :** $\frac{a+b}{b} = \frac{c+d}{d}$

[b] Represent graphically the curve of the function $f : f(X) = 2 - X^2$, taking $X \in [-3, 3]$ and from the graph deduce the equation of the axis of symmetry, the maximum value or the minimum value of the function.

4 **[a]** If $X \times Y = \{(1, 1), (1, 3), (1, 5)\}$, find Y^2 and represent it by a Cartesian diagram.

[b] Find the positive number which if we add its square to each of the two terms of the ratio 5 : 11, it becomes 3 : 5

5 **[a]** If the straight line representing the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = 6X - \ell$ cuts the y-axis at the point (m, 3), **find the value of each of :** ℓ and m

[b] Calculate the arithmetic mean and the standard deviation for the following data :

23, 12, 17, 13, 15 rounding the result of the standard deviation to one decimal place.

8 El-Dakahlia Governorate

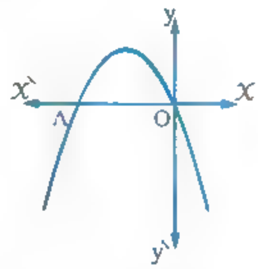
Answer the following questions : *(Calculator is permitted)*

1 **[a] Choose the correct answer from those given :**

1 If $5X = 9y$, then $\frac{3X}{2y} = \dots\dots$

(a) 27 : 10 (b) 9 : 5 (c) 5 : 9 (d) 81 : 25

2 **In the opposite figure :**



The curve of a quadratic function, A(– 4, 0), then the equation of the axis of symmetry is $X = \dots\dots$

(a) 1 (b) – 1

(c) – 2 (d) 0

3 The number added to each of the numbers 1, 3, 6 to be proportional is ……

(a) 4 (b) 3 (c) 1 (d) 2

[b] If b is the middle proportional between a and c, **prove that :** $\frac{a^2}{b^2} + \frac{b^2}{c^2} = \frac{2a}{c}$

2 **[a] Choose the correct answer from those given :**

1 If $f(X + 3) = X - 3$, then $f(7) = \dots\dots$

(a) 4 (b) 1 (c) 7 (d) 10

2 If $\Sigma(X - \overline{X})^2 = 36$ for 9 values, then the standard deviation $= \dots\dots$

(a) 2 (b) 18 (c) 27 (d) 4

3 If $f(X) = 3$, then $f(2) - f(7) = \dots\dots$

(a) 5 (b) – 5 (c) 0 (d) – 4

[b] If $X = \{4, 5, 7\}$, R is a function on X and $R = \{(a, 5), (b, 5), (4, 7)\}$, **find :**

1 The value of 3 a + 3 b

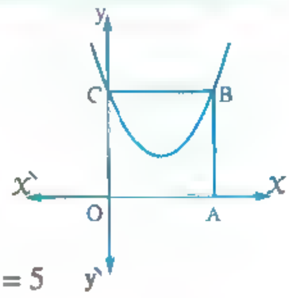
2 The range of the function.

3 [a] If $\frac{a}{4X+y}=\frac{b}{X-4y}$, **prove that :** $\frac{a+b}{5X-3y}=\frac{a-b}{3X+5y}$

[b] **Calculate the standard deviation of values :** 12 , 13 , 16 , 18 , 21

4 [a] The opposite figure represents the curve of the function f where $f(X)=X^2-(k-2)X-k+4$, the figure OABC is a square.

Find the value of : k



[b] If $y = 1 + b$, b varies inversely with the square of X , $X = 1$ at $y = 5$, find the relation between X and y , then find the value of y at $X = 2$

5 [a] If $f(X) = a + X^2$, $\ell(X) = c$ are two polynomial functions where $3f(2) + 3\ell(X) = 6$, **find the numerical value of :** $2f(0) + 2\ell(7)$ where a and c are constants.

[b] If $X = \{3, 5, 7\}$, $Y = \{X : X \in \mathbb{N}, 10 < X < 30\}$ and the function f from $X \longrightarrow Y$ is $f = \{(3, 9), (5, 15), (7, 21)\}$

1 Find the domain of f 2 Write the rule of the function.

9 Ismailia Governorate

Answer the following questions : *(Calculators are allowed)*

1 **Choose the correct answer from those given :**

1 The expectation of the match of Ismaili club in mathematics is called ………

(a) probability. (b) equations. (c) inequalities. (d) relations.

2 The third proportional of the quantities 2 , 3 and 6 is ………

(a) 1 (b) 4 (c) 9 (d) 12

3 The number $\frac{2X}{X-5}$ is a rational number if $X \neq$ ………

(a) zero (b) $\frac{1}{5}$ (c) $\frac{2}{5}$ (d) 5

4 If the point (b − 4 , 2 − b) lies in the third quadrant, then b = ………

(a) 2 (b) 3 (c) 4 (d) 6

5 If $17X + 8 = 11$, then $17X + 11 =$ ………

(a) 8 (b) 11 (c) 14 (d) 17

6 If a set of values are equal , then the dispersion of these values is ………

(a) > zero (b) < zero (c) = 1 (d) = zero

2 [a] If $X = \{2, 3\}$, $Y = \{3, 4, 5\}$, **find :**

1. $X \times Y$ 2. X^2 3. $n(Y^2)$

[b] If $3a = 4b$, **find the value of :** $\frac{2a + b}{5a - 3b}$

3 [a] If y varies inversely with the square of X and $y = 5$ when $X = 3$, find the value of y when $X = 2$

[b] If the straight line which represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = 3X - a$ cuts the y-axis at the point $(b, 5)$, **find the values of :** a and b

4 [a] If we add double the number X to each of the numbers 1 , 3 and 7 , it becomes proportional quantities. **Find the value of :** X

[b] If $X = \{-1, 1, 2\}$, $Y = \{2, 4, 6, 8\}$ and R is a relation from X to Y where "a R b" means "$b = 2a + 4$" for all $a \in X$, $b \in Y$

1. Find the relation R and represent it by an arrow diagram.
2. Is R a function ? and why ?

5 [a] Represent graphically the curve of the function $f : f(X) = 2 - X^2$ where $X \in [-3, 3]$, **then from the graph find :**

1. The coordinates of the vertex of the curve.
2. The equation of the axis of symmetry.
3. The maximum or minimum value of the function.

[b] **Calculate the standard deviation of the values :** 12 , 13 , 16 , 18 , and 21

10 Suez Governorate

Answer the following questions : ***(Calculators are allowed)***

1 **Choose the correct answer from those given :**

1. If 2 , 3 , 6 and X are proportional , then $X =$ ....

(a) 9 (b) 18 (c) 12 (d) 3

2. If $3ak = 12a^2$, then $k =$ ....

(a) $4a^2$ (b) $3a$ (c) $4a$ (d) $3a^2$

3. If $X = \{1, 2\}$, $Y = \{3, 4\}$, then $(3, 4) \in$ ....

(a) $X \times Y$ (b) $Y \times X$ (c) X^2 (d) Y^2

4 If (a , 5) = (6 , b) , then a + b = ...

(a) 5 (b) 11 (c) 6 (d) 1

5 $\frac{\text{Sum of the values}}{\text{Their number}}$ = ...

(a) the range. (b) the standard deviation.

(c) the arithmetic mean. (d) the mode.

6 If the point (2 , y) lies on the X-axis , then y + 4 = ...

(a) 5 (b) 4 (c) 2 (d) 3

2 [a] If 4 a = 3 b , **then find the value of :** $\frac{4a+b}{2a-b}$

[b] If X = {0 , 3 , 4} , Y = {1 , 2 , 3 , 4 , 5} and R is a relation from X to Y where "a R b" means "a + b = 5" for all a ∈ X , b ∈ Y

1 Find the relation R

2 Represent the relation R by an arrow diagram.

3 Is R a function ?

3 [a] If X × Y = {(2 , 6) , (2 , 9) , (3 , 6) , (3 , 9)} , **find :**

1 X , Y 2 Y × Y

[b] Draw the curve of the function $f : f(X) = 1 + X^2$ at the interval [– 3 , 3] **and from the graph find :**

1 The coordinates of the vertex of the curve.

2 The equation of the axis of symmetry.

3 The minimum value.

4 [a] If X , y , z , r are proportional quantities , **then prove that :** $\frac{X^2 + 2z^2}{y^2 + 2r^2} = \frac{Xz}{yr}$

[b] From the data of the following table , answer the following questions :

1 Show the kind of variation between y and X

2 Find the constant proportion.

3 Find the value of y when $X = 3$

X	2	4	6
y	6	3	2

5 [a] If $f(X) = X^2 - 3X$, $g(X) = X - 3$

1 **Find :** $f(2) + g(2)$ 2 **Prove that :** $f(3) + g(3) = 0$

[b] Calculate the standard deviation for the values : 12 , 13 , 16 , 18 , 21

11 Port Said Governorate

Answer the following questions :

1 Choose the correct answer from those given :

1 If $(3 , 5) \in \{3 , 6\} \times \{X , 8\}$, then $X =$ ....

(a) 8 (b) 6 (c) 5 (d) 3

2 The linear function given by the rule $y = 2X - 1$ is represented graphically by a straight line intersecting the y-axis at the point ....

(a) $\left(\frac{1}{2} , 0\right)$ (b) $(0 , -1)$ (c) $(-1 , 0)$ (d) $\left(0 , \frac{1}{2}\right)$

3 The difference between the greatest value and the smallest value in a set of individuals is called ....

(a) the standard deviation. (b) the arithmetic mean.

(c) the median. (d) the range.

4 If the point $(X - 4 , 2 - X)$ where $X \in \mathbb{Z}$ is located in the fourth quadrant , then $X =$ ....

(a) 2 (b) 3 (c) 4 (d) 6

5 Which of the following tables represents the direct variation between X and y ?

(a)

X	y
2	9
4	18

(b)

X	y
3	20
5	12

(c)

X	y
3	6
−2	−9

(d)

X	y
10	9
5	18

6 If $(X - 1 , 11) = (8 , y + 3)$, then $\sqrt{X + 2y} =$ ....

(a) 5 (b) ± 5 (c) $\sqrt{17}$ (d) 25

2 [a] If $X = \{1 , 2\}$, $Y = \{2 , 5\}$, $Z = \{4 , 5\}$, **then find :**

1 $n(X \times Z)$ 2 $(X - Y) \cap Z$

[b] Represent graphically $f : f(X) = X^2 + 2X + 1$, consider $X \in [-4 , 2]$

From the graph deduce :

1 The coordinates of the vertex of the curve.

2 The minimum or the maximum value of the function.

3 [a] If $f(X) = 4X + b$ and $f(3) = 15$, **find the value of :** b

[b] If $y \propto \frac{1}{X}$ and $y = 6$ when $X = 2.5$, **find :**

1 The relation between X , y 2 The value of y when $X = 5$

4 **[a]** If $X = \{1 , 2 , 3\}$, $Y = \{12 , 21 , 47 , 52\}$ and R is a relation from X to Y where "a R b" means "a is a digit from the digits of b" for each $a \in X$, $b \in Y$

1. Write R and represent it by an arrow diagram.
2. Which of the following relations is correct and why ? 1 R 52 , 2 R 21 , 3 R 47

[b] If 7 , x and $\frac{1}{y}$ are in continued proportion , **then find the value of :** $x^4 y^2$

5 **[a]** If $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$, **then prove that :** $\frac{2y - z}{3x - 2y + z} = \frac{1}{2}$

[b] Calculate the arithmetic mean and the standard deviation for the values :

3 , 6 , 7 , 9 , 15

12 Damietta Governorate

Answer the following questions : *(Calculators are allowed)*

1 **Choose the correct answer from the given ones :**

1. $\sqrt{36} = \cdots\cdots\cdots$
   (a) 6 (b) – 6 (c) ± 6 (d) 18
2. The point (– 2 , 5) lies in the …………… quadrant.
   (a) first (b) second (c) third (d) fourth
3. The commonest measure of dispersions and the most accurate is ……………
   (a) the median. (b) the arithmetic mean.
   (c) the mode. (d) the standard deviation.
4. $\mathbb{R} = \cdots\cdots\cdots$
   (a) $\mathbb{Q} \cap \mathbb{Q}'$ (b) $\mathbb{R}_+ \cap \mathbb{R}_-$ (c) $\mathbb{R}_+ \cup \mathbb{R}_-$ (d) $\mathbb{Q} \cup \mathbb{Q}'$
5. If $(x - 3 , 2^y) = (2 , 32)$, then $(x , y) = \cdots\cdots\cdots$
   (a) (5 , 2) (b) (2 , 5) (c) (5 , 5) (d) (2 , 2)
6. If $x y = 8$, then $y \propto \cdots\cdots\cdots$
   (a) $x - 8$ (b) $\frac{1}{x}$ (c) x (d) $x + 8$

2 **[a]** If $X = \{2 , 5\}$, $Y = \{1 , 2\}$, $Z = \{3\}$, **find :**

1. $n(X \times Y)$
2. $(X - Y) \times Z$
3. Y^2

[b] If b is the middle proportional between a and c , **prove that :** $\frac{a - b}{a - c} = \frac{b}{b + c}$

3 [a] If $X = \{1, 3, 4, 5\}$, $Y = \{1, 2, 3, 4, 5, 6\}$ and R is a relation from X to Y where "a R b" means "a + b = 7" for all $a \in X$, $b \in Y$

1. Write R
2. Show giving reasons that R is a function and find its range.

[b] If $\frac{21x - y}{7x - z} = \frac{y}{z}$, **then prove that :** $y \propto z$

4 [a] **Calculate the standard deviation for the values :** 12 , 13 , 16 , 18 , 21

[b] If $y \propto x$, $y = 6$ when $x = 3$, **find :**

1. The relation between x , y
2. The value of y when $x = 5$

5 [a] If $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$, **then prove that :** $\sqrt{3x^2 + 3y^2 + z^2} = 2x + y$

[b] Represent graphically the function $f : f(x) = x^2 + 3$, $x \in [-2, 2]$

From the graph deduce :

1. The equation of symmetry line.
2. The minimum value of the function.

13 Kafr El-Sheikh Governorate

Answer the following questions : ***(Calculator is allowed)***

1 [a] **Choose the correct answer from the given ones :**

1. The third proportional of the numbers 4 , 12 , ... , 48 is ......

(a) 7 (b) 32 (c) 16 (d) 36

2. $\varnothing$ .... $\{1, 2\}$

(a) $\in$ (b) $\notin$ (c) $\not\subset$ (d) $\subset$

3. The range of the set of the values : 7 , 3 , 6 , 9 and 5 equals ....

(a) 3 (b) 6 (c) 4 (d) 12

[b] Represent graphically the function $f : f(x) = (x - 2)^2$, where $x \in [-1, 5]$, then from the graph deduce the vertex of the curve , the equation of the symmetry axis and the minimum value of the function.

2 [a] **Choose the correct answer from the given ones :**

1. $\left(\sqrt{7} + \sqrt{5}\right)\left(\sqrt{7} - \sqrt{5}\right) =$ ....

(a) 2 (b) 12 (c) 35 (d) -2

2 $|-5| + |5| = \ldots\ldots\ldots\ldots$

(a) zero (b) 25 (c) 10 (d) -10

3 If $(x-2, 3) = (5, x+y)$, then $x - y = \ldots\ldots\ldots$

(a) 7 (b) 3 (c) -11 (d) 11

[b] If y is the middle proportional between x and z , **prove that :** $\frac{x}{x-z} = \frac{y}{y+z}$

3 **[a]** If $X = \{1, 3, 4, 5\}$, $Y = \{1, 2, 3, 4, 5\}$ and R is a relation from X to Y where "a R b" means "$b = 6 - a$" for all $a \in X$, $b \in Y$

1 Write R and represent it by an arrow diagram.

2 Show that R is a function and find its range.

[b] If $3x = 2y$, **find the value of the ratio :** $\frac{3x + 2y}{6y - x}$

4 **[a]** If $X = \{2, -1\}$, $Y = \{4, 0\}$, $Z = \{4, 5, -2\}$, **find :**

1 $X \times Y$ 2 $(Y \cap Z) \times X$ 3 $n(Y^2)$

[b] If $f(x) = 2x + a$ and $f(2) = 1$, **find the value of : a**

5 **[a]** If y changes inversely with x^2 and $y = 2$ when $x = 4$

1 Find the relation between y and x

2 Deduce the value of y when $x = 16$

[b] **Calculate the arithmetic mean and the standard deviation of the set of values :** 8 , 9 , 7 , 6 and 5

14 El-Beheira Governorate

Answer the following questions : *(Calculator is permitted)*

1 **Choose the correct answer from the given ones :**

1 The solution set in $\mathbb{R}$ for the equation $x^2 + 9 = 0$ is $\ldots\ldots\ldots$

(a) $\{-3\}$ (b) $\{3\}$ (c) $\{-3, 3\}$ (d) $\varnothing$

2 If the point $(k - 4, 2 - k)$ where $k \in \mathbb{Z}$ is located in the third quadrant , then $k = \ldots\ldots\ldots$

(a) 2 (b) 3 (c) 4 (d) 6

3 The multiplicative inverse of the number $\frac{\sqrt{3}}{6}$ is $\ldots\ldots\ldots$

(a) $-\frac{\sqrt{3}}{6}$ (b) $6\sqrt{3}$ (c) $2\sqrt{3}$ (d) $-2\sqrt{3}$

4 If 7 , x , $\frac{1}{y}$ are in continued proportion , then $x^2 y = \dots\dots$

(a) 7 (b) $\frac{1}{7}$ (c) 14 (d) 49

5 If $a + 3b = 7$, $c = 3$, then the value of $a + 3(b + c) = \dots\dots$

(a) 10 (b) 16 (c) 21 (d) 30

6 The difference between the greatest value and the smallest value in a set of values is called ....

(a) the arithmetic mean. (b) the median.

(c) the range. (d) the standard deviation.

2 [a] If $X = \{1\}$, $Y = \{2, 3\}$, $Z = \{2, 5, 6\}$, **find :**

1 $X \times (Y \cap Z)$ 2 $n(Z^2)$

[b] Find the positive number which if its square is added to each of the two terms of the ratio $5 : 11$, it becomes $3 : 5$

3 [a] If the point $(a, 3)$ is located on the straight line which represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = 4x - 5$, **find the value of :** a

[b] If $\frac{a+b}{3} = \frac{b+c}{6} = \frac{c+a}{5}$, **then prove that :** $\frac{a+b+c}{a} = 7$

4 [a] If $X = \{1, 3, 5\}$ and R is a relation on X where "a R b" means "$a + b = 6$" for each $a \in X$, $b \in X$

1 Write R

2 Show that R is a function and find its range.

[b] **Calculate the standard deviation for the values :** 17 , 22 , 20 , 23 , 18

5 [a] If $y \propto x$, $y = 6$ when $x = 3$, **find :**

1 The relation between y , x

2 The value of y when $x = 5$

[b] Represent graphically the quadratic function f where $f(x) = x^2 - 3$, where $x \in [-3, 3]$ **and from the graph deduce :**

1 The equation of the axis of symmetry.

2 The minimum value of the function.

15 El-Fayoum Governorate

Answer the following questions : *(Using calculators is allowed)*

1 Choose the correct answer :

1 The positive square root to the average of squares deviations of values from the mean is called the ....

(a) median. (b) mode. (c) range. (d) standard deviation.

2 If $f(3X) = 6$, then $f(-2) =$ ....

(a) -12 (b) -3 (c) 6 (d) -18

3 $[-5, 3] -]-5, 3[=$ ....

(a) $\{-5, 3\}$ (b) $]-5, 3]$ (c) $[-5, 3[$ (d) $\varnothing$

4 The fifth of the number 5^{10} equals ....

(a) 5^2 (b) 5^9 (c) 5^5 (d) 5^8

5 If $\frac{X}{2} = \frac{y}{3} = \frac{z}{5}$, then each ratio equals ....

(a) $\frac{X + y + z}{30}$ (b) $\frac{X + 2y - z}{3}$ (c) $\frac{X - y + z}{10}$ (d) $\frac{X - y}{5}$

6 If X is an odd number , then the odd number of the following is ..

(a) $X - 1$ (b) $X + 1$ (c) $X + 2$ (d) $X + 3$

2 [a] If $3a = 2b$, **then find the value of the ratio :** $\frac{3a - b}{a + 2b}$

[b] If $f(X) = aX + 5$ and $f(-3) = 8$, **then find the value of :** a

3 [a] If X , y , z are in continued proportion , **prove that :** $\frac{X^2 + y^2}{y^2 + z^2} = \frac{X}{z}$

[b] If $X = \{-1, 1, 2\}$, $Y = \{2, 4, 6, 8\}$ and R is a relation from X to Y where "a R b" means "$b = 2a + 4$" for each $a \in X$, $b \in Y$, write R and represent it by an arrow diagram , show that R is a function from X to Y , why ?

4 [a] If $y \propto X$ and $y = 20$ when $X = 7$, then find the relation between X and y , then find the value of y when $X = 14$

[b] If $(5 - 2X , y^3) = (1 , 27)$, **then find the value of :** $\sqrt{3X + y}$

5 [a] Represent graphically the function $f : f(X) = X^2 - 2$ where $X \in [-3, 3]$, and from the drawing deduce the coordinates of the vertex of the curve and the minimum value of the function.

[b] **Find the standard deviation of the values :** 7 , 16 , 13 , 5 , 9

16 Beni Suef Governorate

Answer the following questions : *(Calculator is allowed)*

1 **Choose the correct answer from those given :**

1 The point (– 4 , – 2) lies in the .... quadrant.

(a) first (b) second (c) third (d) fourth

2 If x represents a negative number , then the positive number from the following is ....

(a) $2x$ (b) $3x^2$ (c) $4x^3$ (d) $6x^5$

3 If $xy = 1$, then y varies with ....

(a) $\frac{1}{x}$ (b) $x - 1$ (c) x (d) $x + 1$

4 The simplest and easiest method of measuring dispersion is ....

(a) the median. (b) the mean.

(c) the standard deviation. (d) the range.

5 If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$ where $k \in \mathbb{R}$, then $\frac{a\,c\,e}{b\,d\,f} =$ ....

(a) k^3 (b) k^2 (c) k (d) 3

6 If $3x = 2y$, then $\frac{2x}{3y} =$ ....

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{9}{4}$ (d) $\frac{4}{9}$

2 [a] Find the number that if we add it to each term of the ratio 7 : 11 , it becomes 2 : 3

[b] If X = {1 , 2 , 3} , Y = {1 , 3 , 4 , 9} and R is a relation from X to Y where "a R b" means "$b = a^2$" for all $a \in X$, $b \in Y$, write R and represent it by an arrow diagram and show whether R is a function or not.

3 [a] If $\frac{x}{2} = \frac{y}{3} = \frac{z}{4} = \frac{3x - 2y + 5z}{5k}$, **find the numerical value of :** k

[b] Represent graphically the function $f : f(x) = 2 - x^2$, $x \in [-2 , 2]$, from the graph deduce the vertex point of the curve and the maximum value of the function.

4 [a] If y varies directly with x and y = 3 when $x = 15$, find the relation between y and x , then find the value of x when y = 100

[b] If X = {1 , 2} , Y = {3 , 4 , 5} , **find :**

1 X × Y 2 Y × X 3 X^2

5 [a] If $f(x) = 3x + k$, $g(x) = k$ where f and g are polynomial functions , **find the value of k if :** $f(3) + g(5) = 15$

[b] **Calculate the standard deviation of the set of values :** 12 , 13 , 16 , 18 , 21

17 El-Menia Governorate

Answer the following questions : ***(Calculators are allowed)***

1 **Choose the correct answer from those given :**

1. $\sqrt{5} + \sqrt{20} =$ ....
   (a) $\sqrt{25}$ (b) $5\sqrt{5}$ (c) $9\sqrt{5}$ (d) $3\sqrt{5}$
2. If three times a number = 45 , then $\frac{1}{5}$ this number = ...
   (a) 15 (b) 5 (c) 3 (d) 9
3. $5^2 \times 5^{-2} =$ ....
   (a) 5 (b) 1 (c) zero (d) -5
4. If n (X) = 3 , n (X × Y) = 12 , then n (Y) = ...
   (a) 4 (b) 9 (c) 15 (d) 36
5. The relation which represents direct variation between the two variables x and y is ....
   (a) $xy = 5$ (b) $y = x + 3$ (c) $\frac{x}{3} = \frac{5}{y}$ (d) $\frac{x}{5} = \frac{y}{3}$
6. The range is the .... measure of dispersions.
   (a) simplest (b) greatest (c) difficult (d) otherwise

2 [a] If $X = \{1, 2, 3\}$, $Y = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{5}\}$ and R is a relation from X to Y where "a R b" means "a is the multiplicative inverse of b" for all $a \in X$, $b \in Y$, write R and represent it by an arrow diagram. Is R a function ? Why ?

[b] If b is the middle proportional between a and c , **prove that :** $\frac{a+b}{a-c} = \frac{b}{b-c}$

3 [a] If $2y = 3x$, **find the value of :** $\frac{3x + 2y}{6y - x}$

[b] If $X = \{3, 4\}$, $Y = \{4, 5\}$, $Z = \{6, 5\}$, **find :**

1. $X \times (Y \cap Z)$
2. $(X - Y) \times Z$

4 [a] If $y \propto \frac{1}{x}$ and $y = 3$ when $x = 2$, **find :**

1. The relation between x and y
2. The value of x when $y = 4$

[b] **Calculate the standard deviation for the values :** 12 , 13 , 16 , 18 , 21

5 [a] Mention the degree of the function $f : f(X) = 3 - 2X^3$, **then find :** $f(0)$ and $f(-2)$

[b] Represent graphically the function $f : f(X) = X^2 + 2X + 1$, consider $X \in [-4 , 2]$ **and from the drawing deduce :**

1. The equation of the symmetry axis.
2. The maximum or the minimum value of the function.

18 Assiut Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer :**

1. $X^5 \div X^2 =$ .... (where $X \neq 0$)
   (a) X^7 (b) X^3 (c) X^{10} (d) X^5
2. If $X = \{1\}$, $Y = \{3\}$, then $n(X \times Y) =$ ....
   (a) $\{(1 , 3)\}$ (b) $\{(3 , 1)\}$ (c) 3 (d) 1
3. The multiplicative inverse of the number 0.25 is ....
   (a) 4 (b) – 0.25 (c) $\frac{1}{4}$ (d) – 0.5
4. The middle proportional between 4 , 16 is ....
   (a) – 8 (b) 8 (c) ± 8 (d) 64
5. $0.12 \div 0.3 =$ ....
   (a) 0.42 (b) 0.15 (c) 0.24 (d) 0.36
6. The range of the set of the values : 4 , 14 , 25 and 34 equals ....
   (a) 4 (b) 30 (c) 38 (d) 34

2 [a] If $X = \{6 , 7\}$, $Y = \{3 , 7\}$, **find :**

1. $(X \cap Y) \times X$
2. $n(Y^2)$

[b] If $\frac{a}{2} = \frac{b}{3} = \frac{c}{4}$, **then prove that :** $\frac{3c - b}{a + b} = \frac{9}{5}$

3 [a] If $X = \{-1 , 2 , 3\}$, $Y = \{1 , 4 , 6 , 9\}$ and R is a relation from X to Y where "a R b" means "$a^2 = b$" for each of $a \in X$, $b \in Y$, write R and represent it by an arrow diagram and show that R is a function from X to Y and find its range.

[b] If $y \propto \frac{1}{X}$ and $y = 3$ when $X = 4$

1. Find the relation between y and X
2. Find the value of y when $X = \frac{3}{4}$

4 [a] Find the positive number which if its square is added to each of the two terms of the ratio 7 : 11 , it becomes 2 : 3

[b] Represent graphically the function f : $f(X) = X^2 - 4$, $X \in [-3 , 3]$, from the graph deduce the vertex of the curve , the maximum value or the minimum value of the function and the equation of the axis of symmetry.

5 **[a] If $f(X) = X^2 - 2$, $g(X) = 3$, find : $f\left(\sqrt{2}\right) + g(5)$**

[b] Calculate the arithmetic mean and the standard deviation of the set of the values :

11 , 12 , 15 , 17 , 20

19 Souhag Governorate

Answer the following questions : *(Calculators are allowed)*

1 **Choose the correct answer :**

1. $2^8 + 2^8 + 2^8 + 2^8 =$ ....

(a) 2^{32} (b) 8^8 (c) 2^{10} (d) 4^{12}

2. If n (X) = 2 , n (Y^2) = 9 , then n (X × Y) = ...

(a) 6 (b) 18 (c) 11 (d) 7

3. If $\sqrt{3}\, X - 1 = 2$ ($X \in \mathbb{R}$) , then $X =$ ....

(a) 3 (b) $3\sqrt{3}$ (c) -3 (d) $\sqrt{3}$

4. If 8 , 6 , X , 12 are proportional quantities , then $X =$ ....

(a) 4 (b) 16 (c) 5 (d) 25

5. If the median of the values : a + 3 , a + 2 , a + 4 (a $\in \mathbb{Z}^+$) is 8 , then a = ...

(a) 2 (b) 5 (c) 3 (d) 4

6. .... is a measure for dispersion.

(a) The median (b) The mode (c) The range (d) The mean

2 **[a]** If X × Y = {(1 , 1) , (1 , 3) , (1 , 5)}

, find : 1. X , Y 2. Y × X

[b] If $\frac{X}{y} = \frac{2}{3}$ **, then find the value of :** $\frac{3X + 2y}{6y - X}$

3 **[a]** If X = {0 , 1 , 2 , 3} , Y = {2 , 3 , 4 , 5 , 6} , R is a relation from X to Y where "a R b" means "a + b = 5" for each a ∈ X , b ∈ Y

1. Write R and represent it by an arrow diagram.
2. Show that R is a function from X to Y and find its range.

[b] Find the number that if we add it to the two terms of the ratio 7 : 11 , the result will be 2 : 3

4 [a] If the straight line $y = 4x - 5$ passes through the point $(a , 3)$, **find the value of :** a

[b] If $y \propto x$ and $y = 6$ when $x = 3$, **find :**

1 The relation between x and y

2 The value of y when $x = 5$

5 [a] Represent graphically the function $f : f(x) = x^2 - 4x + 4 , x \in [-1 , 5]$

, from the graph deduce :

1 The vertex of the curve.

2 The equation of the axis of symmetry.

[b] **Calculate the mean and the standard deviation of the values :** 12 , 13 , 16 , 18 , 21

20 Qena Governorate

Answer the following questions : *(Calculators are permitted)*

1 **Choose the correct answer :**

1 If $x y = 5$, then $y \propto$ ....

(a) x^{-1} (b) x (c) $5x$ (d) x^2

2 $\sqrt{3} + \sqrt{3} + \sqrt{3} =$ ....

(a) 3 (b) 9 (c) $3\sqrt{3}$ (d) 27

3 The middle proportional between 3 , 12 is ....

(a) 6 (b) -6 (c) ± 6 (d) 9

4 The point $(-2 , 3)$ lies in the .... quadrant.

(a) first (b) second (c) third (d) fourth

5 All of the following are polynomial functions except ....

(a) $f_1(x) = x^3 + x^2 + 3$ (b) $f_2(x) = x^3 + \frac{1}{x} + 7$

(c) $f_3(x) = 5 - x^2$ (d) $f_4(x) = x^2 (x - 3)^2$

6 The range of the values : 51 , 24 , 43 , 55 , 28 is ....

(a) 55 (b) 24 (c) 21 (d) 31

2 [a] If $X = \{1 , 3 , 4 , 5\}$, $Y = \{1 , 2 , 3 , 4 , 5 , 6 , 7\}$ and R is a relation from X to Y where "a R b" means "a + b = 7" , write R and represent it by an arrow diagram. Is R a function ? Why ? and if it's a function , find its range.

[b] If b is the middle proportional between a and c , **prove that :** $\frac{a^2 + b^2}{b^2 + c^2} = \frac{a}{c}$

3 [a] If $f(x) = x^2 - 3x$, $g(x) = x - 3$

1 **Find :** $f\left(\sqrt{2}\right) + 3\,g\left(\sqrt{2}\right)$

2 **Prove that :** $f(3) = g(3)$

[b] Find the number which if added to each of the two terms of the ratio 7 : 11 , it becomes 2 : 3

4 [a] If 5 a = 3 b , find : $\frac{7a+9b}{4a+2b}$

[b] **The following table shows the frequency distribution for the ages of 10 students :**

Ages in years	5	8	9	10	12	Total
Number of students	1	2	3	3	1	10

Calculate the standard deviation to age in years.

5 [a] If $y \propto X$ and y = 40 when X = 14 , find X when y = 80

[b] Represent graphically $f : f(X) = 2X^2 - 3$, $X \in [-2 , 2]$ **From the graph find :**

1 The vertex of the curve.

2 The equation of the axis of symmetry.

3 The maximum or minimum value of the function.

21 Luxor Governorate

Answer the following questions :

1 **Choose the correct answer :**

1 The sum of the factors of the number 15 equals ....

(a) 3 (b) 4 (c) 15 (d) 24

2 If $f(X) = 4X + a$ and $f(2) = 15$, then a = ...

(a) 2 (b) 4 (c) 7 (d) 15

3 The smallest expression in value when $X = 7$ is ....

(a) $\frac{6}{X}$ (b) $\frac{6}{X+1}$ (c) $\frac{6}{X-1}$ (d) $\frac{X}{6}$

4 The third proportional of the two numbers – 6 , 12 is ....

(a) – 24 (b) 6 (c) 18 (d) 72

5 If $3X - 1 = 1 - 3X$, then X = ...

(a) zero (b) $\frac{1}{3}$ (c) – 1 (d) 3

6 Which of the following values for the number X makes the range of the values X , 15 , 20 , 24 equal 14 ?

(a) 30 (b) 25 (c) 19 (d) 10

2 [a] If $f = \{(1 , 3) , (2 , 5) , (3 , 7) , (4 , 9) , (5 , 11)\}$, write :

1 The domain of f 2 The range of f 3 The rule of f

[b] Two integers , the ratio between them is 2 : 3 , if 7 is subtracted from each of them, the ratio becomes 1 : 2 Find the two numbers.

3 [a] If $X = \{-2 , 2 , 5\}$, $Y = \{3 , 7 , \ell\}$ and R is a function from X to Y where "a R b" means "$b = a^2 - 1$" for each $a \in X$, $b \in Y$

1. Find the value of ℓ
2. Write R
3. Represent the function by an arrow diagram.

[b] If $y = a - 9$, $y \propto \frac{1}{x^2}$ and $a = 18$ when $x = \frac{2}{3}$

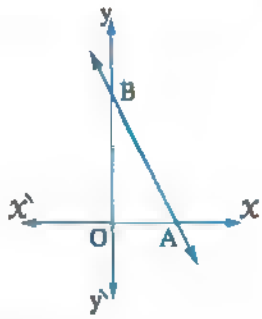
, find the relation between x , y and find the value of y when $x = 1$

4 [a] The opposite figure represents the function f where

$f(x) = 4 - 2x$

Find the coordinates of A , B

and the area of Δ AOB



[b] If $\frac{x}{7} = \frac{y}{3}$, **prove that :** $(2x - 3y)$, $(x + 2y)$, 10 , 26 are proportional.

5 [a] **Calculate the standard deviation of the values :** 72 , 53 , 61 , 70 , 59

[b] Graph the function $f : f(x) = 1 - 4x + x^2$ where $x \in [0 , 4]$ **and from the graph find :**

1. The vertex of the curve.
2. The equation of the axis of symmetry.
3. The maximum or the minimum value of the function.

22 Aswan Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer :**

1. If $X = \{1 , 2\}$, $Y = \{0\}$, then $n(X \times Y) = \dots\dots\dots$

   (a) 0 (b) 1 (c) 2 (d) 3

2. $(\sqrt{5} - 2)(\sqrt{5} + 2) = \dots\dots\dots$

   (a) 5 (b) 3 (c) 2 (d) 1

3. The range of the set of the values : 16 , 32 , 5 , 27 and 20 is ....

   (a) 27 (b) 20 (c) 16 (d) 13

4. The third proportional for the numbers 8 , 6 , ... and 12 is ....

   (a) 24 (b) 20 (c) 16 (d) 8

5 If $X = 3$, $y = 5$, then $y^X = \dots\dots\dots$

(a) 135 (b) 125 (c) 115 (d) 95

6 If $5X = 12$, then $10X = \dots\dots\dots$

(a) 12 (b) 22 (c) 24 (d) 34

2 [a] If $X \times Y = \{(2, 2), (2, 5), (2, 7)\}$

, **find :** 1 Y 2 $Y \times X$

[b] If b is the middle proportional between a and c , **prove that :** $\frac{a-b}{a-c} = \frac{b}{b+c}$

3 [a] If $X = \{2, 3, 5\}$, $Y = \{4, 6, 8, 10\}$ and R is a relation from X to Y where "a R b" means "2 a = b" for each $a \in X$, $b \in Y$

1 Write R and represent it by an arrow diagram.

2 Is R a function ?

[b] If y varies inversely as X and $y = 2$ when $X = 4$

, find the relation between y and X , then find the value of y when $X = 16$

4 [a] If the point (a , 3) is located on the straight line which represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = 4X - 5$, **find the value of :** a

[b] If $\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a - b + 5c}{3X}$, **find the value of :** X

5 [a] Represent graphically the function $f : f(X) = (X - 3)^2$, taking $X \in [0, 6]$ and from the graph deduce the coordinates of the vertex point of the curve , the maximum or minimum value of the function and the equation of the axis of symmetry.

[b] **The following frequency distribution shows the number of children of some families in a new city :**

Number of children	Zero	1	2	3	4
Number of families	8	16	50	20	6

Calculate the mean and the standard deviation of the number of children.

23 New Valley Governorate

Answer the following questions : *(Calculator is allowed)*

1 **Choose the correct answer from those given :**

1. If $\sqrt[3]{X} = \sqrt{16}$, then $X =$ ....
   (a) 4 (b) 8 (c) 16 (d) 64
2. If 2 , X , 4 and 6 are proportional , then $X =$ ....
   (a) 1 (b) 3 (c) 5 (d) 8
3. If $y = 2X$, then $y \propto$ ....
   (a) $\frac{1}{X}$ (b) X (c) $X + 2$ (d) $X - 2$
4. $2^{X-5} = 1$ where $X \in$ ....
   (a) 5 (b) $\mathbb{R} - \{5\}$ (c) $\mathbb{R}$ (d) $\{5\}$
5. The middle proportional between 3 and $\frac{1}{3}$ is ....
   (a) ± 1 (b) 9 (c) $\frac{1}{9}$ (d) ± 9
6. If $\Sigma (X - \overline{X})^2 = 36$ for a set of values whose number equals 9 , then the standard deviation = ...
   (a) 2 (b) 3 (c) 4 (d) 6

2 **[a]** If $X = \{2 , 3\}$, $Y = \{3 , 4 , 5\}$, **then find :**

1. $X \times Y$ and represent it by an arrow diagram.
2. $n (X \times Y)$

[b] If $X^2 y^2 - 14 X y + 49 = 0$, **then prove that :** $y \propto \frac{1}{X}$

3 **[a]** Find the negative number which if its square is added to each of the two terms of the ratio 7 : 11 , it becomes 4 : 5

[b] If $X = \{2 , 4 , 8\}$ and R is a relation on X where "a R b" means "a is double b" for each $a \in X$, $b \in X$, write R Is R a function ? and why ?

4 **[a]** If $\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a - 5b + 3c}{X}$, **then find the value of each of :**

1. X
2. $\frac{a + b + c}{b}$

[b] If the function $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(X) = 2X - 3$, **then find the value of k if :**

1. $f(k) = 5$
2. $(2 , k) \in f$

5 [a] **The following frequency distribution shows the numbers of children of some families in a new city :**

Number of children (X)	3	5	7	9	11
Number of families (k)	3	12	21	10	4

Calculate the mean and the standard deviation to the number of children.

[b] Represent graphically the curve of the function f where $f(X) = (X+1)^2$, $X \in [-3, 1]$ **and from the drawing deduce :**

1 The coordinates of the vertex of the curve.

2 The equation of the symmetry axis.

3 The minimum value of the function.

24 South Sinai Governorate

Answer the following questions :

1 **Choose the correct answer :**

1 The degree of the polynomial function $f : f(X) = X^4 - 2X^2 + 5$ is ....

(a) fourth. (b) third. (c) second. (d) first.

2 The fourth proportional of 3 , 6 , 6 is ....

(a) 9 (b) 12 (c) 6 (d) 1

3 If n (X) = 5 and n (X × Y) = 15 , then n (Y) = ...

(a) 20 (b) 10 (c) 3 (d) 2

4 The arithmetic mean of the values : 3 , 4 , 6 , 7 equals ....

(a) 40 (b) 20 (c) 10 (d) 5

5 If $y^2 + 4X^2 = 4Xy$, then ....

(a) $y \propto X$ (b) $y \propto X^2$ (c) $y \propto \frac{1}{X}$ (d) $y \propto \frac{1}{X^2}$

6 If X is an odd number , then the next odd number is ....

(a) X^2 (b) $X^2 + X$ (c) $X + 6$ (d) $X + 2$

2 If X = {2 , 3 , 4} and Y = {y : y ∈ ℕ , 2 ≤ y < 9} where ℕ is the set of natural numbers and R is a relation from X to Y where "a R b" means "a = $\frac{1}{2}$ b" , a ∈ X and b ∈ Y , write R Is R a function from X to Y ? Then find the range.

3 [a] Find the number which if we added it to each of the two terms of the ratio 7 : 11 , it becomes 2 : 3

[b] If $y \propto X$ and $y = 14$ when $X = 42$, find the relation between X and y , then find the value of y when $X = 60$

4 **[a]** Represent graphically the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = 2X - 3$

[b] If b is the middle proportional between a and c , **prove that :** $\frac{a^2 + b^2}{b^2 + c^2} = \frac{a}{c}$

5 **[a]** If $(X^3 , y + 1) = \left(27 , \sqrt[3]{125}\right)$, **find the value of each of :** X and y

[b] **Calculate the arithmetic mean and the standard deviation for the values :**
20 , 17 , 22 , 23 , 18

25 North Sinai Governorate

Answer the following questions :

1 **Choose the correct answer from those given :**

1 If $f(X) = 5$, then $f(5) + f(-5) =$ ....

(a) zero (b) 5 (c) -5 (d) 10

2 If $(X - 2 , 3) = (5 , 3)$, then $X =$ ....

(a) 5 (b) 3 (c) 7 (d) 8

3 If f is an odd number , then the next odd number is ....

(a) f^2 (b) $f + 6$ (c) $f + 2$ (d) $f^2 + 1$

4 The fourth proportional of the quantities 4 , 8 , 8 equals ....

(a) 4 (b) 8 (c) 12 (d) 16

5 The sum of the two square roots of $2\frac{1}{4}$ equals ....

(a) $1\frac{1}{2}$ (b) zero (c) $\frac{1}{2}$ (d) $\sqrt{2}$

6 The difference between the greatest value and the smallest value of a set of individuals is called ....

(a) the range. (b) the arithmetic mean.
(c) the median. (d) the standard deviation.

2 **[a]** If $X = \{1 , 2 , 3\}$, $Y = \left\{1 , \frac{1}{2} , \frac{1}{3} , \frac{1}{5}\right\}$ and R is a relation from X to Y where "a R b" means "a is the multiplicative inverse of b" for all $a \in X$, $b \in Y$, write R and represent it by an arrow diagram. Is R a function or not ?

[b] If $y \propto \frac{1}{X}$ and $y = 3$ when $X = 2$

1 Find the relation between X and y

2 Find the value of y when $X = 1.5$

3 [a] If $f(X) = 5X + 4$ is represented graphically by a straight line passing through the point $(3, b)$, **then find the value of :** b

[b] If $\frac{X}{y} = \frac{3}{4}$, **find the value of :** $\frac{3X + y}{X + 5y}$

4 [a] If $X \times Y = \{(1, 2), (4, 2), (5, 2)\}$, **then find :** X , Y , Y^2

[b] If b is the middle proportional between a and c , **prove that :** $\frac{5c^2 - 2b^2}{5b^2 - 2a^2} = \frac{c}{a}$

5 [a] **Calculate the standard deviation to the following data :** 12 , 13 , 16 , 18 and 21

[b] Represent graphically the function $f : f(X) = 2 - X^2$, $X \in [-3, 3]$

From the graph deduce :

1. The coordinates of the vertex point of the curve.
2. The equation of the axis of symmetry.
3. The maximum or minimum value of the function.

26 Red Sea Governorate

Answer the following questions : *(Calculators are allowed)*

1 **Choose the correct answer from those given :**

1. If the point $(a - 3, 5)$ lies on y-axis , then a = ...

   (a) 5 (b) 3 (c) 2 (d) 0

2. If 2 , 3 , 6 , X are proportional quantities , then X = ...

   (a) 9 (b) 18 (c) 12 (d) 3

3. The range of the set of the values : 3 , 5 , 6 , 7 , 9 equals ....

   (a) 3 (b) 4 (c) 6 (d) 12

4. If $f(X) = 3$, then $f(5) + f(-5)$ = ...

   (a) -1 (b) 0 (c) 1 (d) 6

5. If $X - y = 5$, $X + y = 1$, then $X^2 - y^2$ = ...

   (a) $\frac{1}{25}$ (b) 1 (c) 5 (d) 25

6. If $X y = 7$, then $y \propto$ ....

   (a) $\frac{1}{X}$ (b) $X - 7$ (c) $X + 7$ (d) X

2 [a] If $X \times Y = \{(1, 1), (1, 5), (1, 7)\}$, **find :**

1. X 2. n (Y) 3. Y × X

[b] If b is the middle proportional between a and c , **prove that :** $\frac{a^2 + b^2}{b^2 + c^2} = \frac{a}{c}$

3 [a] If $f(x) = 4x + a$, $f(2) = 15$, **find the value of :** a

[b] If $X = \{1, 2, 3\}$, $Y = \{1, 2, 3, 4, 5\}$ and R is a relation from X to Y where "a R b" means "a + b = 5" for each $a \in X$, $b \in Y$

1 Write R and represent it by a Cartesian diagram.

2 Is R a function or not ?

4 [a] If $\frac{x}{y} = \frac{2}{3}$, **find the value of :** $\frac{3x + 2y}{6y - x}$

[b] If $y \propto x$ and $y = 2$ when $x = 6$, **find :**

1 The relation between y and x

2 The value of y when $x = 15$

5 [a] Represent graphically the curve of the function f where $f(x) = 4 - x^2$ and $x \in [-3, 3]$, **from the graph deduce :**

1 The coordinates of the vertex of the curve.

2 The equation of the axis of symmetry of the curve.

[b] **Find the standard deviation for the values :** 12 , 13 , 16 , 18 , 26

27 Matrouh Governorate

Answer the following questions : ***(Calculators are allowed)***

1 **Choose the correct answer from those given :**

1 If a , b , 2 and 3 are proportional , then $\frac{a}{b} =$ ....

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

2 $[1, 4] -]1, 4[=$ ....

(a) $\{0\}$ (b) $\{1, 4\}$ (c) $[1, 4[$ (d) $\varnothing$

3 If $(2, 5) \in \{3, 2\} \times \{1, x\}$, then $x =$ ....

(a) 2 (b) 3 (c) 1 (d) 5

4 If $(x - 1, 2^y) = (1, 8)$, then $(x, y) =$ ....

(a) (2 , 3) (b) (3 , 2) (c) (0 , 3) (d) (0 , − 3)

5 The point (3 , − 4) lies in the .... quadrant.

(a) first (b) second (c) third (d) fourth

6 If $\Sigma (x - \overline{x})^2 = 36$ for a set of values whose number is 9 , then $\sigma =$ ....

(a) 2 (b) 4 (c) 18 (d) 27

2 [a] If $X = \{1, 2, 3\}$, $Y = \{1, 3, 6, 9, 12\}$ and R is a relation from X to Y where "a R b" means "$a = \frac{1}{3}$ b" for all $a \in X$, $b \in Y$, write R and show whether it is a function or not, and if it is a function , write the range.

[b] If $\frac{a}{b} = \frac{2}{5}$, **find the value of :** $\frac{2a-2b}{3a+2b}$

3 [a] If $X \times Y = \{(1, 1), (1, 3), (1, 5)\}$

, **find :** 1 X , Y 2 Y^2

[b] If $\frac{x}{2a+b} = \frac{y}{2b-c} = \frac{z}{2c-a}$, **prove that :** $\frac{2x+y}{4a+4b-c} = \frac{2x+2y+z}{3a+6b}$

4 [a] If the point (a , 3) is located on the straight line which represents the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = 4x - 5$, **find the value of :** a

[b] **The following frequency distribution shows the number of children of some families in a new city :**

Number of children	Zero	1	2	3	4
Number of families	8	16	50	20	6

Calculate the mean and the standard deviation to the number of children.

5 [a] If y varies inversely as x and $y = 10$ when $x = 3$, find the relation between x and y , then find the value of y when $x = 5$

[b] Represent graphically the function $f : f(x) = (x-3)^2$, $x \in [0, 6]$
From the graph deduce the vertex of the curve , the maximum or minimum value of the function.

2021 (5) El-Sharkia Governorate

Answer the following questions : (Calculators are allowed)

1 Choose the correct answer from those given :

1 If $(X + 2 , y) = (2 , 3)$, then $X^5 y + 1 =$ ....

(a) 3 (b) 2 (c) zero (d) 1

2 If $a \in X^2$ where $X = \{X : 5 < X < 7 , X \in \mathbb{N}\}$, then a is ....

(a) 36 (b) $\{36\}$ (c) $(6 , 6)$ (d) $[5 , 7]$

3 If y varies directly as X , then ....

(a) X varies inversely as y (b) X varies directly as y

(c) $y = X + 5$ (d) $\frac{X}{3} = \frac{2}{y}$

4 If $\frac{a}{b} = \frac{c}{d} = \frac{h}{m}$, then $\frac{a + c + h}{b + d + m} =$ ....

(a) $\frac{a}{b} + \frac{c}{d} + \frac{h}{m}$ (b) $\frac{c}{h}$ (c) $\frac{c}{a}$ (d) $\frac{c}{d}$

5 If $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = X^{k-2} + 3$ and $f(2) = 11$, then $k =$ ....

(a) 5 (b) 3 (c) 2 (d) -3

6 If the range of the values $6 + k , 6 - k , 6 + 5k , 6 - 2k$ is 14 where $k \in \mathbb{N}$, then $k =$ ....

(a) 1 (b) 2 (c) 3 (d) 4

---

2 [a] If $X = \{2 , 3 , 4\}$, $Y = \{3 , 4 , 5\}$, $Z = \{3 , 4\}$, **find :**

1 $(Y \cap X) - Z$ 2 $X \times Z$ 3 $n(X \cup Z)$

[b] If a , b , c and d are in continued proportion , **prove that :** $\frac{2a^3 + 3b^3}{2a + 3d} = a^2$

---

3 [a] If $X = \{3 , 4 , 5\}$, $Y = \{1 , 2 , 3\}$ and R is a relation from X to Y where "a R b" means that "a + b is a prime number" for all $a \in X$ and $b \in Y$, write R and represent it by an arrow diagram. Is R a function or not ? And why ?

[b] If y varies inversely as $\sqrt{X}$, and $y = 3$ when $X = 9$, find X at $y = 4$

---

4 [a] Represent graphically the curve of the function $f : f(X) = (X - 2)^2 + 1$ where $X \in [\text{zero} , 4]$ **From the graph find :**

1 The equation of the axis of symmetry of the curve.

2 The minimum value of the function.

[b] If $\frac{a}{4} = \frac{b}{3}$, **find the value of :** $\frac{ab + a^2}{ab - b^2}$

5 **[a] Find the arithmetic mean and the standard deviation of the values :** 6 , 3 , 9 , 2 , 5

[b] If the function $f : f(X) = X^{m-3} + X^{4-m}$ is a polynomial function where $m \in \mathbb{Z}$

Find : 1 The value of $f(1)$ 2 The value of m

6 El-Monofia Governorate

Answer the following questions : *(Using calculator is permitted)*

1 **Choose the correct answer :**

1 If $\frac{5}{4} + \frac{5}{X} = \frac{5}{2}$, then $X =$ ....

(a) 2 (b) 4 (c) 5 (d) $\frac{5}{2}$

2 If $X + y = X y = 5$, then $X^2 y + X y^2 =$ ....

(a) 10 (b) 15 (c) 20 (d) 25

3 If $1 < X < 3$, $X \in \mathbb{R}$, then $(3X - 1) \in$ ....

(a) $[2 , 8[$ (b) $[2 , 8]$ (c) $]2 , 8[$ (d) $\{2 , 8\}$

4 If $\frac{a + 2b}{a - b} = \frac{2}{3}$, then $\frac{b}{a} =$ ....

(a) $\frac{1}{8}$ (b) 8 (c) $\frac{-1}{8}$ (d) -8

5 Which of the following values of the number X makes the range of the set of the values X , 15 , 20 , 24 equal to 14 ?

(a) 30 (b) 25 (c) 19 (d) 10

6 If $X \in \mathbb{R}_-$, then the point $\left(-X , \sqrt[3]{X}\right)$ lies in the .... quadrant.

(a) first (b) second (c) third (d) fourth

2 **[a]** If $X = \{4 , 3\}$, $Y = \{5 , 4\}$ and $Z = \{5 , 6\}$, **find :**

1 $X \times (Y \cap Z)$ 2 $(X - Y) \times Z$ 3 $n(Z^2)$

[b] If a , b , c and d are in continued proportion , **prove that :** $\frac{ab - cd}{b^2 - c^2} = \frac{a + c}{b}$

3 **[a]** If $X = \{-2 , -1 , 1 , 2\}$, $Y = \left\{8 , \frac{1}{3} , -1 , 1 , -8\right\}$ and R is a relation from X to Y where "a R b" means "$b = a^3$" for each $a \in X$, $b \in Y$

1 Write R and represent it by an arrow diagram.

2 Show that R is a function and find its range.

[b] If the straight line that represents the function f where $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = aX + b$ cuts y-axis at the point (0 , 3) and $f(2) = 7$, **find :** the values of a and b

27 Matrouh Governorate

Answer the following questions : ***(Calculators are allowed)***

1 Choose the correct answer from those given :

1 The relation which represents the inverse variation between the two variables X and y is ....

(a) $y = 5X$ (b) $y = \frac{1}{5}X$ (c) $y = \frac{5}{X}$ (d) $y = X + 5$

2 If $2^X = 4^3$, then $X =$ ....

(a) 3 (b) 4 (c) 6 (d) 46

3 $[1, 5] - \{0, 1\} =$ ....

(a) $]1, 5[$ (b) $]1, 5]$ (c) $[1, 5[$ (d) $\{5\}$

4 If the arithmetic mean of the set of the values a, 5, 8, 7, 6 equals 6, then a = ...

(a) 4 (b) 6 (c) 8 (d) 30

5 If $\frac{3}{4} + \frac{3}{X} = \frac{3}{2}$, then $X =$ ....

(a) 2 (b) 4 (c) 3 (d) $\frac{3}{2}$

6 The linear function $f : f(X) = 2X - 1$ is represented by a straight line cutting y-axis at the point ....

(a) (0, 1) (b) (0, – 1) (c) (1, 0) (d) (– 1, 0)

---

2 [a] If $X = \{13, 14, 43, 84\}$, and R is a relation on X such that "a R b" means "the two numbers a and b have the same unit digit" for all $a \in X$, $b \in X$, write R and represent it by an arrow diagram. Is R a function ? Why ?

[b] If y varies as X and y = 10 when $X = 7$, find X when y = 20

---

3 [a] If $X = \{3, 4\}$, $Y = \{4, 5\}$, $Z = \{6, 5\}$, **then find :**

1 $X \times (Y \cap Z)$ 2 $(X - Y) \times Z$

[b] If $\frac{X + y}{\ell + m} = \frac{y + z}{m + n} = \frac{z + X}{n + \ell}$, **then prove that :** $\frac{X}{\ell} = \frac{y - X}{m - \ell}$

---

4 [a] If $f : \mathbb{R} \longrightarrow \mathbb{R}$ is represented by a straight line cutting X-axis at (3, b) where $f(X) = 2X - a$, **find :** "3 a + 5 b"

[b] The following is the frequency distribution for a number of defective units which are found in 100 boxes of manufactured units :

Number of defective units	zero	1	2	3	4	5
Number of boxes	3	16	17	25	20	19

Find the standard deviation to the defective units.

5 [a] If $\frac{a}{b} = \frac{2}{5}$ **, then find the value of :** $\frac{b - a}{b + a}$

[b] Represent graphically the function $f : f(X) = X^2 - 3$, $X \in [-3 , 3]$ and from the drawing deduce the coordinates of the vertex of the curve , the equation of the symmetry axis and the minimum or the maximum value of the function.

2020

Governorates' Examinations on Trigonometry and Geometry

1 Cairo Governorate

Answer the following questions : *(Calculator is allowed)*

1 Choose the correct answer from those given :

1. If $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = \frac{1}{2}$, then the slope of $\overleftrightarrow{CD} =$ ....
   (a) 2 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) -2
2. The number of symmetry axes of an isosceles triangle equals ....
   (a) 1 (b) 2 (c) 3 (d) 4
3. $\tan 60° \tan 30° =$ ....
   (a) $\sin 30°$ (b) $\tan 30°$ (c) $\tan 45°$ (d) $\cos 60°$
4. The sum of the measures of the interior angles of the quadrilateral equals ....
   (a) 540° (b) 360° (c) 180° (d) 90°
5. The equation of the straight line which passes through the point (2 , 3) and is parallel to X-axis is ....
   (a) $X = 2$ (b) $X = 3$ (c) $y = 2$ (d) $y = 3$
6. The perimeter of the square whose surface area is 100 cm^2 equals .... cm.
   (a) 10 (b) 20 (c) 40 (d) 50

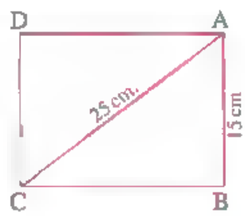
2 [a] If $X \sin 45° \cos 45° = \sin 30°$, find the value of X (Showing the steps of the solution).

[b] Find the equation of the straight line which its slope is 2 and passes through the point (1 , 0)

3 [a] XYZ is a right-angled triangle at Y in which XY = 6 cm. , YZ = 8 cm.
Find the value of : $\cos X \cos Z - \sin X \sin Z$

[b] ABCD is a quadrilateral , where A (2 , 4) , B (– 3 , 0) , C (– 7 , 5) , D (– 2 , 9)
Prove that : The figure ABCD is a square.

4 [a] **In the opposite figure :**



ABCD is a rectangle , AB = 15 cm.
, AC = 25 cm.
Find : 1 The length of $\overline{BC}$ 2 m (∠ ACB)
3 The area of the rectangle ABCD

[b] If C (6 , – 4) is the midpoint of $\overline{AB}$ where A (5 , 3) , find the coordinates of the point B

5 [a] If the straight line whose equation is a $X + 2y - 7 = 0$ is parallel to the straight line which makes an angle of measure 45° with the positive direction of X-axis , find the value of a

[b] Find the equation of the straight line which passes through the two points (4 , 2) , (– 2 , – 1) , then prove that it passes through the origin point.

2 Giza Governorate

Answer the following questions :

1 **Choose the correct answer :**

1 If $\sin X = \frac{1}{2}$ where X is an acute angle , then $\sin 2X =$ ....

(a) $\frac{1}{4}$ (b) 1 (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{2}$

2 The distance between the point (4 , 3) and y-axis equals .... length unit.

(a) – 3 (b) – 4 (c) 3 (d) 4

3 The points (8 , 0) , (0 , 6) , (0 , 0) ....

(a) form a right-angled triangle. (b) form an obtuse-angled triangle.

(c) form an acute-angled triangle. (d) are collinear.

4 If A (5 , 7) , B (1 , – 1) , then the midpoint of $\overline{AB}$ is ....

(a) (2 , 3) (b) (3 , 3) (c) (3 , 2) (d) (3 , 4)

5 The equation of the straight line which passes through the point (1 , – 3) and is parallel to X-axis is ....

(a) $X = 3$ (b) $y = 1$ (c) $y = -3$ (d) $X = -3$

6 The opposite figure represents a quarter of a circle with radius 2 cm. long , then its perimeter = ... cm.


2 cm.

(a) 2π (b) 5π

(c) $\pi + 4$ (d) $4\pi + 4$

2 [a] Find the equation of the straight line which its slope is 2 and passes through the point (1 , – 1)

[b] ABC is a right-angled triangle at C in which AC = 3 cm. , BC = 4 cm. **Find :**

1 cos A cos B – sin A sin B 2 m (∠ B)

3 [a] **Without using calculator , prove that :** sin 60° = 2 sin 30° cos 30°

[b] If the straight line L_1 passes through the two points (3 , 1) , (2 , k) and the straight line L_2 makes with the positive direction of the X-axis an angle of measure 45° , find the value of k if $L_1 \perp L_2$

4 [a] If $\cos E \tan 30^\circ = \cos^2 45^\circ$, then find m ($\angle$ E) where E is an acute angle.

[b] Show the type of the triangle whose vertices are the points :

A (3 , 3) , B (1 , 5) , C (1 , 3) with respect to its side lengths.

5 [a] Find the slope of the straight line $5X + 4y + 10 = 0$, then find the length of the y-intercept.

[b] Prove that the points A (3 , – 1) , B (– 4 , 6) , C (2 , – 2) which belong to a perpendicular coordinates plane passing through the circle whose centre is the point M (– 1 , 2) , then find the area of the circle.

3 Alexandria Governorate

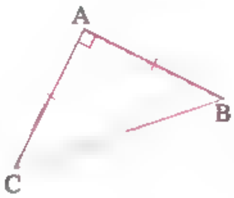
Answer the following questions : (Calculators are permitted)

1 **Choose the correct answer from those given :**

1 If $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = \frac{2}{3}$, then the slope of $\overleftrightarrow{CD}$ = ...

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$ (c) $\frac{-3}{2}$ (d) $\frac{-2}{3}$

2 **In the opposite figure :**



ABC is an isosceles triangle and a right-angled triangle at A

, then tan C = ...

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{\sqrt{3}}$ (c) 1 (d) $\frac{1}{2}$

3 If A , B are two acute angles and m ($\angle$ A) + m ($\angle$ B) = 90° , m ($\angle$ A) ≠ m ($\angle$ B)

, then ....

(a) sin A = cos B (b) sin A = sin B

(c) tan A = tan B (d) cos A = cos B

4 A circle of centre at the origin point and its radius length is 2 length unit , then the point .... belongs to it.

(a) (1 , – 2) (b) $\left(-2 , \sqrt{5}\right)$ (c) (0 , 1) (d) $\left(\sqrt{3} , 1\right)$

5 If X , Y are two supplementary angles and m ($\angle$ X) = m ($\angle$ Y) , then m ($\angle$ X) = ...°

(a) 30 (b) 45 (c) 60 (d) 90

6 The parallelogram whose diagonals are equal in length and perpendicular is the ......

(a) square. (b) rhombus. (c) rectangle. (d) trapezium.

2 [a] **Find the value of X which satisfies :** $X \sin 30° \cos^2 45° = \sin^2 60°$

[b] ABCD is a parallelogram where A (3 , 2) , B (4 , – 5) , C (0 , – 3) Find the two coordinates of the point at which the two diagonals intersect , then find the coordinates of the point D

3 [a] Prove that the points A (3 , - 1) , B (– 4 , 6) and C (2 , - 2) are located on a circle whose centre is the point M (1 , 2) , then find the circumference of the circle. ($\pi = 3.14$)

[b] Find the equation of the straight line which is perpendicular to the straight line whose equation is $X + 2y + 5 = 0$ and intercepts a positive part from y-axis that is equal to 7 units.

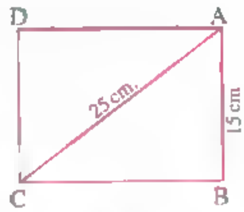
4 [a] Prove that the straight line passing through the two points (– 3 , – 2) , (4 , 5) is parallel to the straight line that makes with the positive direction of the X-axis an angle of measure 45°

[b] ABC is a right-angled triangle at C , AC = 6 cm. , BC = 8 cm.

Find the value of : cos A cos B – sin A sin B

5 [a] Let A (4 , – 6) , B (3 , 7) and C (1 , – 3) Find the equation of the straight line which passes through A and the midpoint of $\overline{BC}$

[b] **In the opposite figure :**



ABCD is a rectangle where AB = 15 cm.

, AC = 25 cm.

Find : 1 m (∠ ACB)

2 The surface area of the rectangle ABCD

4 El-Kalyoubia Governorate

Answer the following questions :

1 **Choose the correct answer :**

1 If $\cos \frac{X}{2} = \frac{1}{2}$ where $\frac{X}{2}$ is the measure of a positive acute angle , then $X =$ ....°

(a) 30 (b) 90 (c) 60 (d) 120

2 The triangle whose area is 24 cm.^{2} and its height is 8 cm. , then the length of the base corresponding to this height is .... cm.

(a) 16 (b) 6 (c) 3 (d) 2

3 If $\overleftrightarrow{CD}$ is parallel to y-axis where C (k , 4) , D (– 5 , 7) , then k = ……………

(a) 5 (b) 7 (c) – 5 (d) 4

4 The equation of the straight line passing through the origin point and its slope = 1 is ……

(a) $y = x$ (b) $y = -x$ (c) $y = 2x$ (d) $y = 0$

5 If the point (0 , a) belongs to the straight line $3x - 4y + 12 = 0$, then a = ……

(a) 4 (b) – 3 (c) 3 (d) – 4

6 In Δ ABC , if $(AB)^2 > (BC)^2 + (AC)^2$, then ∠ C is …………… angle.

(a) an acute (b) a right (c) an obtuse (d) a straight

2 [a] If the distance of the point $(x , 5)$ from the point (6 , 1) equals $2\sqrt{5}$ length unit , then find the value of x

[b] **Without using the calculator , find the numerical value of the expression :**

$\sin 45° \cos 45° + \sin 30° \cos 60° - \cos^2 30°$

3 [a] ABCD is a parallelogram where A (3 , 2) , B (4 , – 5) , C (0 , – 3)

Find the two coordinates of the point at which the two diagonals intersect , then find the coordinates of the point D

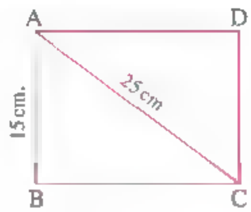
[b] ABC is a right-angled triangle at B in which AC = 10 cm. , BC = 8 cm.

Prove that : $\sin^2 A + 1 = 2 \cos^2 C + \cos^2 A$

4 [a] If the straight line L_1 passes through the two points (3 , 1) and (2 , k) and the straight line L_2 makes with the positive direction of the x-axis an angle of measure 45° , then find k if $L_1 // L_2$

[b] Find the equation of the straight line passing through the point (1 , 2) and perpendicular to the straight line $x + 3y + 7 = 0$

5 [a] **In the opposite figure :**



ABCD is a rectangle in which

AB = 15 cm. and AC = 25 cm.

Find : 1 m (∠ ACB)

2 The surface area of the rectangle ABCD

[b] Find the equation of the straight line which intersects from the x and y axes two positive parts whose lengths are 4 and 9 length units respectively.

5 El-Sharkia Governorate

Answer the following questions : ***(Calculator is allowed)***

1 Choose the correct answer from those given :

1 If $\cos(X + 25°) = \frac{1}{2}$, X is the measure of an acute angle , then $X =$ .... °

(a) 20 (b) 35 (c) zero (d) 5

2 The straight line whose equation is $3y = 2X - 6$, its slope = ...

(a) 2 (b) $\frac{2}{3}$ (c) 6 (d) $\frac{3}{2}$

3 The equation of the straight line which passes through the origin point and makes with the positive direction of X-axis an angle of measure 60° is ....

(a) $X = 3y$ (b) $y = \sqrt{3}X + 2$ (c) $y = 3X$ (d) $y = \sqrt{3}X$

4 If ABC is a right-angled triangle at B and $\sin A = \frac{2}{7}$, then $\cos C =$ ....

(a) $\frac{2}{7}$ (b) $\frac{3}{7}$ (c) $\frac{4}{7}$ (d) $\frac{5}{7}$

5 The distance between the point $A(\sqrt{2}, 4)$ and the origin point equals .... length unit.

(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $3\sqrt{2}$ (d) $4\sqrt{2}$

6 If the slope of the straight line L_1 is $\frac{a}{5}$ and the slope of the straight line L_2 is $\frac{-b}{3}$ where $a, b \neq 0$ and $L_1 \perp L_2$, then $ab =$ ....

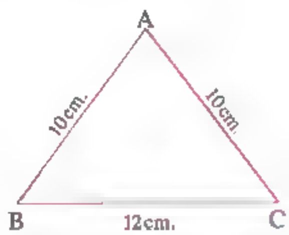
(a) $\frac{3}{5}$ (b) $\frac{-3}{5}$ (c) 15 (d) – 15

2 [a] **Without using the calculator , prove that :** $\frac{\sin 30° \sin 60°}{\sin 45° \cos 45°} = \cos 30°$

[b] Prove that the points A (3 , – 1) , B (– 4 , 6) , C (2 , – 2) which belong to an orthogonal Cartesian coordinates plane lie on the circle whose centre is M (– 1 , 2) , then find the circumference of the circle.

3 [a] If A (5 , 1) , B (3 , – 7) , C (1 , 3) are three noncollinear points , find the equation of the straight line which passes through the point A and is parallel to $\overleftrightarrow{BC}$

[b] **In the opposite figure :**



ABC is an isosceles triangle where

AB = AC = 10 cm. , BC = 12 cm.

Find : 1 sin B

2 The area of the triangle ABC

4 [a] If ABCD is a parallelogram , A (3 , 3) , B (2 , – 2) , C (5 , – 1)

, **find :** 1 The coordinates of the point of intersection of the two diagonals.

2 The coordinates of the point D

[b] Find the equation of the straight line which passes through the two points (4 , 5) , (0 , 3) , then find the coordinates of the intersection point of the straight line with X-axis.

5 [a] If $\cos X = \sin 30° \cos 60°$

, **find :** 1 The measure of angle X (where X is an acute angle).

2 $\tan X$

[b] Find the equation of the straight line which cuts 3 units from the positive part of y-axis and is perpendicular to the straight line $\frac{X}{2} + \frac{y}{3} = 1$

6 El-Monofia Governorate

Answer the following questions : ***(Using calculator is permitted)***

1 Choose the correct answer :

1 If $\cos (X + 15)° = \frac{1}{2}$, then $\sin (75 - X)° =$ ....

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 1

2 A circle is drawn inside a square where the circle touches its four sides. If the perimeter of the square is 56 cm. , then the surface area of the circle is .... cm^2.

(a) $\frac{77}{2}$ (b) 77 (c) 112 (d) 154

3 The number of sides of the regular polygon in which the measure of one of its interior angles is 144° equals .... sides.

(a) 7 (b) 8 (c) 9 (d) 10

4 An isosceles triangle , the lengths of its sides may be 4 cm. , 9 cm. , ... cm.

(a) 4 (b) 9 (c) 13 (d) 36

5 The distance between the point (– 2 , – 3) and X-axis equals .... length units.

(a) 2 (b) 3 (c) – 2 (d) – 3

6 The equation of the straight line which its slope = $\frac{1}{2}$ and cuts the y-axis at the point (0 , 3) is ....

(a) $2y = \frac{1}{2}X + 6$ (b) $y = \frac{1}{2}X$

(c) $y = \frac{1}{2}X + 3$ (d) $2y = \frac{1}{2}X + 3$

2 [a] **Without using calculator , find the numerical value of the expression :**

$\sin 30° \cos 60° + \cos 30° \sin 60° - \tan^2 45°$

[b] $\overline{AB}$ is a diameter in circle M , if A (7 , – 3) and B (5 , 1) where $\pi = 3.14$, **find :**

1 The surface area of the circle.

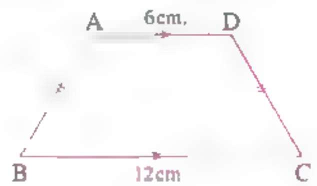
2 The coordinates of the centre of circle M

3 [a] ABC is a right-angled triangle at A , AB = 5 cm. and BC = 13 cm.

Find the numerical value of the expression : sin C cos B + cos C sin B

[b] Find the equation of the straight line which passes through the point (1 , 3) and is perpendicular to the straight line passing through the two points (5 , 0) and (2 , 1)

4 [a] **In the opposite figure :**



ABCD is an isosceles trapezium , its area = 36 $cm.^2$

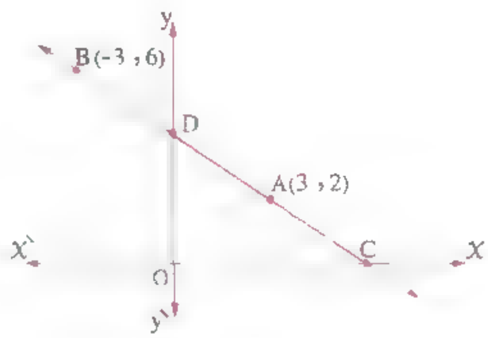
, $\overline{AD}$ // $\overline{BC}$, AD = 6 cm. and BC = 12 cm.

Find the value of : sin B + cos C

[b] Show the type of the triangle ABC according to its angles measures if its vertices are A (– 1 , 3) , B (5 , 1) and C (6 , 4)

5 [a] Find the slope of the straight line and the length of the intercepted part from y-axis where its equation is $4x + 5y - 10 = 0$

[b] **In the opposite figure :**



$\overleftrightarrow{CD}$ passes through the two points A (3 , 2) , B (– 3 , 6) and cuts the two axes at C and D respectively.

Find with the proof :

1 The equation of $\overleftrightarrow{CD}$

2 The area of the triangle DOC where O is the origin point.

7 El-Gharbia Governorate

Answer the following questions : (Calculator is allowed)

1 **Choose the correct answer :**

1 The perpendicular distance between the two straight lines y – 4 = 0 and y + 5 = 0 equals .... length units.

(a) 1 (b) 5 (c) 9 (d) 4

2 The equation of the straight line passing through the point $(3 , -2)$ and parallel to X-axis is ....

(a) $X = 3$ (b) $y = 2$ (c) $y = -2$ (d) $X + y = 1$

3 If the straight line whose equation is $y = k X + 1$ is parallel to the straight line whose equation is $2y - X = 0$, then $k =$ ....

(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) -2

4 If the lengths 3 , 7 , ℓ are lengths of sides of a triangle , then ℓ can be equal to ....

(a) 3 (b) 7 (c) 4 (d) 10

5 The image of the point $(-3 , 5)$ by reflection on the y-axis is ....

(a) $(3 , 5)$ (b) $(5 , 3)$ (c) $(-5 , 3)$ (d) $(-3 , -5)$

6 If ABC is a right-angled triangle at B , then $\frac{\sin A}{\cos C} =$ ....

(a) $\frac{3}{5}$ (b) $\frac{4}{3}$ (c) $\frac{3}{4}$ (d) 1

---

2 [a] If $\tan X = 4 \cos 60° \sin 30°$, then find the value of X where X is the measure of an acute angle.

[b] If the triangle XYZ whose vertices are X $(3 , 5)$, Y $(4 , 2)$, Z $(-5 , a)$ is a right-angled triangle at Y

, **find :** 1 The value of a

2 The surface area of the triangle XYZ

---

3 [a] If the ratio between the two measures of two supplementary angles is 3 : 5 , find the degree measure for each of them by degrees and minutes.

[b] Find the equation of the straight line passing through the point $(-1 , 2)$ and perpendicular to the straight line $X + y = 5$

---

4 [a] Prove that the points A $(3 , -1)$, B $(-4 , 6)$, C $(2 , -2)$ which belong to an orthogonal Cartesian coordinates plane lie on one circle whose centre is the point M $(-1 , 2)$, then find the circumference in terms of π

[b] ABCD is a trapezium in which $\overline{AD} // \overline{BC}$, m $(\angle B) = 90°$, AB = 3 cm. , AD = 6 cm. , BC = 10 cm. **Find the value of :** $\cos(\angle DCB) - \tan(\angle ACB)$

---

5 [a] ABCD is a parallelogram in which A $(3 , 2)$, B $(4 , -5)$, C $(0 , -3)$

Find : 1 The coordinates of the intersection point of the two diagonals.

2 The coordinates of the vertex point D

[b] In the opposite figure :

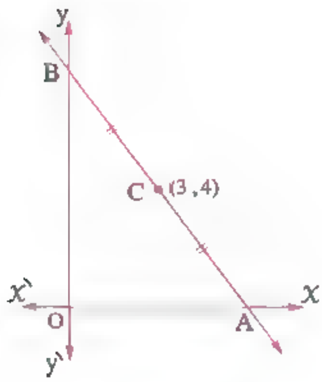
The point C is the midpoint of $\overline{AB}$

where C (3 , 4) , O is the origin point

in the perpendicular coordinate system.

Find : 1 The coordinates of the two points A and B

2 The equation of $\overleftrightarrow{AB}$



8 El-Dakahlia Governorate

Answer the following questions : ***(Calculator is permitted)***

1 [a] **Choose the correct answer from those given :**

1 ABC is a triangle , m (∠ A) = 85° , sin B = cos B , then m (∠ C) = ...

(a) 30° (b) 45° (c) 50° (d) 60°

2 The area of the triangle bounded by the straight lines $x = 0$, $y = 0$

, $3x + 2y = 12$ equals .... square units.

(a) 6 (b) 12 (c) 4 (d) 5

3 If the straight line passing through the two points (1 , y) , (3 , 4) its slope equals tan 45° , then y = ...

(a) 1 (b) 2 (c) – 1 (d) 4

[b] ABCD is an isosceles trapezium such that $\overline{AD} \mathbin{/\!/} \overline{BC}$, AD = 4 cm.

, AB = 5 cm. , BC = 12 cm. **Find the value of :** $\dfrac{\tan B \times \cos C}{\sin^2 C + \cos^2 B}$

2 [a] **Choose the correct answer from those given :**

1 The straight line $a\,x + (2 - a)\,y = 5$ is parallel to the straight line passing through the two points (1 , 4) , (3 , 5) , then a = ...

(a) 3 (b) – 2 (c) 6 (d) 4

2 ABC is a triangle , 2 m (∠ C) = m (∠ A) + m (∠ B) , then m (∠ C) = ...°

(a) 30 (b) 60 (c) 45 (d) 90

3 The straight line $\frac{x}{2} - \frac{y}{3} = 6$ cuts the x-axis at a part with length .... units.

(a) 3 (b) 2 (c) 6 (d) 12

[b] $\overline{AB}$ is a diameter of circle M , B (8 , 11) , M (5 , 7) **Find :**

1 The circumference of the circle.

2 The equation of the straight line perpendicular to $\overline{AB}$ from point A

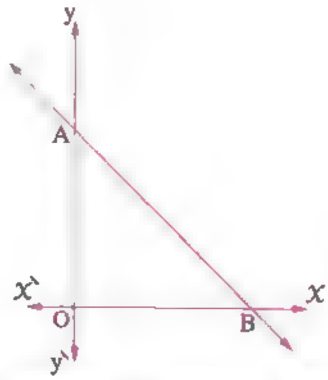
3 [a] **Prove that the quadrilateral ABCD whose vertices are :**

A (– 1 , 3) , B (5 , 1) , C (7 , 4) , D (1 , 6) is a parallelogram.

[b] The opposite figure represents the straight line $\overleftrightarrow{AB}$ whose equation is $y = k\,x + c$ and cuts the two axes with two equal parts and passes through the point (2 , 3) **Find :**

1 The values of k , c

2 The area of the triangle ABO



4 [a] **In the opposite figure :**

The straight line $\overleftrightarrow{AB}$ is parallel to y-axis.

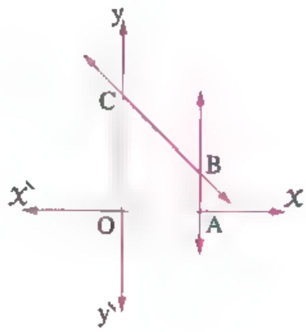
The straight line $\overleftrightarrow{BC}$ its equation is $y = -x + 3$

, the point B (2 , 1) **Find :**

1 The length of $\overline{BC}$

2 The area of the figure OABC

3 m (∠ OCB)



[b] ABC is a right-angled triangle at B

1 **Prove that :** $\sin^2 A + \cos^2 A = 1$

2 If AB = 5 cm. , AC = 13 cm. , **find :** m (∠ C) to the nearest minute.

5 [a] Find the equation of the straight line passing through the point (3 , 4) and makes with the positive direction of x-axis an angle of measure 135°

[b] **Without using calculator , prove that :**

$\tan^2 60° - \tan^2 45° = \sin^2 60° + \cos^2 60° + 2 \sin 30°$

9 Ismailia Governorate

Answer the following questions : *(Calculator is allowed)*

1 Choose the correct answer from those given :

1. The number of axes of symmetry of the scalene triangle equals ....
   (a) zero (b) 1 (c) 2 (d) 3

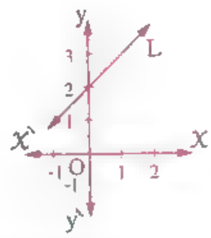
2. The midpoint of $\overline{AB}$ where A (6 , 0) , B (0 , 4) is ....
   (a) (6 , 4) (b) (4 , 6) (c) (3 , 2) (d) (2 , 3)

3. If the lengths of two sides of a triangle are 3 cm. and 4 cm. , then the length of the third side may be .... cm.
   (a) 1 (b) 6 (c) 7 (d) 8

4. If $\tan 2X = \frac{1}{\sqrt{3}}$ where $2X$ is the measure of an acute angle , then $X =$ .... °
   (a) 15 (b) 30 (c) 45 (d) 60

5. When you stand in front of the mirror and see your image , this is called in mathematics ....
   (a) rotation. (b) translation. (c) reflection. (d) similarity.

6. **In the opposite figure :**
   Which of the following represents the equation of the straight line L ?
   (a) $y = X$
   (b) $y = 2$
   (c) $y + X = 2$
   (d) $y - X = 2$



2 [a] **Without using the calculator , find the value of X if :**
$X \cos^2 30° = \tan^2 60° \cos^2 45°$

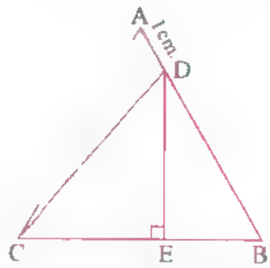
[b] If A (5 , − 1) , B (3 , 7) , C (1 , − 3) , find the equation of the straight line which passes through the midpoint of $\overline{BC}$ and the point A

3 [a] Prove that the points A (1 , − 2) , B (− 4 , 2) , C (1 , 6) are the vertices of an isosceles triangle.

[b] ABC is a right-angled triangle at B , find the value of : $\frac{\sin A}{\cos C}$ and if $\tan D = \frac{\sin A}{\cos C}$ where D is an acute angle , **find :** m (∠ D)

4 [a] If the straight line L_1 passes through the two points $(k , 1)$, $(2 , 4)$ and the straight line L_2 makes with the positive direction of X-axis an angle of measure 45°
, find the value of k if the two straight lines are parallel.

[b] **In the opposite figure :**

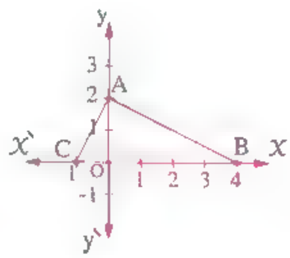


ABC is an equilateral triangle of side length 5 cm.
, $D \in \overline{AB}$ where AD = 1 cm. , $\overline{DE} \perp \overline{BC}$

Find : tan (∠ DCE)

5 [a] If ABCD is a rhombus where A (3 , 3) , C (– 3 , – 3)
, **find :** 1 The intersection point of the diagonals.
2 The equation of $\overleftrightarrow{BD}$

[b] **In the opposite figure :**



A triangle ABC is drawn in the orthogonal Cartesian coordinates plane.

Prove that : Δ ABC is a right-angled triangle and find its area.

10 Suez Governorate

Answer the following questions : (Calculator is allowed)

1 **Choose the correct answer from those given :**

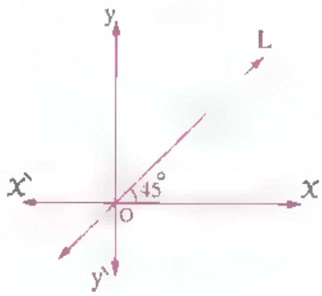
1 $\sin^2 60° + \cos^2 60° =$ ....

(a) 0 (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) 1

2 ABCD is a parallelogram in which m (∠ A) + m (∠ C) = 200°
, then m (∠ B) = ... °

(a) 80 (b) 50 (c) 100 (d) 160

3 **In the figure opposite :**



The equation of the straight line L is ....

(a) $X = 1$

(b) $y = - X$

(c) $y = X$

(d) $y = 1$

4 If a , b are the measures of two complementary angles

where a : b = 1 : 2 , then b = ...°

(a) 180 (b) 90 (c) 30 (d) 60

5 The perpendicular distance between the straight lines

$x - 2 = 0$, $x + 3 = 0$ equals .... length units.

(a) 1 (b) 5 (c) 2 (d) 3

6 If A (0 , 0) , B (5 , 7) , C (5 , h) are the vertices of a right-angled triangle at C

, then h = ...

(a) 0 (b) 5 (c) 7 (d) – 5

2 [a] **Without using calculator , prove that :**

$2 \sin 30° + 4 \cos 60° = \tan^2 60°$

[b] If A (– 1 , – 1) , B (2 , 3) , C (6 , 0) , D (3 , – 4) are four points on an orthogonal Cartesian coordinates plane

, **prove that :** $\overline{AC}$ and $\overline{BD}$ bisect each other.

3 [a] If $\cos 3x = \frac{\sin 60° \sin 30°}{\tan 45° \sin^2 45°}$, find the value of x where $3x$ is an acute angle.

[b] Find the equation of the straight line passing through the point (1 , 2) and perpendicular to the straight line passing through the two points A (2 , – 3) , B (5 , – 4)

4 [a] ABC is a right-angled triangle at C where AB = 5 cm. , BC = 4 cm.

Prove that : sin A cos B + cos A sin B = 1

[b] Find the equation of the straight line whose slope is equal to the slope of the straight line

$\frac{y-1}{x} = \frac{1}{3}$ and intersects a part from the negative direction of y-axis of length 3 units.

5 [a] ABC is a triangle where A (0 , 0) , B (3 , 4) , C (– 4 , 3)

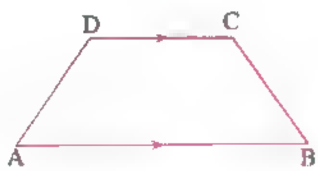
Find the perimeter of Δ ABC

[b] **In the opposite figure :**

ABCD is a trapezoid , $\overline{AB} // \overline{CD}$

, A (9 , – 2) , B (3 , 2) , C (– x , – x) , D (4 , – 3)

Find the coordinates of the point C



11 Port Said Governorate

Answer the following questions

1 Choose the correct answer from those given :

1 If $\frac{-2}{3}$, $\frac{k}{6}$ are the slopes of two perpendicular straight lines , then k = ...

(a) 9 (b) 4 (c) – 9 (d) – 4

2 The distance between the two points (15 , 0) , (6 , 0) equals .... unit length.

(a) – 9 (b) 9 (c) 3 (d) – 3

3 ABC is a right-angled triangle at C , AB = 25 cm. , AC = 15 cm.

, then the area of the surface of the triangle ABC is .... cm^2.

(a) 300 (b) 75 (c) 150 (d) 375

4 If $\overleftrightarrow{CD}$ is parallel to the y-axis where C (m , 4) , D (– 5 , 7) , then m = ...

(a) 5 (b) – 5 (c) – 7 (d) 7

5 If the point of the origin is the midpoint of $\overline{AB}$, where A (5 , – 2) , then the point B is ....

(a) (2 , 5) (b) (5 , – 2) (c) (– 2 , – 5) (d) (– 5 , 2)

6 If tan (X + 10) = $\sqrt{3}$ where X is the measure of an acute angle , then X = ...

(a) 40° (b) 50° (c) 60° (d) 70°

2 [a] Prove that the straight line which passes through the points (– 1 , 3) , (2 , 4) is parallel to the straight line 3 y – X – 1 = 0

[b] Without using calculator , prove that :

sin 90° = sin 60° cos 30° + cos 60° sin 30°

3 [a] If cos E = $\frac{\cos^2 45°}{\tan 30°}$, find m (∠ E) , E is an acute angle.

[b] Prove that the points A (– 3 , 0) , B (3 , 4) , C (1 , – 6) are the vertices of an isosceles triangle.

4 [a] Find the equation of the straight line whose slope is equal to the slope of the straight line $\frac{y-1}{X} = \frac{1}{3}$ and intercepts a negative part from the y-axis that is equal to 3 units.

[b] ABCD is a quadrilateral , where A (2 , 3) , B (6 , 2) , C (– 2 , – 2) , D (– 2 , 1)
Prove that the figure ABCD is a trapezoid.

5 [a] If A $(5, -6)$, B $(3, 7)$ and C $(1, -3)$, then find the equation of the straight line passing through the point A and the midpoint of $\overline{BC}$

[b] XYZ is a right-angled triangle at Y , where XY = 5 cm. , XZ = 13 cm.
, **find the value of :** $\sin X \cos Z + \cos X \sin Z$

12 Damietta Governorate

Answer the following questions : ***(Calculator is allowed)***

1 Choose the correct answer from the given answers :

1 The complement of the angle whose measure is 40° is of measure ....

(a) 50° (b) 80° (c) 90° (d) 140°

2 If D $(6, -4)$ is the midpoint of $\overline{AB}$ where A $(5, -3)$, then B is ....

(a) $(-5, 7)$ (b) $(5, 7)$ (c) $(7, 5)$ (d) $(7, -5)$

3 The length of the radius of the circle of centre $(0, 0)$ and passes through $(3, 4)$ equals .... length units.

(a) 7 (b) 1 (c) 12 (d) 5

4 The slope of the straight line $X - 5 = 0$ is ....

(a) 5 (b) $\frac{1}{5}$ (c) undefined. (d) zero

5 If $\tan(X + 10) = 1$, X is the measure of an acute angle , then $X =$ ....

(a) 45° (b) 35° (c) 80° (d) 50°

6 The perpendicular distance between the two straight lines $X - 3 = 0$, $X + 4 = 0$ equals .... length units.

(a) 1 (b) 5 (c) 2 (d) 7

2 [a] Find the equation of the straight line which passes through the points $(5, 0)$, $(0, 5)$

[b] ABC is a right-angled triangle at B where AB = 7 cm. , AC = 25 cm.

Find the value of the following : $\sin^2 A + \sin^2 C$

3 [a] If the points $(0, 1)$, $(a, 3)$, $(2, 5)$ are located on one straight line
, then find the value of a

[b] Find the equation of the straight line passing through the point $(3, 7)$ and parallel to the straight line $X + 3y + 5 = 0$

4 [a] **Without using the calculator , find the value of X (Where X is the measure of an acute angle) which satisfies that :** $2 \sin X = \sin 30° \cos 60° + \cos 30° \sin 60°$

[b] Find the equation of the straight line whose slope is 2 and intersects a positive part from the y-axis that equals 7 units.

5 [a] **Prove the following equality :** $\tan 60° = \frac{2 \tan 30°}{1 - \tan^2 30°}$

[b] State the kind of the triangle whose vertices are the points A (- 2 , 4) , B (3 , 1) , C (4 , 5) with respect to its sides lenghts.

13 Kafr El-Sheikh Governorate

Answer the following questions : (Calculator is allowed)

1 **Choose the correct answer :**

1 The measure of an exterior angle of the equilateral triangle equals ....

(a) 60° (b) 150° (c) 120° (d) 30°

2 If $\frac{-2}{3}$, $\frac{6}{k}$ are the slopes of two perpendicular straight lines , then k = ...

(a) 4 (b) – 9 (c) – 4 (d) 9

3 If ABCD is a square , then m (∠ CAB) = ...

(a) 90° (b) 45° (c) 60° (d) 630°

4 If $\sin \frac{X}{3} = \frac{1}{2}$, $\frac{X}{3}$ is the measure of an acute angle , then X = ...

(a) 30° (b) 60° (c) 10° (d) 90°

5 The parallelogram whose two diagonals are equal in length and not perpendicular is called a ....

(a) square. (b) rhombus. (c) rectangle. (d) trapezium.

6 The equation of the straight line which passes through the point (2 , 3) and is parallel to X-axis is ....

(a) $X = 2$ (b) $y = 3$ (c) $X = -2$ (d) $y = -3$

2 [a] Show the type of the triangle whose vertices are A (3 , 0) , B (1 , 4) , C (– 1 , 2) due to its side lengths.

[b] **Without using calculator , find the value of the following :**

$\sin^2 45° \cos 60° + \frac{1}{2} \tan 60° \sin 60°$

3 **[a]** If the straight line $L_1 : y = (2 - k) X + 5$ and the straight line L_2 makes with the positive direction of the X-axis an angle of measure 45° , find the value of k if $L_1 \parallel L_2$

[b] If $\sqrt{3} \tan X = 4 \sin 60° \cos 30°$, **find :** X , where X is the measure of an acute angle.

4 **[a]** If the distance between the point $(X , 3)$ and the point $(2 , 5)$ equals $2\sqrt{2}$ length units , then find the values of X

[b] Find the equation of the straight line whose slope is 3 and passes through the point $(5 , -2)$

5 **[a]** If the midpoint of $\overline{BC}$ is A (2 , 3) , and C (−1 , 3) , find the point B

[b] ABC is a right-angled triangle at B , sin A + cos C = 1 , **find :** m (∠ A)

14 El-Beheira Governorate

Answer the following questions : ***(Calculator is permitted)***

1 **Choose the correct answer from the given ones :**

1 If the point of origin is the midpoint of $\overline{AB}$, where A (5 , − 2) , then the point B is ....

(a) (− 5 , − 2) (b) (5 , 2) (c) (− 5 , 2) (d) (0 , 0)

2 The angle of measure 50° is complementary with an angle of measure ....

(a) 50° (b) 40° (c) 30° (d) 130°

3 A circle its centre is (3 , − 4) and its radius length is 5 units. Which of the following points belongs to the circle ?

(a) (− 3 , 4) (b) (0 , 0) (c) (5 , 0) (d) (0 , 4)

4 If $\cos \frac{X}{2} = \frac{1}{2}$ where $\frac{X}{2}$ is the measure of an acute angle , then X = ...

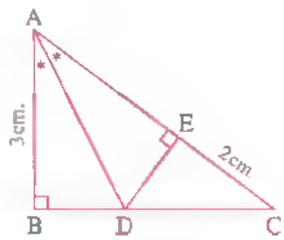
(a) 60° (b) 120° (c) 180° (d) 90°

5 If ABCD is a parallelogram in which m (∠ A) + m (∠ C) = 220° , then m (∠ B) = ...

(a) 110° (b) 70° (c) 140° (d) 80°

6 **In the figure opposite :**

ABC is a right-angled triangle at B , $\overrightarrow{AD}$ bisects ∠ A , $\overline{DE} \perp \overline{AC}$, AB = 3 cm. , CE = 2 cm. , then CB = ... cm.



(a) 2 (b) 3 (c) 4 (d) 5

2 [a] Prove that the straight line which passes through the two points $(-1 , 3)$, $(2 , 4)$ is parallel to the straight line $3y - x - 1 = 0$

[b] ABCD is a trapezium , $\overline{AD} \parallel \overline{BC}$, $m(\angle B) = 90°$, $AB = 3$ cm. , $BC = 6$ cm. , $AD = 2$ cm. Find the length of $\overline{DC}$ and the value of $\cos(\angle BCD)$

3 [a] Find the equation of the straight line whose slope is 3 and passes through the point $(1 , 2)$

[b] Without using the calculator , find the value of x (Where x is the measure of an acute angle) which satisfies that :

$2 \sin x = \tan^2 60° - 2 \tan 45°$

4 [a] If the straight line L_1 passes through the two points $(3 , 1)$, $(2 , k)$ and the straight line L_2 makes with the positive direction of the x-axis an angle of measure $45°$, then find k if the two straight lines L_1 , L_2 are perpendicular.

[b] ABC is a right-angled triangle at B , if $\sqrt{2}\,AB = AC$, find the main trigonometric ratios of the angle C

5 [a] If $A(x , 3)$, $B(3 , 2)$, $C(5 , 1)$ and $AB = BC$, $B \notin \overleftrightarrow{AC}$, then find the value of x

[b] Prove that the points $A(6 , 0)$, $B(2 , -4)$, $C(-4 , 2)$ are the vertices of a right-angled triangle at B , then find the coordinates of the point D that makes the figure ABCD a rectangle.

15 El-Fayoum Governorate

Answer the following questions : (Using calculators is allowed)

1 **Choose the correct answer :**

1. The perpendicular distance between the two straight lines $x - 2 = 0$ and $x + 3 = 0$ equals .... length units.

(a) 1 (b) 5 (c) 2 (d) 3

2. The sum of the measures of the accumulative angles at a point is ....

(a) 90° (b) 180° (c) 270° (d) 360°

3. If $\tan(x + 10) = \sqrt{3}$, where x is the measure of an acute angle , then $x =$ ....

(a) 60° (b) 30° (c) 50° (d) 70°

4. The polygon in which the number of its sides is equal to the number of its diagonals is the ....

(a) quadrilateral. (b) triangle. (c) pentagon. (d) hexagon.

5 A circle of centre at the origin point and its radius length is 2 length units. Which of the following points belongs to the circle ?

(a) $(1 , -2)$ (b) $(-2 , \sqrt{5})$ (c) $(\sqrt{3} , 1)$ (d) $(0 , 1)$

6 The square which the length of its diagonal is $8\sqrt{2}$ cm. , its area equals .... cm^2

(a) 4 (b) 32 (c) 64 (d) 16

2 [a] Prove that the points A $(3 , -1)$, B $(-4 , 6)$, C $(2 , -2)$ which belong to an orthogonal Cartesian coordinates plane lie on the circle whose centre is M $(-1 , 2)$, and find the circumference of the circle where $\pi = 3.14$

[b] **Without using calculator , prove that :**

$\tan^2 60° - \tan^2 45° = \sin^2 60° + \cos^2 60° + 2 \sin 30°$

3 [a] Find the equation of the straight line perpendicular to $\overline{AB}$ from its midpoint where A $(1 , 3)$ and B $(3 , 5)$

[b] ABC is a right-angled triangle at B , where AC = 5 cm. , BC = 4 cm. , **find the value of :** $2 \cos^2 C + \sin^2 A$

4 [a] Prove that the points A $(3 , -2)$, B $(-5 , 0)$, C $(0 , -7)$, D $(8 , -9)$ are the vertices of a parallelogram.

[b] **Find the value of X where :** $4X = \cos^2 30° \tan^2 30° \tan^2 45°$

5 [a] If the two straight lines $3X - 4y - 3 = 0$ and $ky + 4X - 8 = 0$ are both perpendicular , then find the value of k

[b] Find the equation of the straight line which intercepts from the two axes , two positive parts of length 1 and 4 from X and y axes respectively.

16 Beni Suef Governorate

Answer the following questions : (Calculator is allowed)

1 **Choose the correct answer from those given :**

1 $4 \sin 60° \tan 60° =$ ....

(a) 3 (b) 6 (c) 12 (d) $2\sqrt{3}$

2 The image of the point $(4 , 5)$ by the translation $(2 , 3)$ is ....

(a) $(6 , -8)$ (b) $(-8 , 6)$ (c) $(6 , 8)$ (d) $(-6 , -8)$

3 The perpendicular distance between the two straight lines $x - 2 = 0$, $x + 3 = 0$ equals ………… length units.

(a) 1 (b) 2 (c) 4 (d) 5

4 The equation of the straight line which passes through the point $(-5 , 3)$ and is parallel to y-axis is ……………

(a) $x = -5$ (b) $y = -5$ (c) $y = 3$ (d) $x = 3$

5 The number of the axes of symmetry of the circle is ………

(a) zero (b) 1 (c) 2 (d) an infinite number.

6 The points $(0 , 0)$, $(0 , 6)$ and $(8 , 0)$ ………

(a) form an acute-angled triangle. (b) form a right-angled triangle.

(c) form an obtuse-angled triangle. (d) are collinear.

2 [a] If the point C $(6 , -4)$ is the midpoint of $\overline{AB}$ where A $(5 , -3)$, find the coordinates of the point B

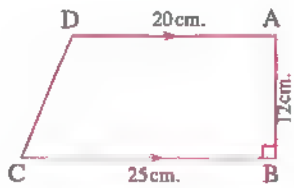
[b] **In the opposite figure :**

ABCD is a trapezium in which

$\overline{AD} \parallel \overline{BC}$, m $(\angle B) = 90°$

, AD = 20 cm. , AB = 12 cm. and BC = 25 cm.

Find the length of $\overline{DC}$ and m $(\angle C)$



3 [a] **Prove that :** $\frac{1}{2} \sin 60° = \sin 30° \cos 30°$

[b] Find the equation of the straight line which passes through the point $(2 , 3)$ and its slope = 2

4 [a] If $\cos E \tan 30° = \sin^2 45°$

, find m $(\angle E)$ where E is an acute angle.

[b] Prove that the straight line which passes through the two points $(2 , -1)$ and $(6 , 3)$ is parallel to the straight line which makes a positive angle of measure 45° with the positive direction of x-axis.

5 [a] Prove that the points A $(3 , -1)$, B $(-4 , 6)$ and C $(2 , -2)$ are located on a circle whose centre is M $(-1 , 2)$

[b] Find the slope of the straight line $3y - 2x + 5 = 0$, then find the length of the intersected part from the y-axis.

17 El-Menia Governorate

Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

1 The angle whose measure is 65° complements an angle of measure .... °

(a) 35 (b) 25 (c) 115 (d) 45

2 ABCD is a parallelogram. If m (∠ A) + m (∠ C) = 200° , then m (∠ B) = ... °

(a) 50 (b) 80 (c) 100 (d) 160

3 The sum of lengths of any two sides in a triangle is .... the length of the third side.

(a) less than (b) equal to (c) greater than (d) twice

4 If sin X = $\frac{1}{2}$, then m (∠ X) = ...° , X is an acute angle.

(a) 45 (b) 60 (c) 90 (d) 30

5 The distance between the two points (3 , 0) , (0 , – 4) equals .... length units.

(a) 4 (b) 5 (c) 6 (d) 7

6 If $X + y = 5$, $k X + 2 y = 0$ are two parallel straight lines , then k = ...

(a) – 2 (b) – 1 (c) 1 (d) 2

2 [a] **Without using calculator , find the value of the expression :**

$\cos 60° \sin 30° - \sin 60° \tan 60° + \cos^2 30°$

[b] Find the equation of the straight line which passes through the point (1 , 2) and is perpendicular to the straight line which passes through the two points A (2 , – 3) , B (5 , – 4)

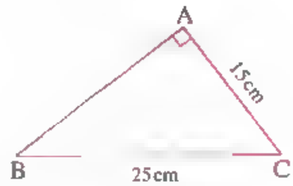
3 [a] **Without using calculator , find the value of X which satisfies :**

$2 \sin X = \tan^2 60° - 2 \tan 45°$ where X is the measure of an acute angle.

[b] In Δ ABC , m (∠ A) = 90° , AC = 15 cm. , BC = 25 cm.

Prove that :

cos C cos B – sin C sin B = zero



4 [a] Prove that the points A (– 1 , – 4) , B (1 , 0) and C (2 , 2) are collinear.

[b] If C (6 , – 4) is the midpoint of $\overline{AB}$ where A (5 , – 3) , find the coordinates of the point B

5 [a] Prove that the straight line that makes an angle of measure 45° with the positive direction of the X-axis is parallel to the straight line whose equation is $X - y - 1 =$ zero

[b] Find the value of a if the distance between the two points (a , 7) and (– 2 , 3) equals 5 length units.

18 Assiut Governorate

Answer the following questions : (Calculator is permitted)

1 **Choose the correct answer :**

1 The measure of the straight angle is ....°

(a) 90 (b) 360 (c) 180 (d) 240

2 If $\tan (X + 20)° = \sqrt{3}$ where $(X + 20)°$ is the measure of an acute angle , then $X =$ ......

(a) 30 (b) 60 (c) 90 (d) 40

3 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.

(a) $\frac{1}{4}$ (b) twice (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

4 If $X + y = 5$, $k X + 2 y = 7$ are perpendicular , then $k =$ ....

(a) – 2 (b) – 1 (c) 1 (d) 2

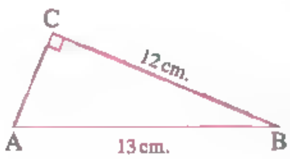
5 The area of the rhombus whose diagonals lengths are 6 cm. and 12 cm. is .... cm^2

(a) 16 (b) 30 (c) 36 (d) 72

6 The perpendicular distance between the two straight lines $X - 3 = 0$, $X + 4 = 0$ equals .... length units.

(a) 2 (b) 7 (c) 12 (d) 6

2 [a] **In the opposite figure :**



ABC is a right-angled triangle at C , AB = 13 cm.

, BC = 12 cm.

Prove that : $\sin A \cos B + \cos A \sin B = 1$

[b] Show the type of the triangle whose vertices are A (1 , 1) , B (5 , 1) , C (3 , 4) due to its side lengths.

3 [a] If $2 \sin X = \tan^2 60° - 4 \sin 30°$, find X , where X is the measure of an acute angle.

[b] ABCD is a parallelogram where A (3 , 2) , B (4 , – 5) , C (1 , 4) , find the two coordinates of the point at which the two diagonals intersect , then find the coordinates of the point D

4 [a] **Without using the calculator , find the value of :** $\cos 60° + \cos^2 30° + \tan^2 45°$

[b] Prove that the straight line passing through the two points $\left(2\sqrt{3} , 3\right)$, $\left(\sqrt{3} , 4\right)$ is perpendicular to the straight line that makes with the positive direction of the X-axis an angle of measure 60°

---

5 [a] Find the equation of the straight line passing through the point (3 , – 5) and parallel to the straight line $X + 3y = 7$

[b] Find the slope of the straight line and the length of the y-intercept by the straight line $\frac{y-1}{X} = \frac{1}{2}$

19 Souhag Governorate

Answer the following questions : ***(Calculator is permitted)***

1 **Choose the correct answer :**

1. The point of concurrence of the medians of the triangle divides each median in the ratio of .... from its base.

   (a) 2 : 3 (b) 2 : 1 (c) 1 : 2 (d) 3 : 2

2. If $\sin X = \cos X$, then $X = \cdots\cdots°$ (X is the measure of an acute angle)

   (a) 30 (b) 45 (c) 60 (d) 90

3. The sum of the measures of the accumulative angles at a point equals ....°

   (a) 30 (b) 60 (c) 180 (d) 360

4. The distance between the two points (3 , 0) , (– 1 , 0) equals .... length units.

   (a) 4 (b) 5 (c) 6 (d) 7

5. The side length of a square is $\sqrt{3}$ cm. , then its area = ... cm^2

   (a) $4\sqrt{3}$ (b) 9 (c) 3 (d) 6

6. If A (5 , – 3) , B (7 , – 5) , then the midpoint of $\overline{AB}$ is ....

   (a) (3 , 5) (b) (2 , 0) (c) (5 , – 5) (d) (6 , – 4)

---

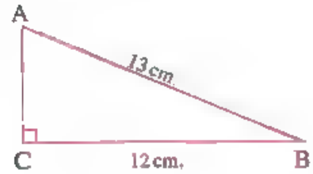
2 [a] If $\cos X = 2\cos^2 30° - 1$ (X is the measure of an acute angle) , **find :** X

[b] Prove that the triangle whose vertices are A (1 , 4) , B (– 1 , – 2) , C (2 , – 3) is right-angled at B

3 [a] **In the opposite figure :**

The triangle ABC is right-angled at C

, AB = 13 cm. , BC = 12 cm.



Find : 1 The length of $\overline{AC}$

2 The value of sin A cos B + cos A sin B

[b] Find the equation of the straight line whose slope equals 2 and passes through the point (1 , 0)

4 [a] **Without using the calculator , prove that :** $2 \sin 30° = \tan^2 60° - 2 \tan 45°$

[b] Find the equation of the straight line passing through the points (1 , 3) , (– 1 , – 3) , then prove that it passes through the origin point.

5 [a] Prove that the points A (– 3 , – 1) , B (6 , 5) , C (3 , 3) are collinear.

[b] Prove that the straight line passing through the two points (– 3 , – 2) , (4 , 5) is parallel to the straight line which makes with the positive direction of the X-axis an angle of measure 45°

20 Qena Governorate

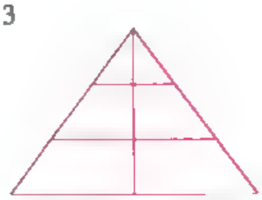
Answer the following questions :

1 **Choose the correct answer :**

1 If $\sin X = \frac{1}{2}$ where X is the measure of an acute angle , then $\sin 2X = \ldots\ldots$

(a) $\frac{1}{4}$ (b) $\frac{\sqrt{3}}{2}$ (c) 60 (d) $\frac{1}{\sqrt{3}}$

2 The number of quadrilaterals in the opposite figure is ....



(a) 3 (b) 6

(c) 9 (d) 12

3 If the two straight lines $X + y = 4$, $a X + 3 y = 0$ are perpendicular , then a = ...

(a) – 3 (b) – 1 (c) 1 (d) 3

4 The number of axes of symmetry of the rhombus equals ....

(a) 1 (b) 2 (c) 3 (d) 4

5 The straight line whose equation is $2 y = 3 X - 6$ intercepted a part equal .... units from y-axis.

(a) 6 (b) 2 (c) 3 (d) $\frac{3}{2}$

6 The image of the point $(-3 , 2)$ by reflection on the origin point is ....

(a) $(3 , 2)$ (b) $(3 , -2)$ (c) $(-3 , -2)$ (d) $(-3 , 2)$

2 [a] Δ ABC is a right-angled triangle at B , AC = 10 cm. , BC = 8 cm.

Prove that : $\sin^2 A + 1 = 2 \cos^2 C + \cos^2 A$

[b] Prove that the points A $(1 , 1)$, B $(0 , -1)$, C $(2 , 3)$ are collinear.

3 [a] If $\sin X \tan 30° = \sin^2 45°$, find the value of X in degrees , where X is the measure of an acute angle.

[b] Prove that the straight line passing through $(-1 , 3)$, $(2 , 4)$ is parallel to the straight line whose equation is $3y - X - 1 = 0$

4 [a] **Without using calculator , prove that :** $\sin 60° = 2 \sin 30° \cos 30°$

[b] ABCD is a quadrilateral in which :

A $(5 , 3)$, B $(6 , -2)$, C $(1 , -1)$, D $(0 , 4)$

Prove that : ABCD is a rhombus and find its area.

5 [a] Prove that the points A $(-3 , 0)$, B $(3 , 4)$, C $(1 , -6)$ are the vertices of an isosceles triangle its vertex A , then find the length of the perpendicular segment from A to $\overline{BC}$

[b] ABCD is a parallelogram in which A $(3 , 2)$, B $(4 , -5)$, C $(0 , -3)$

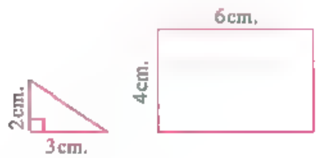
Find the coordinates of the point D

21 Luxor Governorate

Answer the following questions :

1 **Choose the correct answer :**

1 The number of the right triangles which completely cover the surface of the rectangle equals ....



(a) 10 (b) 8

(c) 6 (d) 4

2 If m $(\angle A) = 85°$ and $\sin B = \cos B$ in Δ ABC , then m $(\angle C) =$ ....°

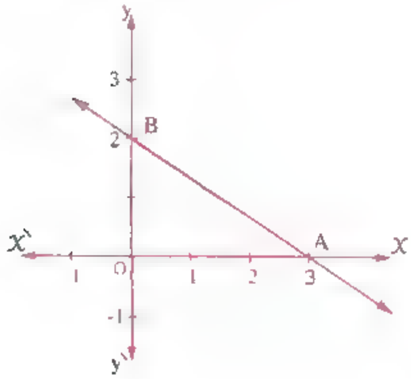
(a) 30 (b) 45 (c) 50 (d) 60

3 The image of the point $(-5 , 6)$ by translation $(3 , -2)$ is ....

(a) $(-4 , -2)$ (b) $(4 , 2)$ (c) $(-2 , 4)$ (d) $(-2 , -4)$

4 **In the opposite figure :**

The slope of $\overleftrightarrow{AB}$ equals ·····



(a) $\frac{2}{3}$

(b) $\frac{-2}{3}$

(c) $\frac{3}{2}$

(d) $\frac{-3}{2}$

5 The measure of the exterior angle at any vertex of an equilateral triangle equals ……………°

(a) 30 (b) 60 (c) 90 (d) 120

6 If C $(-3, y)$ is the midpoint of $\overline{AB}$ where A $(x, -6)$ and B $(9, -12)$, then $y - x =$ ……………

(a) 7 (b) 9 (c) 6 (d) -18

2 [a] If the distance between the two points $(a, 5)$, $(3a-1, 1)$ equals 5 length units, then find a

[b] If $3 \tan x - 4 \sin^2 30° = 8 \cos^2 60°$, find x where x is the measure of an acute angle.

3 [a] Find the equation of the straight line passing by $(1, 2)$ and parallel to the straight line $2x + 3y - 6 = 0$

[b] Find the measure of the angle made by the straight line passing by the two points $\left(-2, \sqrt{3}\right)$, $\left(1, 4\sqrt{3}\right)$ with the positive direction of the x-axis.

4 [a] $\overline{AB}$ is a diameter of the circle M where A $(4, -1)$, B $(-2, 7)$, find the radius length of the circle and find its area.

[b] ABC is a triangle where AB = AC = 10 cm. , BC = 12 cm.

, draw $\overline{AD} \perp \overline{BC}$ and intersects it at D **Prove that :**

1 $\sin^2 C + \cos^2 C = 1$

2 $\sin B + \cos C > 1$

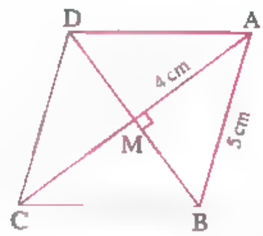
5 [a] If $\overleftrightarrow{AB}$ // the y-axis where A $(x, 7)$, B $(3, 5)$, find the value of x

[b] **In the opposite figure :**

ABCD is a rhombus, its two diagonals intersect at M, if AB = 5 cm. , AM = 4 cm., **find :**

1 m (∠ BAD)

2 The area of the rhombus ABCD



22 Aswan Governorate

Answer the following questions : ***(Calculator is allowed)***

1 Choose the correct answer from those given :

1. The angle with measure 65° is complement of an angle with measure ....

(a) 135° (b) 115° (c) 25° (d) 15°

2. If $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ and the slope of $\overleftrightarrow{AB} = \frac{1}{2}$, then the slope of $\overleftrightarrow{CD} =$ ....

(a) 2 (b) -2 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

3. If $C \in$ the axis of symmetry of $\overline{AB}$, then CA .... CB

(a) $\perp$ (b) < (c) > (d) =

4. If 3 cm. , 7 cm. and y are lengths of sides of a triangle , then y = ... cm.

(a) 3 (b) 4 (c) 7 (d) 10

5. The distance between the two points (6 , 0) and (0 , 8) equals .... length units.

(a) 6 (b) 8 (c) 10 (d) 14

6. If $\tan (X + 10) = \sqrt{3}$ where X is the measure of an acute angle , then $X =$ ....

(a) 80° (b) 50° (c) 35° (d) 20°

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2 [a] If $2 \sin X = \tan^2 60° - 2 \tan^2 45°$, find the value of X where X is the measure of an acute angle.

[b] Find the equation of the straight line which is perpendicular to $\overline{AB}$ from its midpoint where A (1 , 3) and B (3 , 5)

---

3 [a] If C (4 , 2) is the midpoint of $\overline{AB}$ where A (2 , 4) and B (6 , y) , find the value of y

[b] If the points A (– 1 , – 1) , B (2 , 3) , C (6 , 0) are the vertices of a triangle.
, **prove that :** Δ ABC is right-angled at B

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4 [a] XYZ is a right-angled triangle at Y , if XY = 5 cm. , XZ = 13 cm.
, **find :** 1 tan X × tan Z 2 cos X cos Z – sin X sin Z

[b] Find the equation of the straight line which intercepts from the positive parts of the coordinates axes two parts of lengths 1 and 4 from X and y axes respectively.

---

5 [a] Prove that the straight line which passes through the two points (– 1 , 3) and (2 , 4) is parallel to the straight line whose equation is $3y - X - 1 = 0$

[b] Δ ABC is a right-angled triangle at B , if $2 AB = \sqrt{3} AC$
, find the main trigonometric ratios of the angle C

23 New Valley Governorate

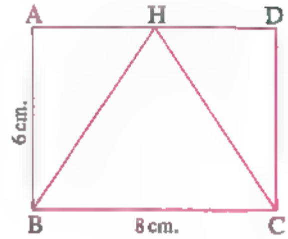
Answer the following questions : *(Calculator is allowed)*

1 **Choose the correct answer from those given :**

1 The quadrilateral ABCD in which AB > CD , $\overline{AB} \parallel \overline{CD}$ is ....

(a) a square. (b) a rectangle. (c) a rhombus. (d) a trapezium.

2 **In the opposite figure :**



ABCD is a rectangle , AB = 6 cm. , BC = 8 cm.

, H ∈ $\overline{AD}$, the area of Δ HBC = ... cm^2

(a) 14 (b) 24

(c) 28 (d) 48

3 For any angle A , $\frac{\sin A}{\cos A}$ = ...

(a) sin A (b) cos A (c) tan A (d) 1

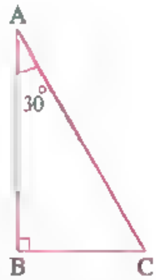
4 If ABCD is a rectangle , A (1 , 0) , C (4 , 4) , then BD = ... length units.

(a) 5 (b) 8 (c) 9 (d) 10

5 If $x + y = 5$ and $k x + 2 y = 1$ are perpendicular , then k = ...

(a) 2 (b) 1 (c) – 1 (d) – 2

6 **In the opposite figure :**



BC : AC : AB = ...

(a) $1 : \sqrt{3} : 2$

(b) $2 : \sqrt{3} : 1$

(c) $1 : 2 : \sqrt{3}$

(d) $\sqrt{3} : 1 : 2$

2 [a] XYZ is a right-angled triangle at Z , XZ = 3 cm. , YZ = 4 cm. **Find the value of :**

1 tan X tan Y 2 $\sin^2 X + \cos^2 X$

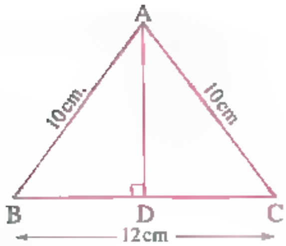
[b] Determine the type of the triangle whose vertices are A (3 , 3) , B (1 , 5) , C (1 , 3) according to its side lengths and according to its angles.

3 [a] If $\tan x = 4 \sin 30° \cos 60°$, x is the measure of an acute angle , **then find the value of each of :**

1 x 2 $\sin x$

[b] Find the equation of the straight line whose slope is 2 and passes through the point (1 , 0)

4 [a] **In the opposite figure :**



ABC is a triangle , AB = AC = 10 cm.

, BC = 12 cm. , $\overline{AD} \perp \overline{BC}$ **Find the value of :**

1 cos B   2 m (∠ B)   3 sin (90° – B)

[b] ABCD is a rhombus , A (– 2 , 3) , B (– 1 , – 2) , C (4 , – 3)

Find : 1 The coordinates of the point of intersection of its diagonals.

2 The coordinates of the point D

5 [a] If the straight line L_1 passes through the points (2 , 1) , (3 , k) and the straight line L_2 makes with the positive direction of the X-axis an angle of measure 45° , find the value of k , if $L_1 \parallel L_2$

[b] Find the equation of the straight line which intersects from the two axes two positive parts of lengths 2 and 4 from X and y axes respectively.

24 South Sinai Governorate

Answer the following questions :

1 **Choose the correct answer from those given :**

1 If $\cos(X + 15°) = \frac{1}{2}$, then $\tan X = \cdots$ where X is the measure of an acute angle.

(a) 1   (b) $\sqrt{3}$ (c) $\frac{\sqrt{3}}{3}$ (d) $\frac{1}{2}$

2 The distance between the two points (– 3 , 0) and (0 , – 4) equals .... length units.

(a) 4   (b) 5   (c) 3   (d) 2

3 If A = (– 4 , 5) and B = (– 2 , – 1) , then the midpoint of $\overline{AB}$ is ....

(a) (0 , 1)   (b) (– 3 , 3)   (c) (– 3 , 2)   (d) (1 , 0)

4 ABC is a triangle in which m (∠ A) = 120° , AB = AC , then m (∠ C) = ...

(a) 60°   (b) 45°   (c) 50°   (d) 30°

5 If $X + y = 5$ and $k X + 2y = 0$ are two straight lines perpendicular , then k = ...

(a) – 2   (b) 2   (c) – 1   (d) 1

6 ABC is a right-angled triangle at A and $\overline{AD} \perp \overline{BC}$, where $D \in \overline{BC}$, then $(AD)^2$ = ...

(a) BD × BC   (b) CD × CB   (c) DB × DC   (d) $(DB)^2 \times (DC)^2$

2 [a] **Without using calculator , prove that :** $\cos 60° = \cos^2 30° - \sin^2 30°$

[b] If the point D = (1 , – 3) is the midpoint of $\overline{AB}$, A = (4 , – 3)

, find the coordinates of the point B

3 [a] Find the equation of the straight line which passes through the points $(1 , 3)$ and $(-1 , -3)$

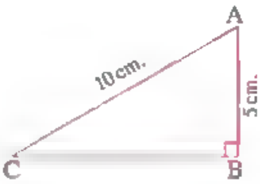
[b] Show the type of the triangle ABC whose vertices are $A = (3 , 3)$, $B = (1 , 5)$ and $C = (1 , 3)$ due to its side lengths.

4 [a] Find the equation of the straight line which passes through the point $(-2 , 3)$ and makes with the positive direction of the X-axis an angle of measure 45°

[b] **Find the value of :** $\frac{2 \tan 45°}{1 + \tan^2 45°}$

5 [a] Find the equation of the straight line which its slope is 2 , and intersects a positive part from y-axis that is equal to 5 units.

[b] **In the opposite figure :**



ABC is a triangle right-angled at B

, in which AC = 10 cm. , AB = 5 cm.

Find : 1 m (∠ C)   2 $\sin^2 C + \cos^2 C$

25 North Sinai Governorate

Answer the following questions :

1 **Choose the correct answer from those given :**

1 If $\sin X = \frac{1}{2}$ where X is the measure of an acute angle , then $X =$ ....

(a) 90°   (b) 60°   (c) 45°   (d) 30°

2 The measure of the exterior angle of the equilateral triangle equals ....

(a) 60°   (b) 90°   (c) 120°   (d) 180°

3 The slope of the straight line which makes with the positive direction of X-axis a positive angle of measure 45° equals ....

(a) 1   (b) -1 (c) zero   (d) 1.4

4 The angle whose measure is 40° complements an angle of measure ....

(a) 30°   (b) 140°   (c) 50°   (d) 40°

5 If $A (2 , -2)$, $B (-2 , 2)$, then the midpoint of $\overline{AB}$ is ....

(a) $(-1 , 1)$ (b) $(1 , -1)$ (c) $(4 , -4)$ (d) $(0 , 0)$

6 If 3 , 7 , ℓ are the lengths of the sides of a triangle , then ℓ can be equal to ....

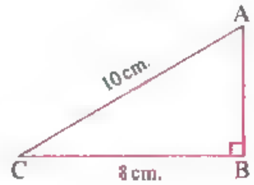
(a) 3   (b) 4   (c) 7   (d) 10

2 [a] **Prove that :** $\cos 60° = 2\cos^2 30° - 1$ (Without using the calculator)

[b] Prove that the triangle whose vertices are A (1 , – 2) , B (– 4 , 2) and C (1 , 6) is an isosceles triangle.

3 [a] Find the equation of the straight line whose slope = 2 and cuts 7 units from the positive part of y-axis.

[b] **In the opposite figure :**



ABC is a right-angled triangle at B in which AC = 10 cm.

, BC = 8 cm.

1 **Find the length of :** $\overline{AB}$

2 **Prove that :** $\sin^2 A + \cos^2 A = 1$

4 [a] If $\cos X = \dfrac{\sin 60° \sin 30°}{\sin^2 45°}$

, find the value of X where X is the measure of an acute angle. (Without using the calculator)

[b] Find the equation of the straight line passing through the point (1 , 2) and perpendicular to the straight line passing through the two points (2 , – 3) , (5 , – 4)

5 If A (3 , – 1) , B (– 4 , 6) , C (2 , – 2) and M (– 1 , 2) :

1 Prove that the points A , B , C lie on the circle whose centre is M

2 Find the circumference of the circle M ($\pi = 3.14$)

26 Red Sea Governorate

Answer the following questions :

1 **Choose the correct answer from those given :**

1 If A (5 , 7) , B (1 , – 1) , then the midpoint of $\overline{AB}$ is ....

(a) (2 , 3) (b) (3 , 3) (c) (3 , 2) (d) (3 , 4)

2 A rhombus whose diagonals lengths are 6 cm. , 8 cm. , then its area is .... cm^2

(a) 48 (b) 28 (c) 24 (d) 14

3 If $\cos X = \dfrac{\sqrt{3}}{2}$ where X is the measure of an acute angle , then $\sin 2X =$ ....

(a) $\dfrac{\sqrt{3}}{2}$ (b) 1 (c) – 2 (d) $\dfrac{1}{\sqrt{3}}$

4 If the lengths of two sides of an isosceles triangle are 5 cm. and 13 cm. , then the length of the third side is .... cm.

(a) 5 (b) 8 (c) 13 (d) 16

5 If the two straight lines $3X - 4y = 3$ and $4X + ky = 8$ are perpendicular , then $k =$ ....

(a) 4 (b) 3 (c) -4 (d) -3

6 The number of axes of symmetry of the equilateral triangle equals ......

(a) zero (b) 1 (c) 2 (d) 3

2 [a] **Without using calculator , prove that :** $\sin 60° = 2 \sin 30° \cos 30° \tan 45°$

[b] Find the equation of the straight line which passes through the two points $(4, 2)$, $(-2, -1)$

3 [a] Find the value of X if $\tan X = 4 \cos 60° \sin 30°$ where X is the measure of an acute angle.

[b] Prove that the points $A(2, 4)$, $B(-3, 0)$ and $C(-7, 5)$ are the vertices of a right-angled triangle , then find its area.

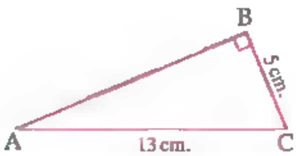
4 [a] Find the equation of the straight line which its slope is 2 and intercepts from the positive part of y-axis 7 length units.

[b] **In the opposite figure :**

ABC is a right-angled triangle at B

, AC = 13 cm. , BC = 5 cm.

Find the value of : $\sin A \cos C + \cos A \sin C$



5 [a] If the distance between the two points $(X, 7)$, $(-2, 3)$ equals 5 length units , find the value of X

[b] If the straight line L_1 passes through the two points $(3, 1)$, $(2, k)$ and the straight line L_2 makes with the positive direction of the X-axis a positive angle its measure is 45° , find the value of k if $L_1 // L_2$

27 Matrouh Governorate

Answer the following questions : *(Calculator is allowed)*

1 **Choose the correct answer from those given :**

1 If $\cos 2X = \frac{1}{2}$, then $m(\angle X) =$ ....

(a) 15° (b) 30° (c) 45° (d) 60°

2 The angle measured 37° is complemented by an angle of measurement ....

(a) 53° (b) 143° (c) 37° (d) 90°

3 If $\frac{-2}{3}$, $\frac{k}{2}$ are the slopes of two parallel straight lines , then k = ...

(a) $\frac{-4}{3}$ (b) $\frac{-3}{4}$ (c) 3 (d) $\frac{1}{3}$

4 The area of the circle equals ....

(a) π r (b) 2 π r (c) πr^2 (d) $2 \pi r^2$

5 In Δ ABC , AB + BC .... AC

(a) > (b) ≥ (c) < (d) ≤

6 If $\overline{AB}$ is a diameter of a circle , where A (3 , – 5) , B (5 , 1) , then the centre of the circle is ....

(a) (8 , – 2) (b) (4 , 2) (c) (2 , 2) (d) (4 , – 2)

2 [a] **Without using calculator , prove that :**

$$\tan 60° = \frac{2 \tan 30°}{1 - \tan^2 30°}$$

[b] Prove that the points A (6 , 0) , B (2 , – 4) , C (– 4 , 2) are the vertices of a right-angled triangle at B

3 [a] If the distance between the two points (a , 7) and (– 2 , 3) equals 5 length units , find the value of a

[b] ABC is a right-angled triangle at B , AB = 3 cm. , BC = 4 cm.

Find the value of : sin A cos C + cos A sin C

4 [a] If A , B are the measures of two complementary angles

, where A : B = 1 : 2

, **find :** sin A + cos B

[b] Find the slope and the intercepted part of y-axis of the straight line whose equation is $\frac{x}{2} + \frac{y}{2} = 1$

5 [a] If C is the midpoint of $\overline{AB}$, where A = (x , – 6) , B = (9 , – 12) and C = (– 3 , y) , find the values of x , y

[b] Find the equation of the straight line passing through the point (3 , – 5) and parallel to the straight line $x + 2y = 7$

4 [a] If the two straight lines $L_1 : 3X - 4y - 3 = 0$, $L_2 : a\,y + 4X - 8 = \text{zero}$ are perpendicular , find the value of a

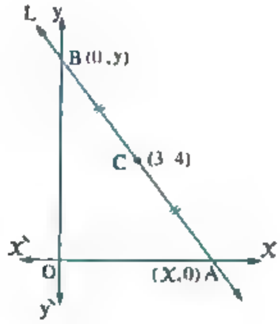
[b] If the points A (3 , 2) , B (4 , − 3) , C (− 1 , − 2) , D (− 2 , 3) are the vertices of a rhombus , find the area of the rhombus ABCD

5 [a] **Prove that :** $\cos^2 60° = \cos^2 30° \tan^2 30° \tan 45°$

[b] **In the opposite figure :**

The point C is the midpoint of $\overline{AB}$ where C (3 , 4)

Find the perimeter of the triangle AOB



2021 — 5 El-Sharkia Governorate

Answer the following questions : (Calculator is allowed)

1 **Choose the correct answer from those given :**

1. In Δ ABC , if m (∠ B) = 90° , then sin A + cos C = ...

(a) 2 sin C (b) 2 cos A (c) 2 cos C (d) tan A

2. If $\sin 2X = \frac{1}{2}$ where $2X$ is the measure of an acute angle , then $X =$ ....°

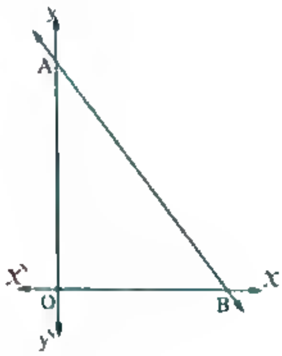
(a) 15 (b) 60 (c) 70 (d) 30

3. **In the opposite figure :**

If AO = 8 length units

, OB = 6 length units

, then the equation of $\overleftrightarrow{AB}$ is ....



(a) $y = \frac{4}{3}X + 8$ (b) $y = -\frac{4}{3}X - 8$

(c) $y = \frac{3}{4}X - 8$ (d) $y = -\frac{4}{3}X + 8$

4. The perpendicular distance between the point (3 , − 4) and X-axis equals .... length units.

(a) 3 (b) − 4 (c) 5 (d) 4

5 In the square XYZL , if the slope of $\overleftrightarrow{XZ} = 1$, then the slope of $\overleftrightarrow{YL} =$ ....

(a) 1 (b) -1 (c) ± 1 (d) $45°$

6 ABC is a right-angled triangle at B , where 3 AC = 5 BC , then tan A = ...

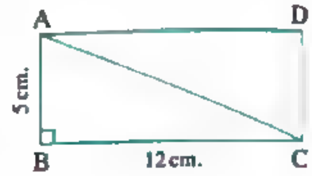
(a) $\frac{3}{5}$ (b) $\frac{5}{3}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

2 [a] If the point C (4 , y) is the midpoint of $\overline{AB}$ where A (X , 3) and B (6 , 5)

, find the value of : $X + y$

[b] Prove that the points A (5 , 3) , B (3 , −2) , C (−2 , −4) are the vertices of a triangle , then prove that the triangle is an obtuse-angled triangle at B

3 [a] **In the opposite figure :**



If ABCD is a rectangle in which AB = 5 cm. , BC = 12 cm.

, find : 1 The length of $\overline{AC}$

2 The value of : 5 tan (∠ ACD) − 13 sin (∠ DAC)

[b] If the two points A (3 , −1) , B (5 , 3)

, find the equation of the axis of symmetry of $\overline{AB}$

4 [a] **Without using the calculator , find the value of :** $\frac{\cos^2 60° + \cos^2 30°}{\sin 60° \tan 60°}$

[b] If the two equations of the two straight lines L_1 and L_2 are :

$L_1 : 6X + ky - 3 =$ zero and $L_2 : 3y = 2X + 6$ respectively.

, find the value of k which makes :

1 The two straight lines parallel. 2 The two straight lines perpendicular.

5 [a] Find the equation of the straight line which passes through the point (1 , 4) and is parallel to the straight line : $X + 2y - 4 =$ zero

[b] If ABCD is a square where : A (2 , 4) , B (−3 , zero) , C (−7 , 5)

, find : 1 The coordinates of the point D 2 the area of the square ABCD

6 El-Monofia Governorate

Answer the following questions : (Using calculator is permitted)

1 **Choose the correct answer :**

1 The surface area of a square is 25 cm², then the length of its diagonal is .... cm.

(a) 5 (b) 10 (c) $5\sqrt{2}$ (d) $10\sqrt{2}$

3 [a] Find the slope of the straight line $3X + 4y - 5 = 0$, then find the length of the intercepted part from y-axis.

[b] **Find the value of X where :** $X \sin 30° \cos^2 45° = \sin^2 60°$

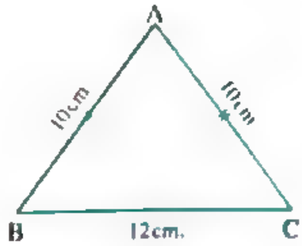
4 **[a] In the opposite figure :**

ABC is a triangle in which AB = AC = 10 cm.

, BC = 12 cm.

1 **Find :** m (∠ B)

2 **Prove that :** $\sin^2 B + \cos^2 B = 1$



[b] Prove that the triangle whose vertices are A (1 , 4) , B (− 1 , − 2) , C (2 , − 3) is right-angled , then find its area.

5 [a] Find the equation of the straight line which passes through the point A (4 , 6) and the midpoint of $\overline{BC}$ where B (3 , 7) , C (1 , − 3)

[b] ABCD is a parallelogram where A (3 , 3) , B (2 , − 2) , C (5 , − 1) , M is the intersection point of its diagonals. **Find :**

1 The coordinates of M   2 The coordinates of D

2021 27 Matrouh Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer from those given :**

1 The area of the square whose perimeter is 16 cm. equals .... cm^2

(a) 4 (b) 8 (c) 16 (d) 256

2 The equation of the straight line whose slope is 1 and passes through the origin point is ....

(a) $X = 1$ (b) $y = 1$ (c) $y = X$ (d) $y = -X$

3 If $\cos 2X = \frac{1}{2}$, then $X =$ ....

(a) 15° (b) 30° (c) 45° (d) 60°

4 A right circular cylinder , if its height equals the length of its base radius = r cm. , then its volume = ... cm^3

(a) πr^3 (b) $2 \pi r^2$ (c) $2 \pi r^3$ (d) $\frac{4}{3} \pi r^3$

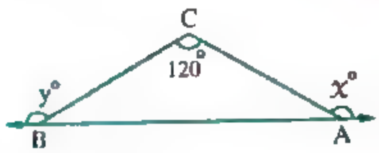
5 The slope of the straight line which is parallel to the X-axis is ....

(a) − 1 (b) zero (c) 1 (d) undefined.

6 **In the opposite figure :**

m (∠ C) = 120°

, then $x° + y° =$ ....



(a) 90° (b) 180° (c) 300° (d) 360°

2 [a] **Without using calculator , find the value of x if :** $4x = \cos^2 30° \tan^2 30° \tan^2 45°$

[b] $\overline{AB}$ is a diameter of the circle M , if B (8 , 11) , M (5 , 7)

Find : 1 The coordinates of A

2 The length of the radius of the circle.

3 [a] Prove that the points A (– 2 , 5) , B (3 , 3) , C (– 4 , 2) are not collinear and if D (– 9 , 4) , prove that the figure ABCD is a parallelogram.

[b] **Explaining the steps and without using calculator , find :**

$$\frac{\cos^2 60° + \cos^2 30° - \tan^2 45°}{\sin 60° \tan 60° - \sin 30°}$$

4 [a] Find the equation of the straight line which passes through the point (3 , 4) and is perpendicular to the straight line $5x - 2y + 7 = 0$

[b] ABCD is an isosceles trapezoid , $\overline{AD} // \overline{BC}$, AD = 4 cm. , AB = 5 cm. where BC = 12 cm.

Prove that : $\frac{5 \tan B \cos C}{\sin^2 C + \cos^2 C} = 3$

5 [a] If the straight line L_1 passes through the two points (3 , 1) , (2 , k) and the straight line L_2 makes with the positive direction of the x-axis an angle whose measure is 45°

, then find k if the two straight lines L_1 , L_2 are :

1 Parallel. 2 Perpendicular.

[b] Find the slope and the intercepted part of y-axis by the straight line : $2x = 3y + 6$

Answers of governorates' examinations of algebra & statistics

1 Cairo

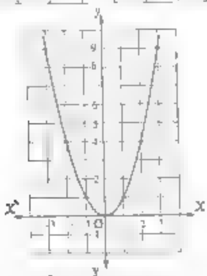
1

1 c 2 a 3 b 4 d 5 a 6 d

2

[a] $f(X) = X^2$

X	−3	−2	−1	0	1	2	3
$f(X)$	9	4	1	0	1	4	9



From the graph :

1 The minimum value = 0

2 The equation of the axis of symmetry is : $X = 0$

[b] Form the table by yourself , then $\sigma \simeq 3.22$

3

[a] 1 $X \times Y = \{(3, 4), (3, 5), (4, 4), (4, 5)\}$

2 $\because X - Y = \{3\}$

$\therefore (X - Y) \times Z = \{3\} \times \{5, 6\}$

$= \{(3, 5), (3, 6)\}$

[b] $\frac{X}{y} = \frac{z}{l} = m$

$\therefore X = y\,m \quad , \quad z = l\,m$

$\frac{y - X}{X} = \frac{y - y\,m}{y\,m} = \frac{y(1 - m)}{y\,m} = \frac{1 - m}{m}$ (1)

$, \frac{l - z}{z} = \frac{l - l\,m}{l\,m} = \frac{l(1 - m)}{l\,m} = \frac{1 - m}{m}$ (2)

From (1) , (2) $\therefore \frac{y - X}{X} = \frac{l - z}{z}$

4

[a] Let the number be X

$\frac{3 + X}{5 + X} = \frac{1}{2}$ $6 + 2X = 5 + X$

$\therefore X = -1$

The number is − 1

[b] 1 $R = \{(1, 1), (2, 1), (3, 1)\}$

2 R is a function , its range = $\{1\}$

5

[a] 1 $y \propto X$ $y = m\,X$

$\because 20 = 4\,m$ $\therefore m = 5$ $\therefore y = 5\,X$

2 At $y = 40$ $\therefore 40 = 5\,X$ $\therefore X = 8$

[b] $f(5) = 13$ $\therefore 13 = 2 \times 5 + k$

$13 = 10 + k$ $\therefore k = 3$

2 Giza

1

1 d 2 c 3 c 4 d 5 a 6 c

2

[a] 1 $n(X \times Z) = 2$

2 $\because Y \cap X = \{2\}$

$\therefore (Y \cap X) \times Z = \{2\} \times \{3\} = \{(2, 3)\}$

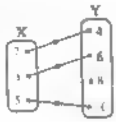
[b] $\because f(2) = 10$ $\therefore 10 = 2 \times 4 + b$

$\therefore 10 = 8 + b$ $\therefore b = 2$

3

[a] $R = \{(2, 4), (3, 6), (5, 10)\}$

R is a function because every element in X has only one image in Y



[b] Let the number be X

$\because \frac{7 + X}{11 + X} = \frac{2}{3}$ $\therefore 21 + 3X = 22 + 2X$

$X = 1$ $\therefore$ The number is 1

4

[a] $2a - 3b = 3c$ $2a = 3b$

$\therefore a = \frac{3}{2}b$

$, 3b = 3c$ $\therefore b = c$

$a : b : c = \frac{3}{2}b : b : b$ (multiplying by 2)

$a : b : c = 3b : 2b : 2b$

$a : b : c = 3 : 2 : 2$

$a = 3\,m \quad , \quad b = 2\,m \quad , \quad c = 2\,m$

$\therefore \frac{6a + b + c}{4a + 6b + 6c} = \frac{18\,m + 2\,m + 2\,m}{12\,m + 12\,m + 12\,m} = \frac{22\,m}{36\,m} = \frac{11}{18}$

[b] Form the table by yourself , then $\sigma \simeq 1.41$

5

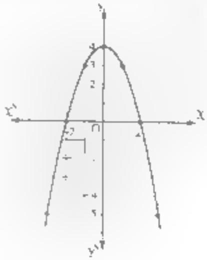
[a] 1 $y \propto x$ $\qquad$ $y = m\,x$

$6 = 3\,m$ $\qquad$ $m = 2$ $\qquad$ $y = 2\,x$

2 At $x = 4$ $\qquad$ $y = 2 \times 4$ $\qquad$ $y = 8$

[b] $f(x) = 4 - x^2$

x	3	2	[illegible]	0	[illegible]	2	3
$f(x)$	5	[illegible]	3	4	3	0	5



From the graph :

* The vertex of the curve is : (0 , 4)
* The equation of the symmetry axis is : $x = 0$

3 Alexandria

1

1 c $\quad$ 2 a $\quad$ 3 b $\quad$ 4 [illegible] $\quad$ 5 d $\quad$ 6 c

2

[a] f is of the first degree

$\therefore f(2) = 3 \times 2 = 6$

$, f(\sqrt{3}) = 3 \times \sqrt{3} = 3\sqrt{3}$

[b] $5\,a = 3\,b$ $\qquad$ $\therefore \frac{a}{b} = \frac{3}{5}$

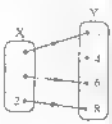
$a = 3\,m$, $b = 5\,m$

$$\frac{7\,a + 9\,b}{4\,a + 2\,b} = \frac{21\,m + 45\,m}{12\,m + 10\,m} = \frac{66\,m}{22\,m} = \frac{3}{2}$$

3

[a] $R = \{(-1 , 2) , (1 , 6) , (2 , 8)\}$

R is a function because every element in X has only one image in Y



[b] $\because x^4 y^2 - 14\,x^2 y + 49 = 0$

$\therefore (x^2 y - 7)^2 = 0$ $\qquad$ $\therefore x^2 y - 7 = 0$

$x^2 y = 7$ $\qquad$ $\therefore y \propto \frac{1}{x^2}$

4

[a] $(x - 2 , 3) = (5 , y + 1)$

$x - 2 = 5$ $\qquad$ $x = 7$

$, y + 1 = 3$ $\qquad$ $y = 2$

[b] Form the tables by yourself , then the $\bar{x} = 2$ children , $\sigma \simeq 1$ children

5

[a] $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$ [illegible]

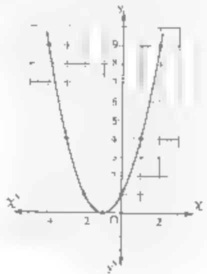
$c = d\,m$, $b = d\,m^2$, $a = d\,m^3$

[illegible]

From (1) , (2) $\qquad$ [illegible]

[b] $f(x) = x^2 + 2\,x + 1$

x	−4	−3	2	[illegible]	0	1	2
$f(x)$	9	4	[illegible]	0	[illegible]	4	9



From the graph :

1 The vertex of the curve is ([illegible] , 0)

2 The equation of the symmetry axis is : $x = -1$

3 The minimum value = 0

4 [illegible]

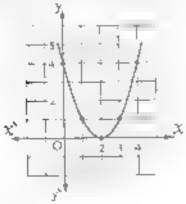
1

1 [illegible] $\quad$ 2 [illegible] $\quad$ 3 d $\quad$ 4 a $\quad$ 5 a $\quad$ 6 a

2

[a] $f(X) = (X-2)^2$

X	0	1	2	3	4
$f(X)$	4	1	0	1	4



From the graph :

1 The equation of the symmetry axis is $X = 2$

2 The minimum value $= 0$

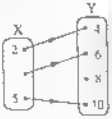
[b] $\because y \propto \frac{1}{X}$ $\therefore \frac{y_1}{y_2} = \frac{X_2}{X_1}$

$\frac{\frac{4}{7}}{y_2} = \frac{3\frac{?}{5}}{2\frac{4}{5}}$ $\therefore y_2 = \frac{\frac{4}{7} \times 2\frac{4}{5}}{3\frac{?}{5}} = \frac{?}{2}$

3

[a] 1 R $= \{(2, 4), (3, 6), (5, 10)\}$

2 Yes, R is a function



[b] $\because \frac{a}{b} = \frac{c}{d} = m$ $\therefore a = b\,m$, $c = d\,m$

$\sqrt[3]{\frac{5a^3 - 3c^3}{5b^3 - 3d^3}} = \sqrt[3]{\frac{5b^3m^3 - 3d^3m^3}{5b^3 - 3d^3}}$

$= \sqrt[3]{\frac{m^3(5b^3 - 3d^3)}{5b^3 - 3d^3}}$

$= \sqrt[3]{m^3} = m$ (1)

, $\frac{a+c}{b+d} = \frac{b\,m + d\,m}{b+d} = \frac{m(b+d)}{b+d} = m$ (2)

From (1), (2) : $\sqrt[3]{\frac{5a^3 - 3c^3}{5b^3 - 3d^3}} = \frac{a+c}{b+d}$

4

[a] 1 $Z - Y = \{5, -2\}$, $X \cap Y = \{4\}$

$(Z - Y) \times (X \cap Y) = \{5, -2\} \times \{4\}$

$= \{(5, 4), (-2, 4)\}$

2 $n(X^2) = 2 \times 2 = 4$

[b] $\because f(3) = 15$ $\therefore 15 = 4 \times 3 + b$

$15 = 12 + b$ $\therefore b = 3$

5

[a] $\frac{a}{2X+y} = \frac{b}{3y-X} = \frac{c}{4X+5y}$

Multiplying the terms of the 2nd ratio by 2 and adding the antecedents and consequents of the 1st and 2nd ratios

$\therefore \frac{a+2b}{2X+y+6y-2X} = \frac{a+2b}{7y}$

= one of the given ratios (1)

Multiplying the terms of the 2nd ratio by 4 and adding the antecedents and consequents of the 2nd and 3rd ratios

$\therefore \frac{4b+c}{12y-4X+4X+5y}$

$= \frac{4b+c}{17y}$ = one of the given ratios (2)

From (1) and (2)

$\frac{a+2b}{7y} = \frac{4b+c}{17y}$ $\therefore \frac{a+2b}{7} = \frac{4b+c}{17}$

[b] Form the tables by yourself , then $\sigma \simeq 1.43$

5 El-Sharkia

1

1 b 2 c 3 c 4 a 5 d 6 c

2

[a] 1 $X - Y = \{1, 2\}$

2 $Y \cap X = \{3\}$

$(Y \cap X) \times Y = \{3\} \times \{3, 4\}$

$= \{(3, 3), (3, 4)\}$

3 $n(Y^2) = 2 \times 2 = 4$

[b] $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$c = d\,m$, $b = d\,m^2$, $a = d\,m^3$

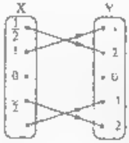
$\frac{b+d}{c^2d+d^3} = \frac{d\,m^2 + d}{d^3m^2 + d^3} = \frac{d(m^2+1)}{d^3(m^2+1)} = \frac{1}{d^2}$ (1)

, $\frac{a}{c} = \frac{d\,m^3}{d\,m} = [illegible]$ (2)

From (1), (2) : $\frac{b+d}{c^2d+d^3} = \frac{1}{[illegible]}$

3

[a] $R=\left\{\left(\frac{1}{2},2\right),(1,1),\left(\frac{-1}{2},-2\right),(-1,-1)\right\}$



R is not a function because the element $0 \in X$ has no image in Y

[b] ① $\because y \propto \frac{1}{x^2}$ $\qquad \therefore x^2 y = m$

$\left(\frac{2}{3}\right)^2 \times 9 = m \qquad \therefore \frac{4}{9} \times 9 = m$

$m = 4 \qquad \therefore x^2 y = 4$

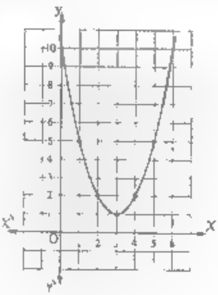
② At $x = \frac{1}{2}$, $\qquad \therefore \left(\frac{1}{2}\right)^2 \times y = 4$

$\frac{1}{4}\, y = 4 \qquad \therefore y = 16$

4

[a] $f(x) = (x-3)^2 + 1$

x	0	1	2	3	4	5	6
$f(x)$	10	5	2	1	2	5	10



From the graph :

① The vertex of the curve is : (3 , 1)

② The minimum value = 1

③ The equation of the axis of symmetry is $x = 3$

[b] $\because \frac{x}{3} = \frac{y}{2} = \frac{z}{5} = m$

$x = 3m$, $y = 2m$, $z = 5m$

$\frac{xy + yz}{x^2 + y^2} = \frac{6m^2 + 10m^2}{9m^2 + 4m^2} = \frac{16m^2}{13m^2} = \frac{16}{13}$

5

[a] Form the table by yourself , then $\sigma \simeq 3.29$

[b] $\because f(a) = b \qquad \therefore b = a^2 + b$

$a^2 = 0 \qquad \therefore a = 0$

$\therefore a\,b^2 + 5 = 0 \times b^2 + 5 = 5$

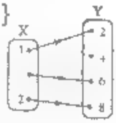
[illegible]

1

① c ② b ③ c ④ d ⑤ b ⑥ a

2

[a] ① $R = \{(-1, 2), (1, 6), (2, 8)\}$



② R is a function because every element in X has only one image in Y , the range = $\{2, 6, 8\}$

[b] $\because$ The straight line which represents the function f cuts the y-axis at (b , 3)

$\therefore b = 0$

$\therefore$ (0 , 3) satisfies the function

$3 = 6 \times 0 - a \qquad \therefore a = -3$

$\therefore 2a - 5b = 2 \times -3 - 5 \times 0 = -6$

3

[a] ① $X \times Y = \{(1, 2), (1, 3)\}$

② $\because Y - Z = \{2\}$

$\therefore X \times (Y - Z) = \{1\} \times \{2\} = \{(1, 2)\}$

③ $n(Z^2) = 3 \times 3 = 9$

[b] $\because$ b is the middle proportional between a and c

$\therefore b^2 = a\,c$

$\therefore \frac{a^2 + b^2}{b^2} = \frac{a^2 + a\,c}{a\,c} = \frac{a(a+c)}{a\,c} = \frac{a+c}{c}$ (1)

$, \frac{b^2 + c^2}{c^2} = \frac{a\,c + c^2}{c^2} = \frac{c(a+c)}{c^2} = \frac{a+c}{c}$ (2)

From (1) , (2) , $\therefore \frac{a^2 + b^2}{b^2} = \frac{b^2 + c^2}{c^2}$

4

[a] $\because a : b : c = 2 : 3 : 5$

$\therefore a = 2m$, $b = 3m$, $c = 5m$

$\because a + b + c = 35$

$\therefore 2m + 3m + 5m = 35$

$10m = 35 \qquad \therefore m = 3.5$

$\therefore a = 7$, $b = 10.5$, $c = 17.5$

[b] 1 $a \propto \frac{1}{x^2}$ $a = \frac{m}{x^2}$

$3 = \frac{m}{2^2}$ $3 = \frac{m}{4}$ $m = 12$

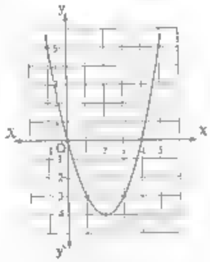
$a = \frac{12}{x^2}$, $y = \frac{12}{x^2} + 7$

2 At $x = \sqrt{3}$ $y = \frac{12}{(\sqrt{3})^2} + 7 = 4 + 7 = 11$

5

[a] $f(x) = x^2 - 4x$

x	-1	0	1	2	3	4	5
$f(x)$	5	0	3	-4	-3	0	5



From the graph :

1 The vertex of the curve is : $(2, -4)$

2 The equation of the line of symmetry is : $x = 2$

3 The minimum value $= -4$

[b] Form the table by yourself , then $\sigma \approx 9.32$

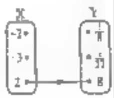
7 - El-Gharbia

1

1 c 2 d 3 a 4 c 5 c 6 b

2

[a] $R = \{(2, 8)\}$



R is not a function because the elements -2 , -3 belonging to X have no images in Y

[b] $\because x^4 y^2 - 14 x^2 y + 49 = 0$

$\therefore (x^2 y - 7)^2 = 0$ $x^2 y - 7 = 0$

$\therefore x^2 y = 7$ $\therefore y \propto \frac{1}{x^2}$

3

[a] $\frac{a}{b} = \frac{c}{d} = m$ $a = b m$, $c = d m$

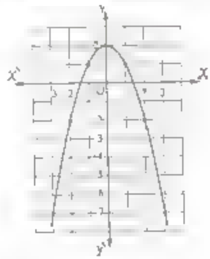
$\frac{a+b}{b} = \frac{b m + b}{b} = \frac{b(m+1)}{b} = m + 1$ (1)

, $\frac{c+d}{d} = \frac{d m + d}{d} = \frac{d(m+1)}{d} = m + 1$ (2)

From (1) , (2) $\frac{a+b}{b} = \frac{c+d}{d}$

[b] $f(x) = 2 - x^2$

x	-3	2	1	0	1	2	3
$f(x)$	-7	2	1	2	1	2	[illegible]



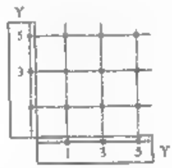
From the graph :

- The equation of the axis of symmetry is $x = 0$
- The maximum value $= 2$

4

[a] $\because Y = \{1, 3, 5\}$

$Y^2 = \{(1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5)\}$



[b] Let the number be x

$\frac{5 + x^2}{11 + x^2} = \frac{3}{5}$ $\therefore 25 + 5x^2 = 33 + 3x^2$

$\therefore 2x^2 = 8$ $\therefore x^2 = 4$

$\therefore x = 2$ or $x = -2$ (refused)

$\therefore$ The number is 2

5

[a] $\because$ The straight line which represents the function cuts the y-axis at $(m, 3)$

$m = 0$

$(0, 3)$ satisfies the function

$3 = 6 \times 0 - \ell$ $\quad \therefore \ell = -3$

[b] Form the table by yourself

, then $\bar{X} = 16$, $\sigma \simeq 3.9$

8 El-Dakahlia

1

[a] $\boxed{1}$ a $\quad$ $\boxed{2}$ c $\quad$ $\boxed{3}$ b

[b] $\because$ b is the middle proportional between a , c

$\therefore b^2 = a c$

$\frac{a^2}{b^2} + \frac{b^2}{c^2} = \frac{a^2}{a c} + \frac{a c}{c^2} = \frac{a}{c} + \frac{a}{c} = \frac{2 a}{c}$

2

[a] $\boxed{1}$ b $\quad$ $\boxed{2}$ a $\quad$ $\boxed{3}$ c

[b] $\boxed{1}$ $\because$ R is a function

$\therefore a = 5$, $b = 7$ or $a = 7$, $b = 5$

$\therefore 3 a + 3 b = 3 (a + b) = 3 \times 12 = 36$

$\boxed{2}$ The range $= \{5 , 7\}$

3

[a] $\frac{a}{4 X + y} = \frac{b}{X - 4 y}$

Adding the antecedents and consequents of 1st and 2nd ratios

$\frac{a + b}{5 X - 3 y}$ = one of the given ratios $\quad$ (1)

Subtracting the antecedent and consequent of 2nd ratio from the antecedent and consequent of 1st ratio

$\therefore \frac{a - b}{3 X + 5 y}$ = one of the given ratios $\quad$ (2)

From (1) , (2) . $\therefore \frac{a + b}{5 X - 3 y} = \frac{a - b}{3 X + 5 y}$

[b] Form the table by yourself

, then $\sigma \simeq 3.29$

4

[a] Let $C = (0 , \ell)$

$\because$ the curve of the function f passes through the point C

$\therefore \ell = 0^2 - (k - 2) \times 0 - k + 4$

$\therefore \ell = 4 - k$ $\quad \therefore B (4 - k , 4 - k)$

$\therefore 4 - k = (4 - k)^2 - (k - 2) \times (4 - k) - k + 4$

$\therefore (4 - k)^2 - (k - 2)(4 - k) = 0$

$\therefore (4 - k)(4 - k - k + 2) = 0$

$\therefore (4 - k)(6 - 2 k) = 0$

$\therefore 4 - k = 0$ $\quad \therefore k = 4$ (refused)

or $6 - 2 k = 0$ $\quad \therefore 6 = 2 k$ $\quad \therefore k = 3$

[b] $(y - 1) \propto \frac{1}{X^2}$ $\quad \therefore y - 1 = \frac{m}{X^2}$

$\therefore y = 1 + \frac{m}{X^2}$ $\quad \therefore 5 = 1 + \frac{m}{1}$

$\therefore m = 4$ $\quad \therefore y = 1 + \frac{4}{X^2}$

, at $X = 2$ $\quad \therefore y = 1 + \frac{4}{2^2} = 1 + 1 = 2$

5

[a] $\because 3 f(2) + 3 \ell (X) = 6$

$\therefore f(2) + \ell (X) = 2$

$\therefore a + 2^2 + c = 2$ $\quad \therefore a + 4 + c = 2$

$\therefore a + c = -2$

$\therefore 2 f(0) + 2 \ell (7) = 2 [f(0) + \ell (7)]$

$= 2 [a + 0^2 + c]$

$= 2 (a + c) = 2 \times -2 = -4$

[b] $\boxed{1}$ The domain $= \{3 , 5 , 7\}$

$\boxed{2}$ The rule of the function is : $f(X) = 3 X$

[illegible]

1

$\boxed{1}$ a $\quad$ $\boxed{2}$ b $\quad$ $\boxed{3}$ d $\quad$ $\boxed{4}$ b $\quad$ $\boxed{5}$ c $\quad$ $\boxed{6}$ d

2

[a] $\boxed{1}$ $X \times Y = \{(2 , 3) , (2 , 4) , (2 , 5) , (3 , 3) , (3 , 4) , (3 , 5)\}$

$\boxed{2}$ $X^2 = \{(2 , 2) , (2 , 3) , (3 , 2) , (3 , 3)\}$

$\boxed{3}$ $n (Y^2) = 3 \times 3 = 9$

[b] $\because \frac{a}{b} = \frac{4}{3}$ $\quad \therefore a = 4 m$, $b = 3 m$

$\therefore \frac{2 a + b}{5 a - 3 b} = \frac{8 m + 3 m}{20 m - 9 m} = \frac{11 m}{11 m} = 1$

3

[a] $y \propto \frac{1}{X^2}$ $\quad \therefore \frac{y_1}{y_2} = \frac{X_2^2}{X_1^2}$

$\therefore \frac{5}{y_2} = \frac{2^2}{3^2}$ $\quad \therefore \frac{5}{y_2} = \frac{4}{9}$ $\quad \therefore y_2 = \frac{5 \times 9}{4} = \frac{45}{4}$

[b] The straight line which represents the function cuts the y-axis at $(b , 5)$

$\therefore b = 0$ $\quad \therefore (0 , 5)$ satisfies the function

$\therefore 5 = 3 \times 0 - a$ $\quad \therefore a = -5$

4

[a] $\frac{1+2X}{3+2X} = \frac{3+2X}{7+2X}$

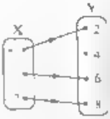
$\therefore (1+2X)(7+2X) = (3+2X)^2$

$7 + 16X + 4X^2 = 9 + 12X + 4X^2$

$\therefore 7 + 16X = 9 + 12X$

$\therefore 4X = 2 \qquad \therefore X = \frac{1}{2}$

[b] 1 $R = \{(-1, 2), (1, 6), (2, 8)\}$

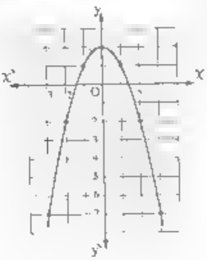


2 R is a function because every element in X has only one image in Y

5

[a] $f(X) = 2 - X^2$

X	-3	-2	-1	0	1	2	3
$f(X)$	7	2	[illegible]	2	1	7	7



From the graph :

1 The vertex of the curve is : $(0, 2)$

2 The equation of the axis of symmetry is : $X = 0$

3 The maximum value = 2

[b] Form the table by yourself , then $\sigma \simeq 3.29$

10 Suez

1

1 a 2 c 3 a 4 b 5 c 6 b

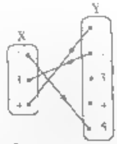
2

[a] $\frac{a}{b} = \frac{3}{4} \qquad a = 3m , \; b = 4m$

$\frac{4a+b}{2a-b} = \frac{12m + 4m}{6m - 4m} = \frac{16m}{2m} = 8$

[b] 1 $R = \{(0, 5), (3, 2), (4, 1)\}$

2



3 Yes , R is a function

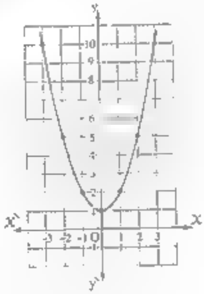
3

[a] 1 $X = \{2, 3\}$, $Y = \{6, 9\}$

2 $Y \times Y = \{(6, 6), (6, 9), (9, 6), (9, 9)\}$

[b] $f(X) = X^2 + 1$

X	-3	-2	-1	0	1	2	3
$f(X)$	10	5	2	1	2	5	10



From the graph :

1 The vertex of the curve is : $(0, 1)$

2 The equation of the axis of symmetry is : $X = 0$

3 The minimum value = 1

4

[a] $\frac{X}{y} = \frac{z}{r} = m \qquad \therefore X = ym , \; z = rm$

$\therefore \frac{X^2 + 2z^2}{y^2 + 2r^2} = \frac{y^2m^2 + 2r^2m^2}{y^2 + 2r^2} = \frac{m^2(y^2 + 2r^2)}{y^2 + 2r^2} = m^2 \quad (1)$

$, \frac{Xz}{yr} = \frac{ym \times rm}{yr} = m^2 \quad (2)$

From (1) , (2) : $\therefore \frac{X^2 + 2z^2}{y^2 + 2r^2} = \frac{Xz}{yr}$

[b] 1 The variation is inverse

2 $\because y \propto \frac{1}{X} \qquad \therefore Xy = m \qquad m = ?$

3 At $X = 3$ $\qquad 3y = 12 \qquad y = 4$

5

[a] 1 $f(2) + g(2) = 2^2 - 3 \times 2 + 2 - 3$

$= 4 - 6 - 1 = -3$

2 $f(3) + g(3) = 3^2 - 3 \times 3 + 3 - 3$

$= 9 - 9 + 3 - 3$

$= 0$

[b] Form the table by yourself , then $\sigma \simeq 3.29$

11 — Port Said —

1

1 c 2 b 3 d 4 d 5 a 6 a

2

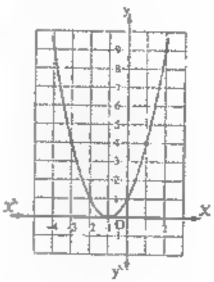
[a] 1 $n(X \times Z) = 2 \times 2 = 4$

2 $\because X - Y = \{1\}$

$\therefore (X - Y) \cap Z = \emptyset$

[b] $f(X) = X^2 + 2X + 1$

X	-4	-3	-2	-1	0	1	2
$f(X)$	9	4	1	0	1	4	9



From the graph :

1 The vertex of the curve is : $(-1 , 0)$

2 The minimum value $= 0$

3

[a] $\because f(3) = 15$ $\therefore 15 = 4 \times 3 + b$

$\therefore 15 = 12 + b$ $\therefore b = 3$

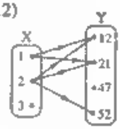
[b] 1 $\because y \propto \frac{1}{X}$ $\therefore X y = m$ $\therefore 2.5 \times 6 = m$

$\therefore m = 15$ $\therefore X y = 15$

2 At $X = 5$ $\therefore 5 y = 15$ $\therefore y = 3$

4

[a] 1 $R = \{(1 , 12) , (1 , 21) , (2 , 12) , (2 , 21) , (2 , 52)\}$



2 2 R 21 because $2 \in X$

its image in Y is 21

[b] $\frac{7}{X} = \frac{X}{\frac{1}{y}}$ $\therefore \frac{7}{X} = X y$

$X^2 y = 7$ $\therefore (X^2 y)^2 = 7^2$ $\therefore X^4 y^2 = 49$

5

[a] $\because \frac{X}{3} = \frac{y}{4} = \frac{z}{5} = m$

$\therefore X = 3m$, $y = 4m$, $z = 5m$

$\therefore \frac{2y - z}{3X - 2y + z} = \frac{8m - 5m}{9m - 8m + 5m} = \frac{3m}{6m} = \frac{1}{2}$

[b] Form the table by yourself

, then $\overline{X} = 8$, $\sigma \simeq 4$

12 — Damietta —

1

1 a 2 b 3 d 4 d 5 c 6 b

2

[a] 1 $n(X \times Y) = 2 \times 2 = 4$

2 $\because X - Y = \{5\}$

$\therefore (X - Y) \times Z = \{5\} \times \{3\} = \{(5 , 3)\}$

3 $Y^2 = \{(1 , 1) , (1 , 2) , (2 , 1) , (2 , 2)\}$

[b] $\frac{a}{b} = \frac{b}{c} = m$ $\therefore b = cm$, $a = cm^2$

$\frac{a - b}{a - c} = \frac{cm^2 - cm}{cm^2 - c} = \frac{cm(m - 1)}{c(m^2 - 1)}$

$= \frac{m(m - 1)}{(m - 1)(m + 1)} = \frac{m}{m + 1}$ (1)

, $\frac{b}{b + c} = \frac{cm}{cm + c} = \frac{cm}{c(m + 1)} = \frac{m}{m + 1}$ (2)

From (1) , (2) $\therefore \frac{a - b}{a - c} = \frac{b}{b + c}$

3

[a] 1 $R = \{(1 , 6) , (3 , 4) , (4 , 3) , (5 , 2)\}$

2 R is a function because every element in X has only one image in Y

, the range $= \{6 , 4 , 3 , 2\}$

[b] $\frac{21X-y}{7X-z}=\frac{y}{z}$

$7Xy-yz=21Xz-yz$

$7Xy=21Xz$

$y=3z \qquad \therefore y\propto z$

4

[a] Form the table by yourself, then $\sigma \simeq 3.29$

[b] 1 $\because y\propto X \qquad \therefore y=mX$

$, 6=3m \qquad \therefore m=2 \qquad \therefore y=2X$

2 At $X=5 \qquad \therefore y=2\times 5=10$

5

[a] $\frac{X}{3}=\frac{y}{4}=\frac{z}{5}=m$

$\therefore X=3m \;,\; y=4m \;,\; z=5m$

$\sqrt{3X^2+3y^2+z^2}=\sqrt{27m^2+48m^2+25m^2}$

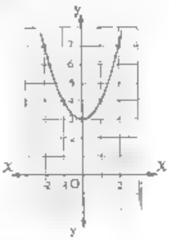
$=\sqrt{100m^2}=10m \qquad (1)$

$, 2X+y=6m+4m=10m \qquad (2)$

From (1), (2): $\therefore \sqrt{3X^2+3y^2+z^2}=2X+y$

[b] $f(X)=X^2+3$

X	-2	-1	0	1	2
$f(X)$	7	4	3	4	7



From the graph :

1 The equation of symmetry line is : $X=0$

2 The minimum value = 3

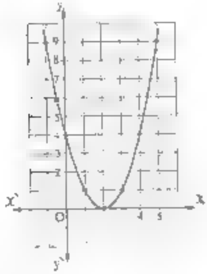
13 Kafr El-Sheikh

1

[a] 1 c   2 d   3 b

[b] $f(X)=(X-2)^2$

X	-1	0	1	2	3	4	5
$f(X)$	9	4	1	0	.	4	9



From the graph :

- The vertex of the curve is (2, 0)
- The equation of the symmetry axis is : $X=2$
- The minimum value = 0

2

[a] 1 a   2 c   3 d

[b] $\because \frac{X}{y}=\frac{y}{z}=m \qquad \therefore y=zm \;,\; X=zm^2$

$\therefore \frac{X-y}{X-z}=\frac{zm^2-zm}{zm^2-z}=\frac{zm(m-1)}{z(m-1)(m+1)}$

$=\frac{m}{m+1} \qquad (1)$

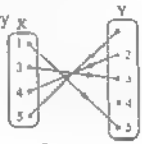
$, \frac{y}{y+z}=\frac{zm}{zm+z}=\frac{zm}{z(m+1)}=\frac{m}{m+1} \qquad (2)$

From (1), (2) $\therefore \frac{X-y}{X-z}=\frac{y}{y+z}$

3

[a] 1 $R=\{(1,5),(3,3),(4,2),(5,1)\}$

2 R is a function because every element in X has only one image in Y, its range $=\{1,2,3,5\}$



[b] $\because \frac{X}{y}=\frac{2}{3} \qquad \therefore X=2m \;,\; y=3m$

$\therefore \frac{3X+2y}{6y-X}=\frac{6m+6m}{18m-2m}=\frac{12m}{16m}=\frac{3}{4}$

4

[a] 1 $X\times Y=\{(2,4),(2,0),(-1,4),(-1,0)\}$

2 $\because Y\cap Z=\{4\}$

$\therefore (Y\cap Z)\times X=\{4\}\times\{2,-1\}$

$=\{(4,2),(4,-1)\}$

3 $n(Y^2)=2\times 2=4$

[b] $\because f(2) = 1$ $\quad\therefore 1 = 2 \times 2 + a$

$\therefore 1 = 4 + a$ $\quad\therefore a = -3$

5

[a] 1 $y \propto \frac{1}{x^2}$ $\quad x^2 y = m$

$(4)^2 \times 2 = m$ $\quad m = 32$

$x^2 y = 32$

2 At $x = 16$ $\quad\therefore (16)^2 \times y = 32$ $\quad\therefore y = \frac{1}{8}$

[b] Form the table by yourself

, then $\bar{X} = 7$, $\sigma \simeq 1.41$

[illegible]

1

1 d 2 b 3 c 4 a 5 b 6 c

2

[a] 1 $\because Y \cap Z = \{2\}$

$X \times (Y \cap Z) = \{1\} \times \{2\} = \{(1, 2)\}$

2 $n(Z^2) = 3 \times 3 = 9$

[b] Let the number be x

$\frac{5 + x^2}{11 + x^2} = \frac{3}{5}$ $\quad\therefore 25 + 5x^2 = 33 + 3x^2$

$2x^2 = 8$ $\quad x^2 = 4$

$x = -2$ (refused) or $x = 2$

The number is 2

3

[a] $\because$ (a , 3) satisfies the function

$3 = 4a - 5$ $\quad\therefore 4a = 8$ $\quad\therefore a = 2$

[b] $\because \frac{a+b}{3} = \frac{b+c}{6} = \frac{c+a}{5}$

Adding the antecedents and consequents of the three ratios

$\frac{a+b+b+c+c+a}{3+6+5} = \frac{2a+2b+2c}{14}$

$= \frac{a+b+c}{7}$

= one of the given ratios (1)

Mutiplying the terms of the 2nd ratio by -1 and adding the antecedents and consequents of the three ratios

$\frac{a+b-b-c+c+a}{3-6+5} = \frac{2a}{2} = a$

= one of the given ratios (2)

From (1) , (2) $\quad \frac{a+b+c}{7} = a$

$\frac{a+b+c}{a} = 7$

4

[a] 1 $R = \{(1, 5), (3, 3), (5, 1)\}$

2 R is a function because every element in X has only one image in X

, its range $= \{5, 3, 1\}$

[b] Form the table by yourself , then $\sigma \simeq 2.28$

5

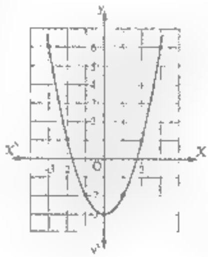
[a] 1 $y \propto x$ $\quad y = m\,x$

$6 = 3m$ $\quad m = 2$ $\quad y = 2x$

2 At $x = 5$ $\quad y = 2 \times 5 = 10$

[b] $f(x) = x^2 - 3$

x	-3	-2	-1	0	1	2	3
$f(x)$	6	1	-2	-3	-2	1	6



From the graph :

1 The equation of the axis of symmetry is : $x = 0$

2 The minimum value $= -3$

15 El-Fayoum

1

1 d 2 c 3 a 4 b 5 b 6 c

2

[a] $\because \frac{a}{b} = \frac{2}{3}$ $\quad\therefore a = 2m$, $b = 3m$

$\therefore \frac{3a-b}{a+2b} = \frac{6m-3m}{2m+6m} = \frac{3m}{8m} = \frac{3}{8}$

[b] $\because f(-3) = 8$ $\quad\therefore 8 = -3a + 5$

$\therefore -3a = 3$ $\quad a = -1$

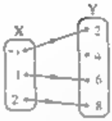
3

[a] $\frac{x}{y} = \frac{y}{z}$ $\qquad$ $y^2 = x z$

$$\frac{x^2 + y^2}{y^2 + z^2} = \frac{x^2 + x z}{x z + z^2} = \frac{x(x+z)}{z(x+z)} = \frac{x}{z}$$

[b] $R = \{(-1, 2), (1, 6), (2, 8)\}$

R is a function because every element in X has only one image in Y



4

[a] $y \propto x$ $\qquad$ $y = m x$ $\qquad$ $20 = 7 m$

$m = \frac{20}{7}$ $\qquad$ $y = \frac{20}{7} x$

At $x = 14$ $\qquad$ $y = \frac{20}{7} \times 14 = 40$

[b] $(5 - 2x, y^3) = (1, 27)$

$5 - 2x = 1$ $\qquad$ $-2x = -4$ $\qquad$ $x = 2$

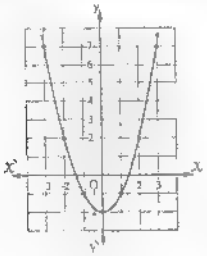
, $y^3 = 27$ $\qquad$ $y = 3$

$\sqrt{3x + y} = \sqrt{3 \times 2 + 3} = 3$

5

[a] $f(x) = x^2 - 2$

x	-3	-2	-1	0	1	2	3
$f(x)$	7	2	-1	-2	-1	2	7



From the graph :

* The vertex of the curve is $(0, -2)$
* The minimum value $= -2$

[b] Form the table by yourself , then $\sigma \simeq 4$

16 Beni Suef

1

1 c $\qquad$ 2 b $\qquad$ 3 a $\qquad$ 4 d $\qquad$ 5 a $\qquad$ 6 d

2

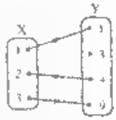
[a] Let the number be x

$\frac{7 + x}{11 + x} = \frac{2}{3}$ $\qquad$ $21 + 3x = 22 + 2x$

$x = 1$ $\qquad$ The number is 1

[b] $R = \{(1, 1), (2, 4), (3, 9)\}$

R is a function because every element in X has only one image in Y



3

[a] $\frac{x}{2} = \frac{y}{3} = \frac{z}{4} = \frac{3x - 2y + 5z}{5k}$

Multiplying the two terms of the 1st ratio by 3 , the two terms of the 2nd ratio by -2 , the two terms of the 3rd ratio by 5 and adding the antecedents and consequents of the three ratios

$\frac{3x - 2y + 5z}{6 - 6 + 20} = \frac{3x - 2y + 5z}{20}$

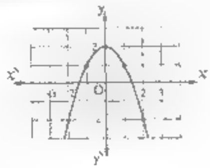
$=$ one of the given ratios

$\frac{3x - 2y + 5z}{5k} = \frac{3x - 2y + 5z}{20}$

$\therefore 5k = 20$ $\qquad$ $\therefore k = 4$

[b] $f(x) = 2 - x^2$

x	-2	-1	0	1	2
$f(x)$	-2	1	2	1	-2



From the graph :

* The vertex of the curve is $(0, 2)$
* The maximum value $= 2$

4

[a] $\because y \propto x$ $\qquad$ $\therefore y = m x$

$\therefore 3 = 15 m$ $\qquad$ $\therefore m = \frac{1}{5}$ $\qquad$ $\therefore y = \frac{1}{5} x$

at $y = 100$ $\qquad$ $\therefore 100 = \frac{1}{5} x$ $\qquad$ $\therefore x = 500$

[b] 1 $X \times Y = \{(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5)\}$

2 $Y \times X = \{(3, 1), (3, 2), (4, 1), (4, 2), (5, 1), (5, 2)\}$

3 $X^2 = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$

5

[a] $\because f(3) + g(5) = 15$, $3 \times 3 + k + k = 15$

, $9 + 2k = 15$ $\therefore 2k = 6$ $\therefore k = 3$

[b] Form the table by yourself , then $\sigma \simeq 3.29$

[illegible]

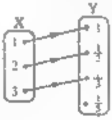
1

1 d 2 c 3 b 4 a 5 d 6 a

2

[a] $R = \left\{(1 , 1) , \left(2 , \frac{1}{2}\right) , \left(3 , \frac{1}{3}\right)\right\}$

R is a function because every element in X has only one image in Y



[b] $\frac{a}{b} = \frac{b}{c} = m$ $b = c\,m$, $a = c\,m^2$

$$\frac{a+b}{a-c} = \frac{c\,m^2 + c\,m}{c\,m^2 - c} = \frac{c\,m(m+1)}{c(m^2-1)}$$

$$= \frac{m(m+1)}{(m+1)(m-1)} = \frac{m}{m-1} \quad (1)$$

$$, \frac{b}{b-c} = \frac{c\,m}{c\,m - c} = \frac{c\,m}{c(m-1)} = \frac{m}{m-1} \quad (2)$$

From (1) and (2) $\therefore \frac{a+b}{a-c} = \frac{b}{b-c}$

3

[a] $\because \frac{X}{y} = \frac{2}{3}$ $\therefore X = 2m$, $y = 3m$

$$\frac{3X + 2y}{6y - X} = \frac{6m + 6m}{18m - 2m} = \frac{12m}{16m} = \frac{3}{4}$$

[b] 1 $\because Y \cap Z = \{5\}$

$\therefore X \times (Y \cap Z) = \{3 , 4\} \times \{5\}$

$= \{(3 , 5) , (4 , 5)\}$

2 $\because X - Y = \{3\}$

$\therefore (X - Y) \times Z = \{3\} \times \{6 , 5\}$

$= \{(3 , 6) , (3 , 5)\}$

4

[a] 1 $\because y \propto \frac{1}{X}$ $\therefore X y = m$

$\therefore 2 \times 3 = m$ $\therefore m = 6$ $\therefore X y = 6$

2 At $y = 4$ $\therefore 4X = 6$ $\therefore X = \frac{3}{2}$

[b] Form the table by yourself , then $\sigma \simeq 3.29$

5

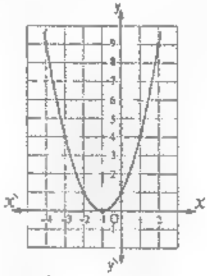
[a] The function f is of the 3rd degree

, $f(0) = 3 - 2 \times (0)^3 = 3$

, $f(-2) = 3 - 2(-2)^3 = 19$

[b] $f(X) = X^2 + 2X + 1$

X	-4	-3	-2	-1	0	1	2
$f(X)$	9	4	1	0	1	4	9



From the graph :

1 The equation of the symmetry axis is : $X = -1$

2 The minimum value $= 0$

18 Assiut

1

1 b 2 d 3 a 4 c 5 a 6 b

2

[a] 1 $\because X \cap Y = \{7\}$

$\therefore (X \cap Y) \times X = \{7\} \times \{6 , 7\}$

$= \{(7 , 6) , (7 , 7)\}$

2 $n(Y^2) = 2 \times 2 = 4$

[b] $\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = m$

$\therefore a = 2m$, $b = 3m$, $c = 4m$

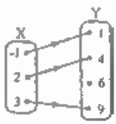
$$\therefore \frac{3c - b}{a + b} = \frac{12m - 3m}{2m + 3m} = \frac{9m}{5m} = \frac{9}{5}$$

3

[a] $R = \{(-1 , 1) , (2 , 4) , (3 , 9)\}$

R is a function because every element in X has only one image in Y

, the range $= \{1 , 4 , 9\}$



[b] 1 $y \propto \frac{1}{X}$ $\quad X y = m$

$\therefore 4 \times 3 = m$ $\quad m = 12$ $\quad X y = 12$

2 At $X = \frac{3}{4}$ $\quad \frac{3}{4} y = 12$ $\quad y = 16$

4

[a] Let the number be X

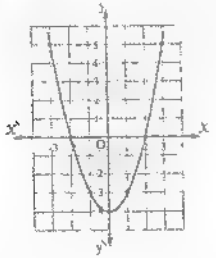
$\therefore \frac{7 + X^2}{11 + X^2} = \frac{2}{3}$ $\quad \therefore 21 + 3 X^2 = 22 + 2 X^2$

$\therefore X^2 = 1$ $\quad \therefore X = -1$ (refused) or $X = 1$

$\therefore$ The number is 1

[b] $f(X) = X^2 - 4$

X	-3	-2	-1	0	1	2	3
$f(X)$	5	0	-3	-4	-3	0	5



From the graph :

* The vertex of the curve is $(0, -4)$
* The minimum value $= -4$
* The equation of the axis of symmetry is : $X = 0$

5

[a] $f\left(\sqrt{2}\right) + g(5) = \left(\sqrt{2}\right)^2 - 2 + 3$

$= 2 - 2 + 3 = 3$

[b] Form the table by yourself

, then $\overline{X} = 15$, $\sigma \simeq 3.29$

19 Souhag

1

1 c $\quad$ 2 a $\quad$ 3 d $\quad$ 4 b $\quad$ 5 b $\quad$ 6 c

2

[a] 1 $X = \{1\}$, $Y = \{1, 3, 5\}$

2 $Y \times X = \{(1, 1), (3, 1), (5, 1)\}$

[b] $\frac{X}{y} = \frac{2}{3}$ $\quad X = 2 m$, $y = 3 m$

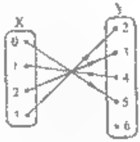
$\therefore \frac{3 X + 2 y}{6 y - X} = \frac{6 m + 6 m}{18 m - 2 m} = \frac{12 m}{16 m} = \frac{3}{4}$

3

[a] 1 $R = \{(0, 5), (1, 4), (2, 3), (3, 2)\}$

2 R is a function because every element in X has only one image in Y

, the range $= \{5, 4, 3, 2\}$



[b] Let the number be X

$\frac{7 + X}{11 + X} = \frac{2}{3}$ $\quad \therefore 21 + 3 X = 22 + 2 X$

$X = 1$ $\quad \therefore$ The number is 1

4

[a] $\because (a, 3)$ satisfies the relation

$3 = 4 a - 5$ $\quad \therefore 4 a = 8$ $\quad \therefore a = 2$

[b] 1 $\because y \propto X$ $\quad \therefore y = m X$

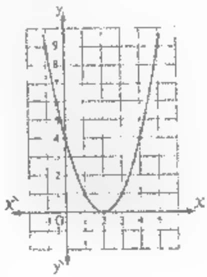
$6 = 3 m$ $\quad \therefore m = 2$ $\quad \therefore y = 2 X$

2 At $X = 5$ $\quad y = 2 \times 5 = 10$

5

[a] $f(X) = X^2 - 4 X + 4$

X	-1	0	1	2	3	4	5
$f(X)$	9	4	1	0	1	4	9



From the graph :

1 The vertex of the curve is : $(2, 0)$

2 The equation of the axis of symmetry is : $X = 2$

[b] Form the table by yourself

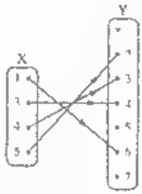
, then $\overline{X} = 16$, $\sigma \simeq 3.29$

20 Qena

1

1 a $\quad$ 2 c $\quad$ 3 c $\quad$ 4 b $\quad$ 5 b $\quad$ 6 d

[a] $R = \{(1 , 6) , (3 , 4) , (4 , 3) , (5 , 2)\}$



R is a function because every element in X has only one image in Y

, its range $= \{2 , 3 , 4 , 6\}$

[b] $\because$ b is the middle proportional between a and c

$b^2 = a\,c$

$$\frac{a^2+b^2}{b^2+c^2} = \frac{a^2+a\,c}{a\,c+c^2} = \frac{a\,(a+c)}{c\,(a+c)} = \frac{a}{c}$$

3

[a] $\because f(\sqrt{2}) + 3\,g(\sqrt{2})$

$= (\sqrt{2})^2 - 3\sqrt{2} + 3(\sqrt{2} - 3)$

$= 2 - 3\sqrt{2} + 3\sqrt{2} - 9 = -7$

[2] $\because f(3) = 3^2 - 3 \times 3 = 0$

$, g(3) = 3 - 3 = 0$

$\therefore f(3) = g(3)$

[b] Let the number be X

$\therefore \frac{7+X}{11+X} = \frac{2}{3}$ $\quad \therefore 21 + 3X = 22 + 2X$

$X = 1$ $\quad \therefore$ The number is 1

4

[a] $\frac{a}{b} = \frac{3}{5}$ $\quad \therefore a = 3\,m \;,\; b = 5\,m$

$$\frac{7a+9b}{4a+2b} = \frac{21\,m+45\,m}{12\,m+10\,m} = \frac{66\,m}{22\,m} = 3$$

[b] Form the tables by yourself , then $\sigma \simeq 1.73$

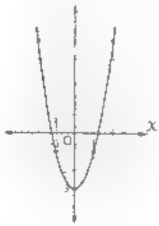
5

[a] $y \propto X$ $\quad \therefore \frac{y_1}{y_2} = \frac{X_1}{X_2}$

$\therefore \frac{40}{80} = \frac{14}{X_2}$ $\quad \therefore X_2 = \frac{80 \times 14}{40} = 28$

[b] $f(X) = 2X^2 - 3$

X	-2	-1	0	1	2
$f(X)$	5	-1	-3	-1	5



From the graph :

[1] The vertex of the curve is $(0 , -3)$

[2] The equation of the axis of symmetry is $X = 0$

[3] The minimum value $= -3$

21 Luxor

1

[1] d [2] c [3] b [4] a [5] b [6] d

[a] [1] The domain $= \{1 , 2 , 3 , 4 , 5\}$

[2] The range $= \{3 , 5 , 7 , 9 , 11\}$

[3] $f(X) = 2X + 1$

[b] Let the two numbers be $2X , 3X$

$\therefore \frac{2X-7}{3X-7} = \frac{1}{2}$ $\quad \therefore 4X - 14 = 3X - 7$

$\therefore X = 7$

The two numbers are $14 , 21$

3

[a] [1] R is a function from X to Y

$\because$ Each element in X has only one image in Y

$, \because$ the image of $-2 = (-2)^2 - 1 = 3 \in Y$

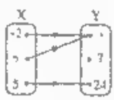
, the image of $2 = (2)^2 - 1 = 3 \in Y$

, the image of $5 = (5)^2 - 1 = 24 \in Y$

$\therefore \ell = 24$

[2] $R = \{(-2 , 3) , (2 , 3) , (5 , 24)\}$

[3]



[b] $y = a - 9 \;,\; y \propto \frac{1}{X^2}$ $\quad \therefore y = \frac{m}{X^2}$

$\therefore \frac{m}{X^2} = a - 9$ $\quad \therefore m = X^2\,(a - 9)$

$\therefore m = \left(\frac{2}{3}\right)^2 (18 - 9) = 4$

$y = \frac{4}{X^2}$

at $X = 1$ $\qquad$ $y = 4$

4

[a] Let A $(X , 0)$

, $\because$ A $(X , 0)$ belongs to the straight line of the function f

$\therefore 4 - 2X = 0$ $\qquad$ $-2X = -4$

$\therefore X = 2$ $\qquad$ $\therefore$ A $(2 , 0)$

, let B $(0 , y)$

, $\because$ B $(0 , y)$ belongs to the straight line of the function f

$\therefore 4 - 2 \times 0 = y$ $\quad$ $\therefore y = 4$

$\therefore$ B $(0 , 4)$

, the area of Δ AOB $= \frac{1}{2} \times 2 \times 4 = 4$ square unit

[b] $\because \frac{X}{7} = \frac{y}{3}$ $\qquad$ $\therefore X = 7m$, $y = 3m$

$, \because \frac{2X - 3y}{X + 2y} = \frac{14m - 9m}{7m + 6m} = \frac{5m}{13m} = \frac{5}{13}$ $\qquad$ (1)

$, \frac{10}{26} = \frac{5}{13}$ $\qquad$ (2)

From (1) , (2) : $\therefore \frac{2X - 3y}{X + 2y} = \frac{10}{26}$

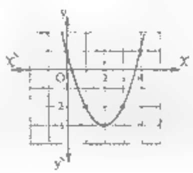
$\therefore (2X - 3y) , (X + 2y) , 10 , 26$ are proportional

5

[a] Form the table by yourself , then $\sigma \simeq 7.07$

[b] $f(X) = 1 - 4X + X^2$

X	0	1	2	3	4
$f(X)$	1	−2	−3	−2	1



From the graph :

1 The vertex of the curve is $(2 , -3)$

2 The equation of the axis of symmetry is : $X = 2$

3 The minimum value $= -3$

22 Aswan

1

1 [illegible] $\quad$ 2 d $\quad$ 3 a $\quad$ 4 c $\quad$ 5 b $\quad$ 6 c

2

[a] 1 $Y = \{2 , 5 , 7\}$

2 $Y \times X = \{(2 , 2) , (5 , 2) , (7 , 2)\}$

[b] $\frac{a}{b} = \frac{b}{c} = m$ $\qquad$ $b = cm$, $a = cm^2$

$\frac{a - b}{a - c} = \frac{cm^2 - cm}{cm^2 - c} = \frac{cm(m - 1)}{c(m^2 - 1)}$

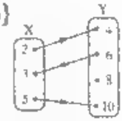
$= \frac{cm(m - 1)}{c(m - 1)(m + 1)} = \frac{m}{m + 1}$ (1)

$, \frac{b}{b + c} = \frac{cm}{cm + c} = \frac{cm}{c(m + 1)} = \frac{m}{m + 1}$ $\qquad$ (2)

From (1) and (2) $\qquad$ $\frac{a - b}{a - c} = \frac{b}{b + c}$

3

[a] 1 $R = \{(2 , 4) , (3 , 6) , (5 , 10)\}$



2 Yes , R is a function

[b] $\because y \propto \frac{1}{X}$ $\qquad$ $\therefore Xy = m$

$\therefore 4 \times 2 = m$

$\therefore m = 8$ $\qquad$ $\therefore Xy = 8$

at $X = 16$ $\qquad$ $\therefore 16y = 8$ $\qquad$ $y = \frac{1}{2}$

4

[a] $\because (a , 3)$ satisfies the function

$\therefore 3 = 4a - 5$ $\qquad$ $\therefore 4a = 8$ $\qquad$ $\therefore a = 2$

[b] $\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a - b + 5c}{3X}$

Multiplying the two terms of the 1st ratio by 2 and the 2nd by -1 and the 3rd by 5 and adding the antecedents and consequents of the three ratios

$\frac{2a - b + 5c}{4 - 3 + 20}$ = one of the given ratios

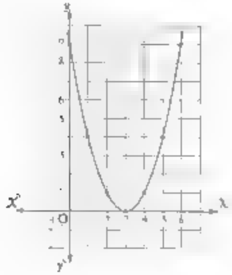
$\frac{2a - b + 5c}{21} = \frac{2a - b + 5c}{3X}$

$3X = 21$ $\qquad$ $X = 7$

5

[a] $f(X) = (X - 3)^2$

X	0	1	2	3	4	5	6
$f(X)$	9	4	1	0	1	4	9



From the graph :

* The vertex of the curve is : (3 , 0)
* The minimum value = 0
* The equation of the axis of symmetry is $X = 3$

[b] Form the tables by yourself , then $\overline{X} = 2$, $\sigma = 0.96$

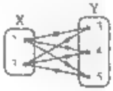
23 New Valley

1

1 d   2 b   3 b   4 d   5 a   6 a

2

[a] 1 $X \times Y = \{(2, 3), (2, 4), (2, 5), (3, 3), (3, 4), (3, 5)\}$



2 $n(X \times Y) = 6$

[b] $X^2 y^2 - 14 X y + 49 = 0$

$(X y - 7)^2 = 0$ $\therefore X y - 7 = 0$

$\therefore X y = 7$ $\therefore y \propto \frac{1}{X}$

3

[a] Let the number be X

$\frac{7 + X^2}{11 + X^2} = \frac{4}{5}$

$\therefore 35 + 5X^2 = 44 + 4X^2$ $\therefore X^2 = 9$

$\therefore X = 3$ (refused) or $X = -3$

The number is -3

[b] $R = \{(4, 2), (8, 4)\}$

R is not a function because $2 \in X$ has no image in X

4

[a] 1 $\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a - 5b + 3c}{X}$

Multiplying the terms of the 1st ratio by 2 and the 2nd ratio by – 5 and the 3rd ratio by 3 and adding the antecedents and consequents of the three ratios

$\therefore \frac{2a - 5b + 3c}{4 - 15 + 12}$ = one of the given ratios

$\therefore \frac{2a - 5b + 3c}{1} = \frac{2a - 5b + 3c}{X}$ $\therefore X = 1$

2 Adding the antecedents and consequents of the three ratios

$\therefore \frac{a + b + c}{2 + 3 + 4}$ = one of the given ratios

$\therefore \frac{a + b + c}{9} = \frac{b}{3}$ $\therefore \frac{a + b + c}{b} = \frac{9}{3} = 3$

[b] 1 $\because f(k) = 5$ $\therefore 5 = 2k - 3$

$\therefore 2k = 8$ $\therefore k = 4$

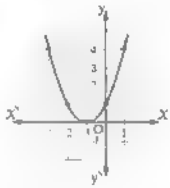
2 $\because (2, k) \in f$ $\therefore k = 2 \times 2 - 3 = 1$

5

[a] Form the tables by yourself , then $\overline{X} = 7$, $\sigma = 2$

[b] $f(X) = (X + 1)^2$

X	-3	-2	-1	0	1
$f(X)$	4	1	0	1	4



From the graph :

1 The vertex of the curve is : $(-1, 0)$
2 The equation of the symmetry axis is : $X = -1$
3 The minimum value = 0

24 South Sinai

1

1 a   2 b   3 c   4 d   5 a   6 d

2

$R = \{(2, 4), (3, 6), (4, 8)\}$

Yes , R is a function , its range = $\{4, 6, 8\}$

3

[a] Let the number be X

$\because \frac{7+X}{11+X}=\frac{2}{3}$ $\quad$ $21+3X=22+2X$

$\therefore X=1$ $\quad$ The number is 1

[b] $y \propto X$ $\quad$ $y=mX$

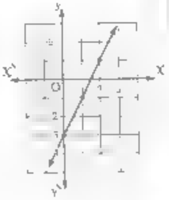
$14=42\,m$ $\quad$ $m=\frac{1}{3}$ $\quad$ $y=\frac{1}{3}X$

at $X=60$ $\quad$ $y=\frac{1}{3}\times 60=20$

4

[a] $f(X)=2X-3$

X	0	1	2
$f(X)$	-3	[illegible]	[illegible]



[b] $\because$ b is the middle proportional between a , c

$b^2=a\,c$

$\therefore \frac{a^2+b^2}{b^2+c^2}=\frac{a^2+a\,c}{a\,c+c^2}=\frac{a(a+c)}{c(a+c)}=\frac{a}{c}$

5

[a] $\because (X^3 , y+1)=\left(27 , \sqrt[3]{125}\right)$

$\therefore X^3=27$ $\quad$ $\therefore X=3$

$, y+1=\sqrt[3]{125}$ $\quad$ $y+1=5$ $\quad$ $y=4$

[b] Form the table by yourself , then $\overline{X}=20$

$, \sigma \simeq 2.28$

25 North Sinai

1

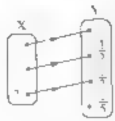
1 d $\quad$ 2 c $\quad$ 3 c $\quad$ 4 d $\quad$ 5 b $\quad$ 6 a

2

[a] R = {[illegible] , $\left(2 , \frac{1}{2}\right)$,

$\left(3 , \frac{1}{3}\right)$}

Yes , R is a function



[b] 1 $y \propto \frac{1}{X}$ $\quad$ $\therefore X\,y=m$

$\because 2\times 3=m$ $\quad$ $\therefore m=6$ $\quad$ $X\,y=6$

2 At $X=1.5$ $\quad$ $\therefore 1.5\,y=6$ $\quad$ $y=4$

3

[a] $\because$ (3 , b) satisfies the function,

$b=5\times 3+4=19$

[b] $\because \frac{X}{y}=\frac{3}{4}$ $\quad$ $\therefore X=3\,m$, $y=4\,m$

$\therefore \frac{3X+y}{X+5y}=\frac{9\,m+4\,m}{3\,m+20\,m}=\frac{13\,m}{23\,m}=\frac{13}{23}$

4

[a] $X=\{1 , 4 , 5\}$, $Y=\{2\}$

$, Y^2=\{(2 , 2)\}$

[b] $\because$ b is the middle proportional between a , c

$b^2=a\,c$

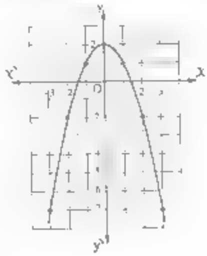
$\frac{5\,c^2 - b^2}{5\,b^2 - 2\,a^2}=\frac{5\,c^2 - 2\,a\,c}{5\,a\,c - 2\,a^2}=\frac{c(5\,c-2\,a)}{a(5\,c-2\,a)}=\frac{c}{a}$

5

[a] Form the table by yourself , then $\sigma \simeq 3.29$

[b] $f(X)=2-X^2$

X	-3	-2	-1	0	1	2	3
$f(X)$	-7	-2	1	2	1	-2	-7



From the graph :

1 The vertex of the curve is (0 , 2)

2 The equation of the axis of symmetry is $X=0$

3 The maximum value = 2

26 Red Sea

1

1 b $\quad$ 2 a $\quad$ 3 c $\quad$ 4 d $\quad$ 5 c $\quad$ 6 a

2

[a] ① $X = \{1\}$ ② $n(Y) = 3$

③ $Y \times X = \{(1, 1), (5, 1), (7, 1)\}$

[b] ∵ b is the middle proportional between a , c

$\therefore b^2 = a\,c$

$\therefore \frac{a^2 + b^2}{b^2 + c^2} = \frac{a^2 + a\,c}{a\,c + c^2} = \frac{a\,(a + c)}{c\,(a + c)} = \frac{a}{c}$

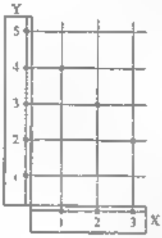
3

[a] ∵ $f(2) = 15$ $\therefore 15 = 4 \times 2 + a$

$\therefore 15 = 8 + a$ $\therefore a = 7$

[b] ① $R = \{(1, 4), (2, 3), (3, 2)\}$

② Yes , R is a function



4

[a] $\because \frac{X}{y} = \frac{2}{3}$ $\therefore X = 2\,m$, $y = 3\,m$

$\therefore \frac{3X + 2y}{6y - X} = \frac{6m + 6m}{18m - 2m} = \frac{12m}{16m} = \frac{3}{4}$

[b] ① $\because y \propto X$ $\therefore y = m\,X$ $\therefore 2 = 6\,m$

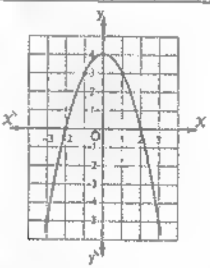
$\therefore m = \frac{1}{3}$ $\therefore y = \frac{1}{3}X$

② At $X = 15$ $\therefore y = \frac{1}{3} \times 15 = 5$

5

[a] $f(X) = 4 - X^2$

X	-3	-2	-1	0	1	2	3
$f(X)$	-5	0	3	4	3	0	-5



From the graph :

① The vertex of the curve is : (0 , 4)

② The equation of the axis of symmetry is $X = 0$

[b] Form the table by yourself , then $\sigma \simeq 4.98$

27 Matrouh

1

① a ② b ③ d ④ a ⑤ d ⑥ a

2

[a] $R = \{(1, 3), (2, 6), (3, 9)\}$

R is a function because every element in X has only one image in Y

, its range = $\{3, 6, 9\}$

[b] $\because \frac{a}{b} = \frac{2}{5}$ $\therefore a = 2\,m$, $b = 5\,m$

$\frac{2a - 2b}{3a + 2b} = \frac{4m - 10m}{6m + 10m} = \frac{-6m}{16m} = \frac{-3}{8}$

3

[a] ① $X = \{1\}$, $Y = \{1, 3, 5\}$

② $Y^2 = \{(1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5)\}$

[b] $\because \frac{X}{2a + b} = \frac{y}{2b - c} = \frac{z}{2c - a}$

Multiplying the two terms of the 1st ratio by 2 and adding the antecedents and consequents of the 1st and the 2nd ratios

$\therefore \frac{2X + y}{4a + 2b + 2b - c} = \frac{2X + y}{4a + 4b - c}$

= one of the given ratios. (1)

Multiplying the terms of the 1st ratio by 2 and the 2nd by 2 and adding the antecedents and consequents of the three ratios

$\therefore \frac{2X + 2y + z}{4a + 2b + 4b - 2c + 2c - a} = \frac{2X + 2y + z}{3a + 6b}$

= one of the given ratios (2)

From (1) and (2) :

$\therefore \frac{2X + y}{4a + 4b - c} = \frac{2X + 2y + z}{3a + 6b}$

4

[a] ∵ (a , 3) satisfies the function

$\therefore 3 = 4\,a - 5$ $\therefore 4\,a = 8$ $\therefore a = 2$

[b] Form the tables by yourself , then $\overline{X} = 2$

, $\sigma \simeq 0.96$

6

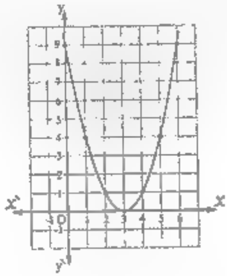
[a] $y \propto \frac{1}{X}$ $\quad X y = m$

$3 \times 10 = m$ $\quad \because m = 30$ $\quad X y = 30$

at $X = 5$ $\quad \therefore 5 y = 30$ $\quad \therefore y = 6$

[b] $f(X) = (X - 3)^2$

X	0	1	2	3	4	5	6
$f(X)$	9	4	1	0	1	4	9



From the graph :

* The vertex of the curve is $(3 , 0)$
* The minimum value $= 0$

Answers of governorates' examinations of trigonometry & geometry

1 — Cairo —

1

1 d 2 a 3 c 4 b 5 d 6 c

2

[a] ∵ $X \sin 45° \cos 45° = \sin 30°$

$\therefore X \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$ $\therefore \frac{1}{2} X = \frac{1}{2}$

$\therefore X = 1$

[b] ∵ The slope of the straight line = 2

∴ Its equation is : $y = 2X + c$

, ∵ (1 , 0) satisfies the equation

∴ $0 = 2 \times 1 + c$ $\therefore c = -2$

∴ The equation is : $y = 2X - 2$

3

[a] ∵ $m(\angle Y) = 90°$

∴ $(XZ)^2 = (6)^2 + (8)^2 = 100$

∴ XZ = 10 cm.

∴ $\cos X \cos Z - \sin X \sin Z$

$= \frac{6}{10} \times \frac{8}{10} - \frac{8}{10} \times \frac{6}{10} = 0$

[b] ∵ $AB = \sqrt{(-3-2)^2 + (0-4)^2} = \sqrt{25+16}$

$= \sqrt{41}$ length units

, $BC = \sqrt{(-7+3)^2 + (5-0)^2} = \sqrt{16+25}$

$= \sqrt{41}$ length units

, $CD = \sqrt{(-2+7)^2 + (9-5)^2} = \sqrt{25+16}$

$= \sqrt{41}$ length units

, $AD = \sqrt{(-2-2)^2 + (9-4)^2} = \sqrt{16+25}$

$= \sqrt{41}$ length units

∴ AB = BC = CD = AD ∴ ABCD is a rhombus

, ∵ $AC = \sqrt{(-7-2)^2 + (5-4)^2} = \sqrt{81+1}$

$= \sqrt{82}$ length units

, $BD = \sqrt{(-2+3)^2 + (9-0)^2} = \sqrt{1+81}$

$= \sqrt{82}$ length units

∴ AC = BD ∴ ABCD is a square.

4

[a] 1 In Δ ABC

$m(\angle B) = 90°$

$(BC)^2 = (25)^2 - (15)^2 = 400$

∴ BC = 20 cm.

2 ∵ $\sin(\angle ACB) = \frac{15}{25}$

$m(\angle ACB) \simeq 36° \, 52' \, 12''$

3 The area = $20 \times 15 = 300$ cm^2.

[b] Let B (X , y)

∴ $(6 , -4) = \left(\frac{5+X}{2} , \frac{-3+y}{2}\right)$

$\frac{5+X}{2} = 6$ ∴ $5 + X = 12$ ∴ $X = 7$

, $\frac{-3+y}{2} = -4$ ∴ $-3 + y = -8$ ∴ $y = -5$

∴ B (7 , −5)

5

[a] ∵ The two straight lines are parallel

∴ $m_1 = m_2$ ∴ $\frac{-a}{2} = \tan 45°$

∴ $\frac{-a}{2} = 1$ ∴ $a = -2$

[b] ∵ The slope of the straight line $= \frac{-1-2}{-2-4} = \frac{1}{2}$

∴ Its equation is : $y = \frac{1}{2} X + c$

, ∵ (4 , 2) satisfies the equation

∴ $2 = \frac{1}{2} \times 4 + c$ ∴ $c = 0$

∴ The equation is : $y = \frac{1}{2} X$

, ∵ $c = 0$

∴ The straight line passes through the origin point.

2 — Giza —

1

1 d 2 d 3 a 4 b 5 c 6 c

2

[a] ∵ The slope = 2

∴ The equation is : $y = 2X + c$

, ∵ (1 , −1) satisfies the equation

∴ $-1 = 2 \times 1 + c$ ∴ $c = -3$

∴ The equation is : $y = 2X - 3$

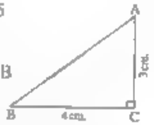
[b] $\boxed{1}$ ∵ m (∠ C) = 90°

$(AB)^2 = (3)^2 + (4)^2 = 25$

AB = 5 cm

cos A cos B - sin A sin B

$= \frac{3}{5} \times \frac{4}{5} - \frac{4}{5} \times \frac{3}{5} = 0$



$\boxed{2}$ tan B $= \frac{3}{4}$ m (∠ B) ≃ 36° 52̂ 12̂

3

[a] $\sin 60° = \frac{\sqrt{3}}{2}$ (1)

, $2 \sin 30° \cos 30° = 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$ (2)

From (1) , (2) $\sin 60° = 2 \sin 30° \cos 30°$

[b] $L_1 \perp L_2$, $m_1 \times m_2 = -1$

$\frac{k}{2} \quad \frac{1}{3} \times \tan 45°$,

$(1 - k) \times 1 = -1$ ∴ k = 2

4

[a] ∵ $\cos E \tan 30° = \cos^2 45°$

$\cos E \times \frac{1}{\sqrt{3}} = \left(\frac{1}{\sqrt{2}}\right)^2$

$\cos E = \frac{\sqrt{3}}{2}$ ∴ m (∠ E) = 30°

[b] ∵ $AB = \sqrt{(1-3)^2 + (5-3)^2} = \sqrt{4+4}$

$= 2\sqrt{2}$ length units

, $BC = \sqrt{(\ldots)^2 + (3-5)^2} = \sqrt{0+4}$

= 2 length units

, $AC = \sqrt{(\ldots 3)^2 + (3-3)^2} = \sqrt{4+0}$

= 2 length units

BC = AC

Δ ABC is isosceles

5

[a] $m = -\frac{5}{8}$

The intercepted part is $\frac{5}{2}$ from the negative part of the y-axis.

[b] ∵ $MA = \sqrt{(3+1)^2 + (-1-2)^2} = \sqrt{16+9}$

= 5 length units

, $MB = \sqrt{(-4+1)^2 + (6-2)^2} = \sqrt{9+16}$

= 5 length units

, $MC = \sqrt{(2+1)^2 + (-2-2)^2} = \sqrt{9+16}$

= 5 length units

MA = MB = MC

A , B , C belong to the circle M

, the area = 3.14 × 5^2 = 78.5 square units

3 Alexandria

1

$\boxed{1}$ b $\boxed{2}$ c $\boxed{3}$ a $\boxed{4}$ d $\boxed{5}$ d $\boxed{6}$ a

2

[a] ∵ $X \sin 30° \cos^2 45° = \sin^2 60°$

$X \times \frac{1}{2} \times \left(\frac{1}{\sqrt{2}}\right)^2 = \left(\frac{\sqrt{3}}{2}\right)^2$

$\frac{1}{4} X = \frac{3}{4}$ $X = 3$

[b] ∵ The two diagonals of the parallelogram bisect each other

Let M be the intersection point of the diagonals

$M = \left(\frac{3+0}{2} , \frac{\ldots 3}{2}\right) = \left(\frac{3}{2} , \ldots\right)$

Let D (X , y)

$\left(\frac{3}{2} , \ldots\right) = \left(\frac{4+X}{2} , \frac{5+y}{2}\right)$

$\frac{4+X}{2} = \frac{3}{2}$ 4 + X = 3 X = -1

, $\frac{5+y}{2} = \ldots$ 5 + y = ... y = 4

D (-1 , 4)

3

[a] ∵ $MA = \sqrt{(3+1)^2 + (-1-2)^2} = \sqrt{16+9}$

= 5 length units

, $MB = \sqrt{(-4+1)^2 + (6-2)^2} = \sqrt{9+16}$

= 5 length units

, $MC = \sqrt{(2+1)^2 + (-2-2)^2} = \sqrt{9+16}$

= 5 length units

MA = MB = MC

A , B , C are located on the circle M

, the circumference = 2 × 3.14 × 5

= 31.4 length units

[b] ∵ The slope of the given straight line $= \frac{-1}{2}$

∴ The slope of the required straight line $= 2$

∴ Its equation is : $y = 2X + c$

, ∵ it intercepts a part of 7 units from the positive part of the y-axis

∴ Its equation is : $y = 2X + 7$

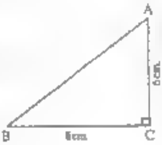
4

[a] ∵ $m_1 = \frac{5+2}{4+3} = 1$, $m_2 = \tan 45° = 1$

∴ $m_1 = m_2$

∴ The two straight lines are parallel

[b] ∵ $m(\angle C) = 90°$



∴ $(AB)^2 = (6)^2 + (8)^2 = 100$

∴ $AB = 10$ cm

∴ $\cos A \cos B - \sin A \sin B$

$= \frac{6}{10} \times \frac{8}{10} - \frac{8}{10} \times \frac{6}{10} = 0$

5

[a] Let D be the midpoint of $\overline{BC}$

$D = \left(\frac{3+1}{2} , \frac{7-3}{2}\right) = (2 , 2)$

∴ The slope of $\overleftrightarrow{AD} = \frac{2+6}{2-4} = -4$

∴ Its equation is : $y = -4X + c$

, ∵ $(4 , -6)$ satisfies the equation.

∴ $-6 = -4 \times 4 + c$ ∴ $c = 10$

∴ The equation is : $y = -4X + 10$

[b] 1 In ΔABC : ∵ $m(\angle B) = 90°$

∴ $\sin(\angle ACB) = \frac{15}{25}$

∴ $m(\angle ACB) \simeq 36° 52' 12''$

2 ∵ $(BC)^2 = (25)^2 - (15)^2 = 400$

∴ $BC = 20$ cm

∴ The area $= 20 \times 15 = 300$ cm^2

4 El-Kalyoubia

1

1 d 2 b 3 c 4 a 5 c 6 c

2

[a] $\sqrt{(X-6)^2 + (5-1)^2} = 2\sqrt{5}$ (Squaring both sides)

∴ $(X-6)^2 + 16 = 20$

∴ $X^2 - 12X + 36 + 16 - 20 = 0$

∴ $X^2 - 12X + 32 = 0$

∴ $(X-8)(X-4) = 0$

∴ $X = 8$ or $X = 4$

[b] $\sin 45° \cos 45° + \sin 30° \cos 60° - \cos^2 30°$

$= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{2} - \left(\frac{\sqrt{3}}{2}\right)^2$

$= \frac{1}{2} + \frac{1}{4} - \frac{3}{4} = 0$

3

[a] ∵ The two diagonals of the parallelogram bisect each other

Let M be the intersection point of the diagonals

∴ $M = \left(\frac{3+0}{2} , \frac{2-3}{2}\right) = \left(\frac{3}{2} , \frac{-1}{2}\right)$

Let $D(X , y)$

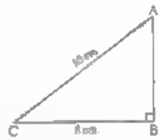
∴ $\left(\frac{3}{2} , \frac{-1}{2}\right) = \left(\frac{4+X}{2} , \frac{-5+y}{2}\right)$

∴ $\frac{4+X}{2} = \frac{3}{2}$ ∴ $4 + X = 3$ ∴ $X = -1$

, $\frac{-5+y}{2} = \frac{-1}{2}$ ∴ $-5 + y = -1$ ∴ $y = 4$

∴ $D(-1 , 4)$

[b] ∵ $m(\angle B) = 90°$



∴ $(AB)^2 = (10)^2 - (8)^2$

$= 36$

∴ $AB = 6$ cm.

∴ $\sin^2 A + 1 = \left(\frac{8}{10}\right)^2 + 1 = \frac{41}{25}$ (1)

, $2\cos^2 C + \cos^2 A = 2 \times \left(\frac{8}{10}\right)^2 + \left(\frac{6}{10}\right)^2$

$= \frac{41}{25}$ (2)

From (1) , (2) ∴ $\sin^2 A + 1 = 2\cos^2 C + \cos^2 A$

4

[a] ∵ $L_1 // L_2$ ∴ $m_1 = m_2$

∴ $\frac{k-1}{2-3} = \tan 45°$

∴ $-k + 1 = 1$ ∴ $k = 0$

[b] ∵ The slope of the given straight line $= \frac{-1}{3}$

∴ The slope of the required straight line = 3

∴ Its equation is : $y = 3x + c$

, ∵ (1 , 2) satisfies the equation

∴ $2 = 3 \times 1 + c$ ∴ $c = -1$

∴ The equation is : $y = 3x - 1$

5

[a] 1 In Δ ABC : ∵ $m(\angle B) = 90°$

$\sin(\angle ACB) = \frac{15}{25}$

∴ $m(\angle ACB) = 36° \, 52' \, 12''$

2 ∵ $(BC)^2 = (25)^2 - (15)^2 = 400$

∴ BC = 20 cm.

∴ The area $= 20 \times 15 = 300 \text{ cm}^2$

[b] ∵ The straight line passes through the two points (4 , 0) , (0 , 9)

∴ The slope of the straight line $= \frac{9-0}{0-4} = -\frac{9}{4}$

and the intercepted part = 9 units from the positive part of y-axis

∴ The equation of the straight line is :

$y = -\frac{9}{4}x + 9$

5 – El-Sharkia –

1

1 b 2 b 3 d 4 a 5 c 6 c

2

[a] ∵ $\frac{\sin 30° \sin 60°}{\sin 45° \cos 45°} = \frac{\frac{1}{2} \times \frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}} = \frac{\sqrt{3}}{2}$ (1)

, $\cos 30° = \frac{\sqrt{3}}{2}$ (2)

From (1) , (2) ∴ $\frac{\sin 30° \sin 60°}{\sin 45° \cos 45°} = \cos 30°$

[b] ∵ $MA = \sqrt{(-1-3)^2 + (2+1)^2} = \sqrt{16+9}$

= 5 length units

, $MB = \sqrt{(-1+4)^2 + (2-6)^2} = \sqrt{9+16}$

= 5 length units

and $MC = \sqrt{(-1-2)^2 + (2+2)^2} = \sqrt{9+16}$

= 5 length units

∴ MA = MB = MC

∴ A , B and C lie on the circle M

, the circumference $= 2 \times 3.14 \times 5$

= 31.4 length units

3

[a] The slope of $\overline{BC} = \frac{3+7}{1-3} = -5$

∴ The slope of the required straight line $= -5$

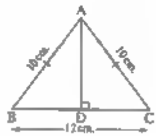
∴ Its equation is : $y = -5x + c$

, ∵ A (5 , 1) satisfies the equation

∴ $1 = -5 \times 5 + c$ ∴ $c = 26$

∴ The equation is : $y = -5x + 26$

[b] Draw $\overline{AD} \perp \overline{BC}$



1 ∵ $\overline{AD} \perp \overline{BC}$, AC = AB

∴ BD = CD = 6 cm

In Δ ADB ,

∵ $m(\angle ADB) = 90°$

∴ $(AD)^2 = (10)^2 - (6)^2 = 64$

∴ AD = 8 cm ∴ $\sin B = \frac{8}{10}$

2 The area of Δ ABC $= \frac{1}{2} \times 12 \times 8 = 48 \text{ cm}^2$

4

[a] 1 ∵ The midpoint of $\overline{AC} = \left(\frac{3+5}{2} , \frac{3-1}{2}\right)$

= (4 , 1)

∴ The point of intersection of the two diagonals is : (4 , 1)

2 Let D (x , y)

∴ $(4 , 1) = \left(\frac{2+x}{2} , \frac{-2+y}{2}\right)$

∴ $\frac{2+x}{2} = 4$ ∴ $x = 6$

, $\frac{-2+y}{2} = 1$ ∴ $y = 4$ ∴ D = (6 , 4)

[b] $\because$ The slope of the straight line $= \frac{3-5}{0-4} = \frac{1}{2}$

Its equation is : $y = \frac{1}{2} X + c$

, $\because$ (0 , 3) satisfies the equation.

$\therefore 3 = \frac{1}{2} \times 0 + c$ $\therefore c = 3$

$\therefore$ The equation is : $y = \frac{1}{2} X + 3$

at $y = 0$ $\therefore 0 = \frac{1}{2} X + 3$ $\therefore X = -6$

$\therefore$ The intersection point of the straight line with the X-axis is : $(-6 , 0)$

5

[a] 1 , $\cos X = \sin 30° \cos 60°$

$\cos X = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$\therefore X \simeq 75° \; 31' \; 21''$

2 $\tan 75° \; 31' \; 21'' \simeq 3.873$

[b] $\because$ The slope of the given straight line $= \frac{-3}{2}$

$\therefore$ The slope of the required straight line $= \frac{2}{3}$

, $\because$ the required straight line cuts 3 units of the positive part of y-axis

Its equation is : $y = \frac{2}{3} X + 3$

6 El-Monofia

1

1 a 2 d 3 d 4 b 5 b 6 c

2

[a] $\sin 30° \cos 60° + \cos 30° \sin 60° - \tan^2 45°$

$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - (1)^2$

$= \frac{1}{4} + \frac{3}{4} - 1 = 0$

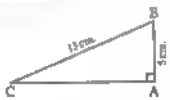
[b] 1 $\because AB = \sqrt{(5-7)^2 + (1+3)^2} = \sqrt{4+16}$

$= 2\sqrt{5}$ length units

$\therefore$ The area $= 3.14 \times \left(\sqrt{5}\right)^2 = 15.7 \text{ cm}^2$

2 $M = \left(\frac{7+5}{2} , \frac{-3+1}{2}\right) = (6 , -1)$

3



[a] $\because m(\angle A) = 90°$

$\therefore (AC)^2 = (13)^2 - (5)^2$

$= 144$

$\therefore AC = 12$ cm.

, $\sin C \cos B + \cos C \sin B$

$= \frac{5}{13} \times \frac{5}{13} + \frac{12}{13} \times \frac{12}{13} = 1$

[b] $\because$ The slope of the given straight line $= \frac{1-0}{2-5} = \frac{-1}{3}$

$\therefore$ The slope of the required straight line $= 3$

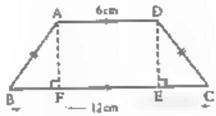
$\therefore$ Its equation is : $y = 3X + c$

, $\because$ (1 , 3) satisfies the equation

$\therefore 3 = 3 \times 1 + c$ $\therefore c = 0$

The equation is : $y = 3X$

4



[a] Draw $\overline{AF} \perp \overline{BC}$

, $\overline{DE} \perp \overline{BC}$

$\because \overline{AD} \parallel \overline{BC}$

, $\overline{AF} \perp \overline{BC}$

, $\overline{DE} \perp \overline{BC}$

$\therefore$ ADEF is a rectangle $\therefore$ EF = AD = 6 cm.

$\therefore$ BF + EC = 6 cm

$\therefore$ BF = EC = 3 cm. (Δ ABF $\equiv$ Δ DCE)

The area of the trapezium $= \frac{1}{2}$ (AD + BC) $\times$ AF

$36 = \frac{1}{2} (6 + 12) \times AF$

AF = 4 cm DE = AF = 4 cm

In Δ ABF $m(\angle AFB) = 90°$

$\therefore (AB)^2 = (3)^2 + (4)^2 = 25$ $\therefore$ AB = 5 cm

$\therefore$ DC = AB = 5 cm

$\therefore \sin B + \cos C = \frac{4}{5} + \frac{3}{5} = \frac{7}{5}$

[b] $AB = \sqrt{(5+1)^2 + (1-3)^2} = \sqrt{36+4}$

$= \sqrt{40}$ length units

, $BC = \sqrt{(6-5)^2 + (4-1)^2} = \sqrt{1+9}$

$= \sqrt{10}$ length units

, $AC = \sqrt{(6+1)^2 + (4-3)^2} = \sqrt{49+1}$

$= \sqrt{50}$ length units

$\because (AC)^2 = 50$

, $(AB)^2 + (BC)^2 = 40 + 10 = 50$

$\therefore (AC)^2 = (AB)^2 + (BC)^2$

$\therefore \Delta$ ABC is a right-angled triangle at B

5

[a] The slope $= \frac{-4}{5}$ and the intercepted part $= 2$ units from the positive part of the y-axis

[b] 1 $\because$ The slope of $\overleftrightarrow{CD} = \frac{6-2}{-3-3} = \frac{-2}{3}$

The equation of $\overleftrightarrow{CD}$ is $y = \frac{-2}{3} x + c$

$\because$ A $(3 , 2)$ satisfies the equation

$2 = \frac{-2}{3} \times 3 + c$ $\quad$ $c = 4$

$\therefore$ The equation is : $y = \frac{-2}{3} x + 4$

2 At $x = 0$ $\quad$, $y = \frac{-2}{3} \times 0 + 4$ $\quad$ $\therefore y = 4$

$\therefore$ OD $= 4$ units

, at $y = 0$ $\quad$ $\therefore 0 = \frac{-2}{3} x + 4$ $\quad$ $x = 6$

$\therefore$ OC $= 6$ units

$\therefore$ The area of Δ DOC $= \frac{1}{2} \times 4 \times 6$

$= 12$ square units

7 El-Gharbia

1

1 c $\quad$ 2 c $\quad$ 3 b $\quad$ 4 b $\quad$ 5 a $\quad$ 6 d

2

[a] $\because \tan x = 4 \cos 60° \sin 30°$

$\therefore \tan x = 4 \times \frac{1}{2} \times \frac{1}{2} = 1$ $\quad$ $\therefore x = 45°$

[b] 1 $\because \overleftrightarrow{XY} \perp \overleftrightarrow{YZ}$

The slope of $\overleftrightarrow{XY} \times$ the slope of $\overleftrightarrow{YZ} = -1$

$\frac{2-5}{4-3} \times \frac{a-2}{-5-4} = -1$

$3 \times \frac{a-2}{-9} = -1$ $\quad$ $a - 2 = -3$

$\therefore a = -1$

2 $\because$ XY $= \sqrt{(4-3)^2 + (2-5)^2} = \sqrt{1+9}$

$= \sqrt{10}$ length units

, YZ $= \sqrt{(-5-4)^2 + (-1-2)^2} = \sqrt{81+9}$

$= \sqrt{90}$ length units

The area of Δ XYZ $= \frac{1}{2} \times \sqrt{10} \times \sqrt{90}$

$= 15$ square units

3

[a] Let the measures be $3x , 5x$

$\therefore 3x + 5x = 180°$ $\quad$ $\therefore 8x = 180°$

$x = 22° 30'$

$\therefore$ The measures are $67° 30' , 112° 30'$

[b] $\because$ The slope of the given straight line $= -1$

The slope of the required straight line $= 1$

$\therefore$ Its equation is : $y = x + c$

$\because (-1 , 2)$ satisfies the equation

$2 = -1 + c$ $\quad$ $c = 3$

$\therefore$ The equation is : $y = x + 3$

4

[a] $\because$ MA $= \sqrt{(-1-3)^2 + (2+1)^2} = \sqrt{16+9}$

$= 5$ length units

, MB $= \sqrt{(-1+4)^2 + (2-6)^2} = \sqrt{9+16}$

$= 5$ length units

and MC $= \sqrt{(-1-2)^2 + (2+2)^2} = \sqrt{9+16}$

$= 5$ length units

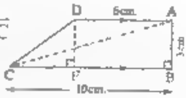
MA = MB = MC

$\therefore$ A , B and C lie on the circle M

, the circumference $= 2 \times 5 \times \pi$

$= 10 \pi$ length units

[b] Draw $\overline{DF} \perp \overline{BC}$



$\because \overline{AD} // \overline{BC} , \overline{AB} \perp \overline{BC}$

, $\overline{DF} \perp \overline{BC}$

ABFD is a rectangle

BF = AD = 6 cm

FC = 4 cm , DF = AB = 3 cm

From Δ DFC which is right-angled at F

$(DC)^2 = 3^2 + 4^2 = 25$ $\quad$ $\therefore$ DC = 5 cm

$\cos (\angle DCB) - \tan (\angle ACB) = \frac{4}{5} - \frac{3}{10} = \frac{1}{2}$

5

[a] 1 $\because$ The midpoint of $\overline{AC} = \left(\frac{3+0}{2} , \frac{2-3}{2}\right)$

$= \left(1\frac{1}{2} , -\frac{1}{2}\right)$

The intersection point of the two diagonals is $\left(1\frac{1}{2} , -\frac{1}{2}\right)$

2 Let D (x , y)

The midpoint of $\overline{AC}$ = the midpoint of $\overline{BD}$

$\therefore \left(1\frac{1}{2} , -\frac{1}{2}\right) = \left(\frac{x+4}{2} , \frac{y-5}{2}\right)$

$\frac{x+4}{2} = 1\frac{1}{2}$

$x + 4 = 3$ $\therefore x = -1$

, $\frac{y-5}{2} = -\frac{1}{2}$ $\therefore y - 5 = -1$ $\therefore y = 4$

$\therefore$ D $(-1 , 4)$

[b] 1 Let A $(x , 0)$, B $(0 , y)$

$(3 , 4) = \left(\frac{x+0}{2} , \frac{0+y}{2}\right)$

$\frac{x}{2} = 3$ $\therefore x = 6$

, $\frac{y}{2} = 4$ $\therefore y = 8$

$\therefore$ A $(6 , 0)$, B $(0 , 8)$

2 $\because$ The slope of $\overleftrightarrow{AB} = \frac{8-0}{0-6} = -\frac{4}{3}$

$\therefore$ The equation of $\overleftrightarrow{AB}$ is : $y = -\frac{4}{3} x + c$

$\because$ $(0 , 8)$ satisfies the equation

$\therefore 8 = \frac{-4}{3} \times 0 + c$ $\therefore c = 8$

$\therefore$ The equation is : $y = -\frac{4}{3} x + 8$

2 Let A (x , y)

$(5 , 7) = \left(\frac{x+8}{2} , \frac{y+11}{2}\right)$

$\frac{x+8}{2} = 5$ $\therefore x + 8 = 10$ $\therefore x = 2$

, $\frac{y+11}{2} = 7$ $\therefore y + 11 = 14$ $\therefore y = 3$

$\therefore$ A $(2 , 3)$

, $\because$ the slope of $\overleftrightarrow{AB} = \frac{11-3}{8-2} = \frac{4}{3}$

$\therefore$ The slope of the required straight line $= \frac{-3}{4}$

$\therefore$ Its equation is : $y = -\frac{3}{4} x + c$

$\because$ A $(2 , 3)$ satisfies the equation

$\therefore 3 = -\frac{3}{4} \times 2 + c$ $\therefore c = \frac{9}{2}$

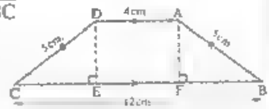
$\therefore$ The equation is : $y = \frac{-3}{4} x + \frac{9}{2}$

8 El-Dakahlia

1

[a] 1 c   2 b   3 b

[b] Draw $\overline{AF} \perp \overline{BC}$, $\overline{DE} \perp \overline{BC}$



$\because \overline{AD} // \overline{BC}$, $\overline{AF} \perp \overline{BC}$, $\overline{DE} \perp \overline{BC}$

$\therefore$ AFED is a rectangle $\therefore$ FE = AD = 4 cm

$\therefore$ BF + EC = 8 cm

$\therefore$ BF = EC = 4 cm. (Δ ABF $\equiv$ Δ DCE)

$\because$ From Δ ABF which is right-angled at F

$\therefore (AF)^2 = (5)^2 - (4)^2 = 9$

$\therefore$ AF = 3 cm

$\therefore$ DE = AF = 3 cm (AFED is a rectangle)

$\therefore \frac{\tan B \cos C}{\sin^2 C + \cos^2 B} = \frac{\frac{3}{4} \times \frac{4}{5}}{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2} = \frac{3}{5}$

2

[a] 1 b   2 b   3 d

[b] 1 $\because$ MB $= \sqrt{(8-5)^2 + (11-7)^2} = \sqrt{9 + 16}$

= 5 length units

$\therefore$ The circumference = 2 × 5 × 3.14

= 31.4 length units

3

[a] $\because$ The midpoint of $\overline{AC} = \left(\frac{-1+7}{2} , \frac{3+4}{2}\right)$

$= \left(3 , \frac{7}{2}\right)$

, the midpoint of $\overline{BD} = \left(\frac{5+1}{2} , \frac{1+6}{2}\right)$

$= \left(3 , \frac{7}{2}\right)$

$\therefore$ The midpoint of $\overline{AC}$ = the midpoint of $\overline{BD}$

$\therefore$ The two diagonals bisect each other

$\therefore$ ABCD is a parallelogram

[b] 1 Let A $(0 , n)$, B $(n , 0)$

$\therefore$ The slope $= \frac{0-n}{n-0} = -1$ $\therefore k = -1$

, $\because$ $(2 , 3)$ satisfies the equation

$\therefore 3 = -1 \times 2 + c$ $\therefore c = 5$

2 $\because$ A $(0 , n)$ satisfies the equation.

$\therefore n = -1 \times 0 + 5$ $\therefore n = 5$

$\therefore$ A $(0 , 5)$, B $(5 , 0)$

$\therefore$ The area of Δ ABO $= \frac{1}{2} \times 5 \times 5$

$= \frac{25}{2}$ square units

4

[a] $\boxed{1}$ $\because$ The intercepted part of the y-axis by $\overline{BC}$ is 3 units

$\therefore C = (0 , 3)$

$\therefore BC = \sqrt{(0-2)^2+(3-1)^2} = \sqrt{4+4}$

$= 2\sqrt{2}$ length units.

$\boxed{2}$ $\because B(2 , 1)$ $\therefore OA = 2$ length units

, $AB = 1$ length unit

, $\because \overline{AB} \parallel \overline{OC}$, $AB \neq OC$

$\therefore$ OABC is a trapezium

$\therefore$ The area of OABC $= \frac{1}{2}(1+3) \times 2$

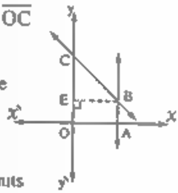
$= 4$ square units

$\boxed{3}$ Draw $\overline{BE} \perp \overline{OC}$

$\because \overline{BE} \perp \overline{OC}$, $\overline{AO} \perp \overline{OC}$

, $\overline{AB} \parallel \overline{OC}$

ABEO is a rectangle



$\therefore OE = AB$

$= 1$ length unit

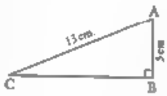
$BE = OA = 2$ length units

, $CE = 3 - 1 = 2$ length units.

In Δ BEC : $\because \tan(\angle BCE) = \frac{2}{2} = 1$

$\therefore m(\angle OCB) = 45°$

[b] $\boxed{1}$ $\because m(\angle B) = 90°$



$\therefore \sin^2 A + \cos^2 A$

$= \frac{(BC)^2}{(AC)^2} + \frac{(AB)^2}{(AC)^2}$

$= \frac{(BC)^2+(AB)^2}{(AC)^2} = \frac{(AC)^2}{(AC)^2} = 1$

$\boxed{2}$ $\because \sin C = \frac{5}{13}$ $\therefore m(\angle C) \simeq 22° \, 37'$

5

[a] $\because$ The slope $= \tan 135° = -1$

$\therefore$ The equation is : $y = -X + c$

, $\because (3 , 4)$ satisfies the equation.

$\therefore 4 = -3 + c$ $\therefore c = 7$

$\therefore$ The equation is $y = -X + 7$

[b] $\because \tan^2 60° - \tan^2 45° = \left(\sqrt{3}\right)^2 - (1)^2$

$= 3 - 1 = 2$ (1)

, $\sin^2 60° + \cos^2 60° + 2 \sin 30°$

$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + 2 \times \frac{1}{2} = \frac{3}{4} + \frac{1}{4} + 1 = 2$ (2)

From (1) , (2)

$\therefore \tan^2 60° - \tan^2 45° = \sin^2 60° + \cos^2 60° + 2 \sin 30°$

9 Ismailia

1

$\boxed{1}$ a $\boxed{2}$ c $\boxed{3}$ b $\boxed{4}$ a $\boxed{5}$ c $\boxed{6}$ d

2

[a] $\because X \cos^2 30° = \tan^2 60° \cos^2 45°$

$\therefore X \times \left(\frac{\sqrt{3}}{2}\right)^2 = \left(\sqrt{3}\right)^2 \times \left(\frac{1}{\sqrt{2}}\right)^2$

$\therefore \frac{3}{4} X = 3 \times \frac{1}{2}$ $\therefore X = 2$

[b] $\because$ The midpoint of $\overline{BC} = \left(\frac{3+1}{2} , \frac{7-3}{2}\right) = (2 , 2)$

$\therefore$ The slope of the required straight line $= \frac{-1-2}{5-2}$

$= -1$

$\therefore$ Its equation is : $y = -X + c$

, $\because A(5 , -1)$ satisfies the equation

$\therefore -1 = -5 + c$ $\therefore c = 4$

$\therefore$ The equation is : $y = -X + 4$

3

[a] $AB = \sqrt{(-4-1)^2+(2+2)^2} = \sqrt{25+16}$

$= \sqrt{41}$ length units

, $BC = \sqrt{(1+4)^2+(6-2)^2} = \sqrt{25+16}$

$= \sqrt{41}$ length units

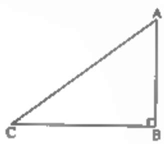
, $AC = \sqrt{(1-1)^2+(6+2)^2} = \sqrt{0+64}$

$= 8$ length units

$\therefore AB = BC$

$\therefore \Delta$ ABC is an isosceles triangle

[b] $\because m(\angle B) = 90°$



$\therefore \frac{\sin A}{\cos C} = \frac{\frac{BC}{AC}}{\frac{BC}{AC}} = 1$

, $\because \tan D = \frac{\sin A}{\cos C} = 1$

$\therefore m(\angle D) = 45°$

4

[a] ∵ $L_1 \parallel L_2$ ∴ $m_1 = m_2$

∴ $\frac{1-4}{k-2} = \tan 45°$ ∴ $\frac{-3}{k-2} = 1$

∴ $k - 2 = -3$ ∴ $k = -1$

[b] In Δ BED :

∵ $m(\angle BED) = 90°$, $m(\angle B) = 60°$

∴ $m(\angle BDE) = 30°$, $BE = \frac{1}{2} BD = 2$ cm

, $\sin 60° = \frac{DE}{BD}$ ∴ $\frac{\sqrt{3}}{2} = \frac{DE}{4}$

$DE = 2\sqrt{3}$ cm

In Δ CDE :

∵ $m(\angle CED) = 90°$, $CE = 5 - 2 = 3$ cm.

∴ $\tan(\angle DCE) = \frac{2\sqrt{3}}{3}$

5

[a] 1 The midpoint of $\overline{AC} = \left(\frac{3-3}{2} , \frac{3-3}{2}\right) = (0 , 0)$

The intersection point of the diagonals is $(0 , 0)$

2 ∵ The slope of $\overleftrightarrow{AC} = \frac{-3-3}{-3-3} = 1$

, ∵ $\overleftrightarrow{AC} \perp \overleftrightarrow{BD}$

∴ The slope of $\overleftrightarrow{BD} = -1$

, $\overleftrightarrow{BD}$ passes through $(0 , 0)$

∴ The equation of $\overleftrightarrow{BD}$ is : $y = -x$

[b] ∵ $A(0 , 2)$, $B(4 , 0)$, $C(-1 , 0)$

∴ The slope of $\overline{AB} = m_1 = \frac{2-0}{0-4} = \frac{-1}{2}$

, the slope of $\overline{AC} = m_2 = \frac{0-2}{-1-0} = 2$

$m_1 \times m_2 = \frac{-1}{2} \times 2 = -1$

∴ $\overline{AB} \perp \overline{AC}$

∴ Δ ABC is a right-angled triangle at A

, its area $= \frac{1}{2} \times 2 \times 5 = 5$ square units.

10 Suez

1

1 d 2 a 3 c 4 d 5 b 6 a

2

[a] ∵ $2 \sin 30° + 4 \cos 60° = 2 \times \frac{1}{2} + 4 \times \frac{1}{2} = 3$ (1)

, $\tan^2 60° = \left(\sqrt{3}\right)^2 = 3$ (2)

From (1) , (2) :

∴ $2 \sin 30° + 4 \cos 60° = \tan^2 60°$

[b] ∵ The midpoint of $\overline{AC} = \left(\frac{-1+6}{2} , \frac{-1+0}{2}\right)$

$= \left(\frac{5}{2} , \frac{-1}{2}\right)$

, the midpoint of $\overline{BD} = \left(\frac{2+3}{2} , \frac{3-4}{2}\right)$

$= \left(\frac{5}{2} , \frac{-1}{2}\right)$

∴ The midpoint of $\overline{AC}$ = the midpoint of $\overline{BD}$

∴ $\overline{AC}$ and $\overline{BD}$ bisect each other.

3

[a] $\cos 3x = \frac{\sin 60° \sin 30°}{\tan 45° \sin^2 45°}$

$= \frac{\frac{\sqrt{3}}{2} \times \frac{1}{2}}{1 \times \left(\frac{1}{\sqrt{2}}\right)^2} = \frac{\frac{\sqrt{3}}{4}}{\frac{1}{2}} = \frac{\sqrt{3}}{2}$

∴ $3x = 30°$ ∴ $x = 10°$

[b] ∵ The slope of $\overline{AB} = \frac{-4+3}{5-2} = \frac{-1}{3}$

∴ The slope of the required straight line $= 3$

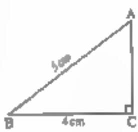
∴ Its equation is $y = 3x + c$

, ∵ $(1 , 2)$ satisfies the equation.

∴ $2 = 3 \times 1 + c$ ∴ $c = -1$

∴ The equation is : $y = 3x - 1$

4



[a] ∵ $m(\angle C) = 90°$

∴ $(AC)^2 = (5)^2 - (4)^2 = 9$

∴ $AC = 3$ cm.

∴ $\sin A \cos B + \cos A \sin B = \frac{4}{5} \times \frac{4}{5} + \frac{3}{5} \times \frac{3}{5}$

$= 1$

[b] ∵ $\frac{y-1}{x} = \frac{1}{3}$ ∴ $y = \frac{1}{3} x + 1$

The slope of the given straight line $= \frac{1}{3}$

The slope of the required straight line $= \frac{1}{3}$

, it intersects a part from the negative direction of the y-axis of length 3 units

The equation is : $y = \frac{1}{3} x - 3$

5

[a] $\because AB = \sqrt{(3-0)^2 + (4-0)^2} = \sqrt{9+16}$

$= 5$ length units

, $BC = \sqrt{(-4-3)^2 + (3-4)^2} = \sqrt{49+1}$

$= 5\sqrt{2}$ length units

, $AC = \sqrt{(-4-0)^2 + (3-0)^2} = \sqrt{16+9}$

$= 5$ length units

The perimeter of $\Delta ABC = 5 + 5\sqrt{2} + 5$

$= 10 + 5\sqrt{2}$ length units.

[b] $\overline{AB} \,/\!/\, \overline{CD}$ $\qquad m_1 = m_2$

$\frac{2+2}{3 \quad 9} = \frac{3 + x}{4 + x}$ $\qquad \frac{2}{3} = \frac{3 + x}{4 + x}$

$9 + 3x = 8 \quad 2x$ $\qquad 5x = 1$

$x = \frac{1}{5}$ $\qquad C\left(\frac{1}{5}, \frac{1}{5}\right)$

11 Port Said

1

1 a 2 b 3 c 4 b 5 d 6 b

2

[a] $m_1 = \frac{4-3}{2+1} = \frac{1}{3}$

, $m_2 = \frac{1}{3}$ $\qquad \therefore m_1 = m_2$

The two straight lines are parallel

[b] $\because \sin 90^\circ = 1$ (1)

, $\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} = 1$ (2)

From (1) , (2) .

$\sin 90^\circ = \sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$

3

[a] $\cos E = \frac{\cos^2 45^\circ}{\tan 30^\circ} = \frac{\left(\frac{1}{\sqrt{2}}\right)^2}{\frac{1}{\sqrt{3}}} = \frac{\sqrt{3}}{2}$

$\therefore m(\angle E) = 30^\circ$

[b] $AB = \sqrt{(3+3)^2 + (4-0)^2} = \sqrt{36+16}$

$= 2\sqrt{13}$ length units

, $BC = \sqrt{(1-3)^2 + (-6-4)^2} = \sqrt{4+100}$

$= 2\sqrt{26}$ length units

, $AC = \sqrt{(1+3)^2 + (-6-0)^2} = \sqrt{16+36}$

$= 2\sqrt{13}$ length units

$AB = AC$ $\qquad \therefore \Delta ABC$ is an isosceles triangle

4

[a] $\frac{y-1}{x} = \frac{1}{3}$ $\qquad y = \frac{1}{3} x + 1$

The slope of the given straight line $= \frac{1}{3}$

$\therefore$ The slope of the required straight line $= \frac{1}{3}$

, it intercepts a part from the negative direction of the y-axis of length 3 units

The equation is : $y = \frac{1}{3} x - 3$

[b] $\because$ The slope of $\overleftrightarrow{AD} = \frac{1-3}{-2-2} = \frac{1}{2}$

, the slope of $\overleftrightarrow{BC} = \frac{2+2}{6+2} = \frac{1}{2}$

The slope of $\overleftrightarrow{AD}$ = the slope of $\overleftrightarrow{BC}$

$\overleftrightarrow{AD} \,/\!/\, \overleftrightarrow{BC}$ (1)

, $\because$ the slope of $\overleftrightarrow{AB} = \frac{2}{6} \cdot \frac{3}{2} = \frac{1}{4}$

, the slope of $\overleftrightarrow{CD} = \frac{1+2}{-2+2}$ is undefined

$\therefore$ The slope of $\overleftrightarrow{AB} \neq$ the slope of $\overleftrightarrow{CD}$

$\overleftrightarrow{AB}$ is not parallel to $\overleftrightarrow{CD}$ (2)

From (1) , (2) .

ABCD is a trapezoid

5

[a] $\because$ The midpoint of $\overline{BC} = \left(\frac{3+1}{2}, \frac{7-3}{2}\right) = (2, 2)$

The slope of the straight line $= \frac{2+6}{2-5} = \frac{-8}{3}$

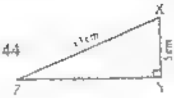
Its equation is : $y = \frac{-8}{3} x + c$

, $\because (5, -6)$ satisfies the equation

$-6 = \frac{-8}{3} \times 5 + c$ $\qquad c = \frac{22}{3}$

The equation is : $y = -\frac{8}{3} x + \frac{22}{3}$

[b] $m(\angle Y) = 90^\circ$



$\therefore (YZ)^2 = (13)^2 - (5)^2 = 144$

$\therefore YZ = 12$ cm

$\sin X \cos Z + \cos X \sin Z$

$= \frac{12}{13} \times \frac{12}{13} + \frac{5}{13} \times \frac{5}{13} = 1$

12 Damietta

1

1 a 2 d 3 d 4 c 5 b 6 d

2

[a] The slope of the straight line $= \frac{5-0}{0-5} = -1$

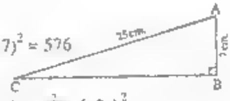
$\therefore$ Its equation is $y = -X + c$

$\because (0, 5)$ satisfies the equation

$\therefore 5 = 0 + c \qquad \therefore c = 5$

$\therefore$ The equation is: $y = -X + 5$

[b] $\because m(\angle B) = 90^\circ$



$\therefore (BC)^2 = (25)^2 - (7)^2 = 576$

$\therefore BC = 24$ cm

$\sin^2 A + \sin^2 C = \left(\frac{24}{25}\right)^2 + \left(\frac{7}{25}\right)^2 = 1$

3

[a] The points are located on one straight line

$\therefore \frac{3-1}{a-0} = \frac{5-1}{2-0} \qquad \therefore \frac{2}{a} = 2 \qquad \therefore a = 1$

[b] The slope of the given straight line $= \frac{1}{3}$

$\therefore$ The slope of the required straight line $= \frac{-1}{3}$

$\therefore$ Its equation is $y = \frac{-1}{3} X + c$

$\because (3, 7)$ satisfies the equation

$\therefore 7 = \frac{-1}{3} \times 3 + c \qquad \therefore c = 8$

$\therefore$ The equation is $y = \frac{-1}{3} X + 8$

4

[a] $2 \sin X = \sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ$

$2 \sin X = \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

$\therefore 2 \sin X = 1 \qquad \therefore \sin X = \frac{1}{2}$

$\therefore X = 30^\circ$

[b] $\because$ The slope of the straight line = 2 and it intersects from the positive part of y-axis 7 units

$\therefore$ Its equation is $y = 2X + 7$

5

[a] $\tan 60^\circ = \sqrt{3}$ (1)

$, \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} = \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}} = \sqrt{3}$ (2)

From (1), (2)

$\therefore \tan 60^\circ = \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ}$

[b] $\because AB = \sqrt{(-2-3)^2 + (4+1)^2} = \sqrt{25 + 25}$

$= 5\sqrt{2}$ length units

$, BC = \sqrt{(3-4)^2 + (-1-5)^2} = \sqrt{1 + 36}$

$= \sqrt{37}$ length units

$, AC = \sqrt{(-2-4)^2 + (4-5)^2} = \sqrt{36 + 1}$

$= \sqrt{37}$ length units

$\therefore BC = AC$

$\therefore \Delta ABC$ is an isosceles triangle

13 Kafr El-Sheikh

1

1 c 2 a 3 b 4 d 5 c 6 d

2

[a] $\because AB = \sqrt{(3-1)^2 + (0-4)^2} = \sqrt{4 + 16}$

$= 2\sqrt{5}$ length units

$, BC = \sqrt{(1+1)^2 + (4-2)^2} = \sqrt{4 + 4}$

$= 2\sqrt{2}$ length units

$, AC = \sqrt{(3+1)^2 + (0-2)^2} = \sqrt{16 + 4}$

$= 2\sqrt{5}$ length units

$\therefore AB = AC$

$\therefore \Delta ABC$ is an isosceles triangle

[b] $\sin^2 45^\circ \cos 60^\circ + \frac{1}{2} \tan 60^\circ \sin 60^\circ$

$= \left(\frac{1}{\sqrt{2}}\right)^2 \times \frac{1}{2} + \frac{1}{2} \times \sqrt{3} \times \frac{\sqrt{3}}{2}$

$= \frac{1}{4} + \frac{3}{4} = 1$

3

[a] $\because L_1 \parallel L_2$ $\quad\therefore m_1 = m_2$

$\therefore 2 - k = \tan 45°$ $\quad\therefore 2 - k = 1$

$\therefore k = 1$

[b] $\because \sqrt{3} \tan X = 4 \sin 60° \cos 30°$

$\therefore \sqrt{3} \tan X = 4 \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

$\therefore \sqrt{3} \tan X = 3$ $\quad\therefore \tan X = \sqrt{3}$

$\therefore X = 60°$

4

[a] $\sqrt{(2-X)^2 + (5-3)^2} = 2\sqrt{2}$ (Squaring both sides)

$(2-X)^2 + (2)^2 = 8$

$\therefore X^2 - 4X + 4 + 4 = 8$ $\quad X^2 - 4X = 0$

$\therefore X(X-4) = 0$ $\quad\therefore X = 0$ or $X = 4$

[b] $\because$ The slope $= 3$

$\therefore$ The equation is : $y = 3X + c$

$, \because (5, -2)$ satisfies the equation

$-2 = 3 \times 5 + c$ $\quad\therefore c = -17$

$\therefore$ The equation is : $y = 3X - 17$

5

[a] Let $B(X, y)$

$\because (2, 3) = \left(\frac{X-1}{2}, \frac{y+3}{2}\right)$

$\frac{X-1}{2} = 2$ $\quad\therefore X - 1 = 4$ $\quad\therefore X = 5$

$, \frac{y+3}{2} = 3$ $\quad\therefore y + 3 = 6$ $\quad\therefore y = 3$

$\therefore B(5, 3)$

[b] $\because \angle A , \angle C$ are complementary angles

$\therefore \sin A = \cos C$

$\because \sin A + \cos C = \sin A + \sin A = 1$

$\therefore \sin A = \frac{1}{2}$ $\quad\therefore m(\angle A) = 30°$

14 El-Beheira

1

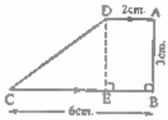
1 c 2 b 3 b 4 b 5 b 6 c

2

[a] $\because m_1 = \frac{4-3}{2+1} = \frac{1}{3}$ $, m_2 = \frac{1}{3}$ $\quad\therefore m_1 = m_2$

$\therefore$ The two straight lines are parallel

[b] Draw $\overline{DE} \perp \overline{BC}$



$\because \overline{AD} \parallel \overline{BC} , \overline{AB} \perp \overline{BC}$

$, \overline{DE} \perp \overline{BC}$

$\therefore ABED$ is a rectangle

$\therefore DE = AB = 3$ cm

$, BE = AD = 2$ cm. $\quad\therefore CE = 6 - 2 = 4$ cm.

In $\triangle DEC$: $\because m(\angle DEC) = 90°$

$\therefore (DC)^2 = (3)^2 + (4)^2 = 25$ $\quad\therefore DC = 5$ cm.

$\cos(\angle BCD) = \frac{4}{5}$

3

[a] $\because$ The slope $= 3$

$\therefore$ The equation is : $y = 3X + c$

$, \because (1, 2)$ satisfies the equation

$\therefore 2 = 3 \times 1 + c$ $\quad\therefore c = -1$

$\therefore$ The equation is : $y = 3X - 1$

[b] $\because 2 \sin X = \tan^2 60° - 2 \tan 45°$

$\therefore 2 \sin X = \left(\sqrt{3}\right)^2 - 2 \times 1$ $\quad\therefore 2 \sin X = 1$

$\therefore \sin X = \frac{1}{2}$ $\quad\therefore X = 30°$

4

[a] $\because L_1 \perp L_2$ $\quad\therefore m_1 \times m_2 = -1$

$\therefore \frac{k-1}{2-3} \times \tan 45° = -1$ $\quad\therefore (1-k) \times 1 = -1$

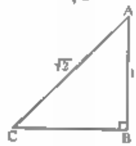
$\therefore 1 - k = -1$ $\quad\therefore k = 2$

[b] $\because \sqrt{2}\, AB = AC$ $\quad\therefore \frac{AB}{AC} = \frac{1}{\sqrt{2}}$

Let $AB = 1$ length unit

$, AC = \sqrt{2}$ length unit

$, \because m(\angle B) = 90°$



$\therefore (BC)^2 = \left(\sqrt{2}\right)^2 - (1)^2 = 1$

$\therefore BC = 1$ length unit

$\sin C = \frac{1}{\sqrt{2}}$ $, \cos C = \frac{1}{\sqrt{2}}$ $, \tan C = 1$

5

[a] $\because AB = BC$

$\therefore \sqrt{(X-3)^2 + (3-2)^2} = \sqrt{(3-5)^2 + (2-1)^2}$

(Squaring both sides)

$\therefore (X-3)^2 + 1 = 4 + 1$

$\therefore X^2 - 6X + 9 + 1 - 4 - 1 = 0$

$\therefore X^2 - 6X + 5 = 0$ $\quad\therefore (X-5)(X-1) = 0$

$\therefore X = 5$ or $X = 1$ $\left(\text{refused because } B \notin \overline{AC}\right)$

[b] $\because AB = \sqrt{(2-6)^2 + (-4-0)^2}$

$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$ length unit

, $BC = \sqrt{(-4-2)^2 + (2+4)^2} = \sqrt{36+36} = \sqrt{72}$

$= 6\sqrt{2}$ length unit

, $CA = \sqrt{(6+4)^2 + (0-2)^2} = \sqrt{100+4} = \sqrt{104}$

$= 2\sqrt{26}$ length unit

$\because (AB)^2 + (BC)^2 = 32 + 72 = 104 = (CA)^2$

$\therefore \Delta ABC$ is right-angled at B

Let E be the midpoint of $\overline{AC}$

$\therefore E = \left(\frac{6-4}{2}, \frac{0+2}{2}\right) = (1, 1)$

$\because$ In the rectangle the two diagonals bisect each other

$\therefore$ E is the midpoint of $\overline{BD}$

Let D (X, y)

$\therefore (1, 1) = \left(\frac{X+2}{2}, \frac{y-4}{2}\right)$ $\quad\therefore \frac{X+2}{2} = 1$

$\therefore X + 2 = 2$ $\quad\therefore X = 0$

, $\frac{y-4}{2} = 1$ $\quad\therefore y - 4 = 2$

$\therefore y = 6$ $\quad\therefore D(0, 6)$

15 El-Fayoum

1

1 b	2 d	3 c	4 c	5 c	6 c

2

[a] $\because MA = \sqrt{(-1-3)^2 + (2+1)^2} = \sqrt{16+9}$

$= 5$ length units

, $MB = \sqrt{(-1+4)^2 + (2-6)^2} = \sqrt{9+16}$

$= 5$ length units

and $MC = \sqrt{(-1-2)^2 + (2+2)^2} = \sqrt{9+16}$

$= 5$ length units

$\therefore MA = MB = MC$

$\therefore$ A, B and C lie on the circle M

, the circumference $= 2 \times 3.14 \times 5$

$= 31.4$ length units

[b] $\because \tan^2 60° - \tan^2 45° = \left(\sqrt{3}\right)^2 - (1)^2 = 2$ $\quad$ (1)

, $\sin^2 60° + \cos^2 60° + 2 \sin 30°$

$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + 2 \times \frac{1}{2} = 2$ $\quad$ (2)

From (1), (2)

$\therefore \tan^2 60° - \tan^2 45° = \sin^2 60° + \cos^2 60° + 2 \sin 30°$

[a] $\because$ The slope of $\overleftrightarrow{AB} = \frac{5-3}{3-1} = 1$

$\therefore$ The slope of the required straight line $= -1$

$\therefore$ Its equation is : $y = -X + c$

, $\because$ the midpoint of $\overline{AB}$

$= \left(\frac{1+3}{2}, \frac{3+5}{2}\right) = (2, 4)$

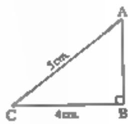
, $\because$ the required straight line passes through the midpoint of $\overline{AB}$

$\therefore 4 = -2 + c$ $\quad\therefore c = 6$

$\therefore$ The equation of the required straight line is :

$y = -X + 6$

[b] $\because m(\angle B) = 90°$



$\therefore (AB)^2 = (5)^2 - (4)^2 = 9$

$\therefore AB = 3$ cm

$\therefore 2\cos^2 C + \sin^2 A$

$= 2\left(\frac{4}{5}\right)^2 + \left(\frac{4}{5}\right)^2 = \frac{48}{25}$

[a] $\because$ The midpoint of $\overline{AC} = \left(\frac{3}{}, \frac{-2-7}{2}\right)$

$= \left(\frac{3}{2} \quad \frac{9}{}\right)$

, the midpoint of $\overline{BD} = \left(\frac{-5+6}{2}, \frac{-9}{2}\right)$

$= \left(\frac{3}{2}, \frac{-9}{2}\right)$

$\therefore$ The midpoint of $\overline{AC}$ = the midpoint of $\overline{BD}$

$\therefore \overline{AC}$ and $\overline{BD}$ bisect each other

$\therefore$ The points A, B, C and D are the vertices of a parallelogram

[b] $\because 4X = \cos^2 30° \tan^2 30° \tan^2 45°$

$\therefore 4X = \left(\frac{\sqrt{3}}{2}\right)^2 \times \left(\frac{1}{\sqrt{3}}\right)^2 \times (1)^2$

$\therefore 4X = \frac{3}{4} \times \frac{1}{3} \times 1$ $\quad\therefore 4X = \frac{1}{4}$ $\quad\therefore X = \frac{1}{16}$

6

[a] The two straight lines are perpendicular

$\therefore m_1 \times m_2 = -1$ $\therefore \frac{3}{4} \times \frac{-4}{k} = -1$

$\therefore \frac{3}{k} = 1$ $\therefore k = 3$

[b] $\because$ The straight line passes through $(1, 0)$, $(0, 4)$

Its slope $= \frac{4-0}{0-1} = -4$

Its equation is $y = -4X + c$

$\because (0, 4)$ satisfies the equation

$4 = -4 \times 0 + c$ $c = 4$

The equation is $y = -4X + 4$

16 Beni Suef

1

1 b 2 c 3 d 4 a 5 d 6 b

2

[a] Let $B(X, y)$

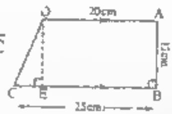
$\therefore (6, -4) = \left(\frac{5+X}{2}, \frac{-3+y}{2}\right)$

$\therefore \frac{5+X}{2} = 6$ $5 + X = 12$ $X = 7$

$, \frac{-3+y}{2} = -4$ $\therefore -3 + y = -8$ $y = -5$

$\therefore B(7, -5)$

[b] Draw $\overline{DE} \perp \overline{BC}$



$\because \overline{AD} \,//\, \overline{BC}$, $\overline{AB} \perp \overline{BC}$

$, \overline{DE} \perp BC$

$\therefore$ ABED is a rectangle

$DE = AB = 12$ cm.

$, BE = AD = 20$ cm. $\therefore CE = 25 - 20 = 5$ cm

In Δ DEC $\because m(\angle DEC) = 90^\circ$

$(DC)^2 = (12)^2 + (5)^2 = 169$

$\therefore DC = 13$ cm. $, \because \tan C = \frac{12}{5}$

$m(\angle C) \simeq 67^\circ\, 22'\, 48''$

3

[a] $\frac{1}{2} \sin 60^\circ = \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$ (1)

$, \sin 30^\circ \cos 30^\circ = \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$ (2)

From (1), (2).

$\therefore \frac{1}{2} \sin 60^\circ = \sin 30^\circ \cos 30^\circ$

[b] $\because$ The slope $= 2$

$\therefore$ The equation of the straight line is , $y = 2X + c$

$, \because (2, 3)$ satisfies the equation

$\therefore 3 = 2 \times 2 + c$ $\therefore c = -1$

The equation is $y = 2X - 1$

4

[a] $\cos E \tan 30^\circ = \sin^2 45^\circ$

$\cos E \times \frac{1}{\sqrt{3}} = \left(\frac{1}{\sqrt{2}}\right)^2$

$\cos E = \frac{\sqrt{3}}{2}$ $m(\angle E) = 30^\circ$

[b] $m_1 = \frac{3+1}{6-2} = 1$, $m_2 = \tan 45^\circ = 1$

$m_1 = m_2$

The two straight lines are parallel

5

[a] $\because MA = \sqrt{(-1-3)^2 + (2+1)^2} = \sqrt{16+9}$

$= 5$ length units

$, MB = \sqrt{(-1+4)^2 + (2-6)^2} = \sqrt{9+16}$

$= 5$ length units

and $MC = \sqrt{(-1-2)^2 + (2+2)^2} = \sqrt{9+16}$

$= 5$ length units

$MA = MB = MC$

A , B and C are located on the circle M

[b] The slope $= \frac{2}{3}$

and the intersected part $= \frac{5}{3}$ units

from the negative direction of the y-axis

17 El-Menia

1

1 b 2 b 3 c 4 d 5 b 6 d

2

[a] $\cos 60^\circ \sin 30^\circ - \sin 60^\circ \tan 60^\circ + \cos^2 30^\circ$

$= \frac{1}{2} \times \frac{1}{2} - \frac{\sqrt{3}}{2} \times \sqrt{3} + \left(\frac{\sqrt{3}}{2}\right)^2$

$= \frac{1}{4} - \frac{3}{2} + \frac{3}{4} = -\frac{1}{2}$

[b] $\because$ The slope of the given straight line

$= \frac{-4+3}{5-2} = \frac{-1}{3}$

The slope of the required straight line = 3

∴ Its equation is : $y = 3X + c$

∵ (1 , 2) satisfies the equation.

$2 = 3 \times 1 + c$ ∴ $c = -1$

The equation is : $y = 3X - 1$

3

[a] ∵ $2 \sin X = \tan^2 60° - 2 \tan 45°$

∴ $2 \sin X = \left(\sqrt{3}\right)^2 - 2 \times 1$ ∴ $2 \sin X = 1$

∴ $\sin X = \frac{1}{2}$ ∴ $X = 30°$

[b] ∵ m (∠ A) = 90°

∴ $(AB)^2 = (25)^2 - (15)^2 = 400$

∴ AB = 20 cm

∴ $\cos C \cos B - \sin C \sin B = \frac{15}{25} \times \frac{20}{25} - \frac{20}{25} \times \frac{15}{25}$
$= 0$

4

[a] The slope of $\overline{AB} = m_1 = \frac{0+4}{1+1} = 2$

, the slope of $\overline{BC} = m_2 = \frac{2-0}{2-1} = 2$

∵ $m_1 = m_2$ ∴ $\overline{AB} \parallel \overline{BC}$

, ∵ B is a common point

∴ A , B , C are collinear

[b] Let B (X , y) ∴ $(6 , -4) = \left(\frac{5+X}{2} , \frac{-3+y}{2}\right)$

∴ $\frac{5+X}{2} = 6$ ∴ $5 + X = 12$ ∴ $X = 7$

, $\frac{-3+y}{2} = -4$ ∴ $-3 + y = -8$ ∴ $y = -5$

∴ B (7 , −5)

5

[a] $m_1 = \tan 45° = 1$, $m_2 = 1$

∴ $m_1 = m_2$

∴ The two straight lines are parallel

[b] ∵ $\sqrt{(a+2)^2 + (7-3)^2} = 5$ (Squaring both sides)

$(a+2)^2 + (4)^2 = 25$

$a^2 + 4a + 4 + 16 - 25 = 0$

$a^2 + 4a - 5 = 0$ ∴ $(a-1)(a+5) = 0$

∴ $a = 1$ or $a = -5$

18 Assiut

1

1 c 2 d 3 c 4 a 5 c 6 b

2

[a] ∵ m (∠ C) = 90°

∴ $(AC)^2 = (13)^2 - (12)^2 = 25$

∴ AC = 5 cm

∴ $\sin A \cos B + \cos A \sin B = \frac{12}{13} \times \frac{12}{13} + \frac{5}{13} \times \frac{5}{13}$
$= 1$

[b] ∵ $AB = \sqrt{(5-1)^2 + (1-1)^2} = \sqrt{16} = 4$ length units

, $BC = \sqrt{(3-5)^2 + (4-1)^2} = \sqrt{4+9}$
$= \sqrt{13}$ length units

, $AC = \sqrt{(3-1)^2 + (4-1)^2} = \sqrt{4+9} = \sqrt{13}$ length units.

∵ BC = AC ∴ Δ ABC is isosceles

3

[a] ∵ $2 \sin X = \tan^2 60° - 4 \sin 30°$

∴ $2 \sin X = \left(\sqrt{3}\right)^2 - 4 \times \frac{1}{2}$ ∴ $2 \sin X = 1$

∴ $\sin X = \frac{1}{2}$ ∴ $X = 30°$

[b] ∵ The midpoint of $\overline{AC} = \left(\frac{3+1}{2} , \frac{2+4}{2}\right) = (2 , 3)$

∴ The point of intersection of the diagonals is : (2 , 3)

Let D (X , y)

∴ $(2 , 3) = \left(\frac{4+X}{2} , \frac{-5+y}{2}\right)$

∴ $\frac{4+X}{2} = 2$ ∴ $4 + X = 4$ ∴ $X = 0$

, $\frac{-5+y}{2} = 3$

∴ $-5 + y = 6$ ∴ $y = 11$ ∴ D (0 , 11)

4

[a] $\cos 60° + \cos^2 30° + \tan^2 45°$

$= \frac{1}{2} + \left(\frac{\sqrt{3}}{2}\right)^2 + (1)^2 = \frac{1}{2} + \frac{3}{4} + 1 = \frac{9}{4}$

[b] $m_1 = \frac{4-3}{\sqrt{3} - 2\sqrt{3}} = \frac{-1}{\sqrt{3}}$, $m_2 = \tan 60° = \sqrt{3}$

∴ $m_1 \times m_2 = \frac{-1}{\sqrt{3}} \times \sqrt{3} = -1$

∴ The two straight lines are perpendicular.

5

[a] $\because$ The slope of the given straight line $= \frac{-1}{3}$

$\therefore$ The slope of the required straight line $= \frac{-1}{3}$

$\therefore$ Its equation is : $y = \frac{-1}{3} X + c$

, $\because$ (3 , – 5) satisfies the equation

$\therefore -5 = \frac{-1}{3} \times 3 + c \qquad \therefore c = -4$

$\therefore$ The equation is : $y = \frac{-1}{3} X - 4$

[b] $\because \frac{y-1}{X} = \frac{1}{2}$

$y = \frac{1}{2} X + 1$

$\therefore$ The slope $= \frac{1}{2}$ and the intercepted part equals 1 unit from the positive direction of the y-axis.

19 Souhag

1

1 c 2 b 3 d 4 a 5 c 6 d

2

[a] $\because \cos X = 2 \cos^2 30° - 1$

$\therefore \cos X = 2 \times \left(\frac{\sqrt{3}}{2}\right)^2 - 1 \qquad \therefore \cos X = 2 \times \frac{3}{4} - 1$

$\cos X = \frac{1}{2} \qquad \therefore X = 60°$

[b] $\because$ The slope of $\overleftrightarrow{AB} = m_1 = \frac{-2-4}{-1-1} = 3$

, the slope of $\overleftrightarrow{BC} = m_2 = \frac{-3+2}{2+1} = \frac{-1}{3}$

$\therefore m_1 \times m_2 = 3 \times \frac{-1}{3} = -1 \qquad \therefore \overleftrightarrow{AB} \perp \overleftrightarrow{BC}$

$\therefore \Delta ABC$ is right-angled at B

3

[a] 1 $\because m(\angle C) = 90°$

$\therefore (AC)^2 = (13)^2 - (12)^2 = 25 \qquad \therefore AC = 5$ cm

2 $\sin A \cos B + \cos A \sin B$

$= \frac{12}{13} \times \frac{12}{13} + \frac{5}{13} \times \frac{5}{13} = 1$

[b] $\because$ The slope = 2

$\therefore$ The equation of the straight line is : $y = 2 X + c$

, $\because$ (1 , 0) satisfies the equation

$\therefore 0 = 2 \times 1 + c \qquad \therefore c = -2$

$\therefore$ The equation is $y = 2 X - 2$

4

[a] $\because 2 \sin 30° = 2 \times \frac{1}{2} = 1$ (1)

, $\tan^2 60° - 2 \tan 45° = \left(\sqrt{3}\right)^2 - 2 \times 1 = 1$ (2)

From (1) , (2) : $\therefore 2 \sin 30° = \tan^2 60° - 2 \tan 45°$

[b] $\because$ The slope of the straight line $= \frac{-3-3}{-1-1} = 3$

$\therefore$ Its equation is : $y = 3 X + c$

, $\because$ (1 , 3) satisfies the equation.

$\therefore 3 = 3 \times 1 + c \qquad \therefore c = 0$

$\therefore$ The equation is : $y = 3 X$

, $c = 0$

$\therefore$ The straight line passes through the origin point.

5

[a] $\because$ The slope of $\overleftrightarrow{AB} = m_1 = \frac{5+1}{6+3} = \frac{2}{3}$

, the slope of $\overleftrightarrow{BC} = m_2 = \frac{3-5}{3-6} = \frac{2}{3}$

$\because m_1 = m_2 \qquad \therefore \overleftrightarrow{AB} // \overleftrightarrow{BC}$

, $\because$ B is a common point

A , B , C are collinear

[b] $m_1 = \frac{5+2}{4+3} = 1$, $m_2 = \tan 45° = 1$

$\therefore m_1 = m_2$

$\therefore$ The two straight lines are parallel

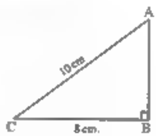
20 Qena

1

1 b 2 c 3 a 4 b 5 c 6 b

2

[a] $\because m(\angle B) = 90°$



$\therefore (AB)^2 = (10)^2 - (8)^2 = 36$

AB = 6 cm.

$\therefore \sin^2 A + 1$

$= \left(\frac{8}{10}\right)^2 + 1 = \frac{41}{25}$ (1)

, $2 \cos^2 C + \cos^2 A = 2 \times \left(\frac{8}{10}\right)^2 + \left(\frac{6}{10}\right)^2 = \frac{41}{25}$ (2)

From (1) , (2) $\quad \sin^2 A + 1 = 2 \cos^2 C + \cos^2 A$

[b] $\because$ The slope of $\overleftrightarrow{AB} = m_1 = \frac{-1-1}{0-1} = 2$

, the slope of $\overleftrightarrow{BC} = m_2 = \frac{3+1}{2-0} = 2$

$\therefore m_1 = m_2 \qquad \therefore \overleftrightarrow{AB} // \overleftrightarrow{BC}$

, ∵ B is a common point

∴ A , B , C are collinear

3

[a] ∵ $\sin X \tan 30° = \sin^2 45°$

∴ $\sin X \times \frac{1}{\sqrt{3}} = \left(\frac{1}{\sqrt{2}}\right)^2$ ∴ $\sin X = \frac{\sqrt{3}}{2}$

∴ $X = 60°$

[b] $m_1 = \frac{4-3}{2+1} = \frac{1}{3}$, $m_2 = \frac{1}{3}$

∵ $m_1 = m_2$

∴ The two straight lines are parallel

4

[a] ∵ $\sin 60° = \frac{\sqrt{3}}{2}$ (1)

, $2 \sin 30° \cos 30° = 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$ (2)

From (1) , (2) : ∴ $\sin 60° = 2 \sin 30° \cos 30°$

[b] ∵ $AB = \sqrt{(6-5)^2 + (-2-3)^2} = \sqrt{1+25}$

$= \sqrt{26}$ length units

, $BC = \sqrt{(1-6)^2 + (-1+2)^2} = \sqrt{25+1}$

$= \sqrt{26}$ length units

, $CD = \sqrt{(0-1)^2 + (4+1)^2} = \sqrt{1+25}$

$= \sqrt{26}$ length units

, $DA = \sqrt{(5-0)^2 + (3-4)^2} = \sqrt{25+1}$

$= \sqrt{26}$ length units

∴ AB = BC = CD = DA

∴ ABCD is a rhombus

, ∵ $AC = \sqrt{(1-5)^2 + (-1-3)^2} = \sqrt{16+16}$

$= 4\sqrt{2}$ length units

, $BD = \sqrt{(0-6)^2 + (4+2)^2} = \sqrt{36+36}$

$= 6\sqrt{2}$ length units

∴ The area of the rhombus $= \frac{1}{2} AC \times BD$

$= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$

= 24 square units

5

[a] ∵ $AB = \sqrt{(3+3)^2 + (4-0)^2} = \sqrt{36+16}$

$= 2\sqrt{13}$ length units

, $BC = \sqrt{(1-3)^2 + (-6-4)^2} = \sqrt{4+100}$

$= 2\sqrt{26}$ length units

, $CA = \sqrt{(-3-1)^2 + (0+6)^2} = \sqrt{16+36}$

$= 2\sqrt{13}$ length units

∴ AB = AC

∴ Δ ABC is an isosceles triangle and its vertex is A

Let D be the midpoint of $\overline{BC}$ (The base of Δ ABC)

∴ $D = \left(\frac{3+1}{2} , \frac{4-6}{2}\right) = (2 , -1)$

∴ $AD = \sqrt{(2+3)^2 + (-1-0)^2} = \sqrt{25+1}$

$= \sqrt{26}$ length units

∴ The length of the segment perpendicular to $\overline{BC}$ from A $= \sqrt{26}$ length units.

[b] ∵ The midpoint of $\overline{AC} = \left(\frac{3+0}{2} , \frac{2-3}{2}\right)$

$= \left(1\frac{1}{2} , -\frac{1}{2}\right)$

∴ The point of intersection of the two diagonals is $\left(1\frac{1}{2} , -\frac{1}{2}\right)$

and let D (X , y)

∵ The midpoint of $\overline{AC}$ = the midpoint of $\overline{BD}$

∴ $\left(1\frac{1}{2} , -\frac{1}{2}\right) = \left(\frac{X+4}{2} , \frac{y-5}{2}\right)$

∴ $\frac{X+4}{2} = 1\frac{1}{2}$ ∴ $X + 4 = 3$ ∴ $X = -1$

, $\frac{y-5}{2} = -\frac{1}{2}$ ∴ $y - 5 = -1$ ∴ $y = 4$

∴ D(−1 , 4)

21 Luxor

1

1 b 2 c 3 c 4 b 5 d 6 c

2

[a] ∵ $\sqrt{(3a-1-a)^2 + (1-5)^2} = 5$ (Squaring both sides)

∴ $(2a-1)^2 + (-4)^2 = 25$

∴ $4a^2 - 4a + 1 + 16 - 25 = 0$

∴ $4a^2 - 4a - 8 = 0$ ∴ $a^2 - a - 2 = 0$

∴ $(a-2)(a+1) = 0$ ∴ $a = 2$ or $a = -1$

[b] ∵ $3 \tan X - 4 \sin^2 30° = 8 \cos^2 60°$

∴ $3 \tan X - 4 \times \left(\frac{1}{2}\right)^2 = 8 \times \left(\frac{1}{2}\right)^2$

∴ $3 \tan X = 2 + 1$ ∴ $\tan X = 1$ ∴ $X = 45°$

3

[a] The slope of the given straight line $= \frac{-2}{3}$

The slope of the required straight line $= \frac{-2}{3}$

Its equation is : $y = \frac{-2}{3} X + c$

, (1 , 2) satisfies the equation.

$2 = \frac{-2}{3} \times 1 + c$ $\therefore c = \frac{8}{3}$

∴ The equation is $y = \frac{-2}{3} X + \frac{8}{3}$

[b] $m = \frac{4\sqrt{3} - \sqrt{3}}{1 + 2} = \sqrt{3}$ $\therefore \tan \theta = \sqrt{3}$ $\therefore \theta = 60°$

4

[a] ∵ $AB = \sqrt{(-2-4)^2 + (7+1)^2} = \sqrt{36 + 64}$

$= 10$ length units

$\therefore r = \frac{1}{2} AB = 5$ length units

∴ The area $= 3.14 \times (5)^2 = 78.5$ square units.

[b] 1 ∵ $AB = AC$, $\overline{AD} \perp \overline{BC}$

$\therefore BD = CD = 6$ cm.

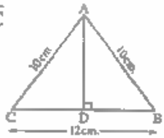
In ΔADC :

∵ $m(\angle ADC) = 90°$

$\therefore (AD)^2 = (10)^2 - (6)^2 = 64$

$\therefore AD = 8$ cm.

$\sin^2 C + \cos^2 C = \left(\frac{8}{10}\right)^2 + \left(\frac{6}{10}\right)^2 = 1$



2 $m(\angle B) = m(\angle C)$ $\therefore \sin B = \sin C$

$\therefore \sin B + \cos C = \sin C + \cos C$

$= \frac{8}{10} + \frac{6}{10} = \frac{14}{10} > 1$

5

[a] $\overleftrightarrow{AB}$ // y-axis ∴ $3 - X = 0$ ∴ $X = 3$

[b] 1 In ΔAMB $m(\angle AMB) = 90°$

$\therefore \cos(\angle BAM) = \frac{4}{5}$

$\therefore m(\angle BAC) \simeq 36° \; 52' \; 12''$

$\therefore m(\angle BAD) \simeq 73° \; 44' \; 24''$

2 ∵ $(BM)^2 = (5)^2 - (4)^2 = 9$ $\therefore BM = 3$ cm

∴ The area $= \frac{1}{2} \times 8 \times 6 = 24 \text{ cm}^2$

22 Aswan

1

1 c 2 b 3 d 4 c 5 c 6 b

2

[a] ∵ $2 \sin X = \tan^2 60° - 2 \tan^2 45°$

$\therefore 2 \sin X = \left(\sqrt{3}\right)^2 - 2 \times (1)^2$ $\therefore 2 \sin X = 1$

$\sin X = \frac{1}{2}$ $\therefore X = 30°$

[b] ∵ The slope of $\overleftrightarrow{AB} = \frac{5-3}{3-1} = 1$

∴ The slope of the required straight line $= -1$

∴ Its equation is : $y = -X + c$

∵ the midpoint of $\overline{AB} = \left(\frac{1+3}{2} , \frac{3+5}{2}\right)$

$= (2 , 4)$

, the required straight line passes through the midpoint of $\overline{AB}$

$\therefore 4 = -2 + c$ $\therefore c = 6$

∴ The equation of the required straight line is $y = -X + 6$

3

[a] ∵ $(4 , 2) = \left(\frac{2+6}{2} , \frac{4+y}{2}\right)$

$\therefore \frac{4+y}{2} = 2$ $4 + y = 4$ $y = 0$

[b] ∵ The slope of $\overleftrightarrow{AB} = m_1 = \frac{3+1}{2+1} = \frac{4}{3}$

, the slope of $\overleftrightarrow{BC} = m_2 = \frac{0-3}{6-2} = \frac{-3}{4}$

$\therefore m_1 \times m_2 = \frac{4}{3} \times \frac{-3}{4} = -1$ $\therefore \overleftrightarrow{AB} \perp \overleftrightarrow{BC}$

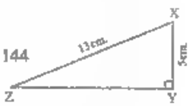
∴ ΔABC is right-angled at B

4

[a] ∵ $m(\angle Y) = 90°$

$\therefore (YZ)^2 = (13)^2 - (5)^2 = 144$

$\therefore YZ = 12$ cm.



1 $\tan X \tan Z = \frac{12}{5} \times \frac{5}{12} = 1$

2 $\cos X \cos Z - \sin X \sin Z$

$= \frac{5}{13} \times \frac{12}{13} - \frac{12}{13} \times \frac{5}{13} = 0$

[b] ∵ The straight line passes through the points (1 , 0) , (0 , 4)

Its slope $= \frac{4-0}{0-1} = -4$

$\therefore$ Its equation is : $y = -4X + c$

, $\because$ the straight line intercepts 4 units from the positive part of y-axis

$\therefore$ Its equation is : $y = -4X + 4$

5

[a] $\because m_1 = \frac{3-4}{-1-2} = \frac{1}{3}$, $m_2 = \frac{1}{3}$ $\therefore m_1 = m_2$

$\therefore$ The two straight lines are parallel.

[b] $\because 2\,AB = \sqrt{3}\,AC$ $\therefore \frac{AB}{AC} = \frac{\sqrt{3}}{2}$

Let $AB = \sqrt{3}$ length units

, $AC = 2$ length units

$\therefore BC = 1$ length units

$\therefore \sin C = \frac{\sqrt{3}}{2}$, $\cos C = \frac{1}{2}$, $\tan C = \sqrt{3}$

23 New Valley

1

1 d 2 b 3 c 4 a 5 d 6 c

2

[a] $\because m(\angle Z) = 90^\circ$

$\therefore (XY)^2 = (3)^2 + (4)^2 = 25$

$\therefore XY = 5$ cm.

1 $\tan X \tan Y = \frac{4}{3} \times \frac{3}{4} = 1$

2 $\sin^2 X + \cos^2 X = \left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2 = 1$

[b] $\because AB = \sqrt{(1-3)^2 + (5-3)^2} = \sqrt{4+4}$

$= 2\sqrt{2}$ length units

, $BC = \sqrt{(1-1)^2 + (3-5)^2} = \sqrt{0+4}$

$= 2$ length units

, $AC = \sqrt{(1-3)^2 + (3-3)^2} = \sqrt{4+0}$

$= 2$ length units

$\therefore BC = AC$

$\therefore \Delta ABC$ is an isosceles triangle

, $\because (AB)^2 = \left(2\sqrt{2}\right)^2 = 8$

, $(BC)^2 + (AC)^2 = (2)^2 + (2)^2 = 8$

$\therefore (AB)^2 = (BC)^2 + (AC)^2$

$\therefore \Delta ABC$ is a right-angled triangle at C

3

[a] 1 $\because \tan X = 4 \sin 30^\circ \cos 60^\circ$

$\therefore \tan X = 4 \times \frac{1}{2} \times \frac{1}{2} = 1$ $\therefore X = 45^\circ$

2 $\sin 45^\circ = \frac{1}{\sqrt{2}}$

[b] $\because$ The slope of the straight line = 2

$\therefore$ Its equation is : $y = 2X + c$

, $\because (1, 0)$ satisfies the equation.

$\therefore 0 = 2 \times 1 + c$ $\therefore c = -2$

$\therefore$ The equation is : $y = 2X - 2$

4

[a] 1 $\because AB = AC$, $\overline{AD} \perp \overline{BC}$

$\therefore BD = CD = 6$ cm.

In ΔADB : $\because m(\angle ADB) = 90^\circ$

$\therefore \cos B = \frac{6}{10} = \frac{3}{5}$

2 $m(\angle B) \simeq 53^\circ\, 7'\, 48''$

3 $\because \sin(90^\circ - B) = \cos B$

$\therefore \sin(90^\circ - B) = \frac{3}{5}$

[b] 1 $\because$ The midpoint of $\overline{AC} = \left(\frac{-2+4}{2}, \frac{3-3}{2}\right)$

$= (1, 0)$

$\therefore$ The point of intersection of the diagonals $= (1, 0)$

2 Let $D(X, y)$

$\therefore (1, 0) = \left(\frac{-1+X}{2}, \frac{-2+y}{2}\right)$

$\therefore \frac{-1+X}{2} = 1$ $\therefore -1 + X = 2$ $\therefore X = 3$

, $\frac{-2+y}{2} = 0$ $\therefore -2 + y = 0$ $\therefore y = 2$

$\therefore D(3, 2)$

5

[a] $\because L_1 \parallel L_2$ $\therefore m_1 = m_2$ $\therefore \frac{k-1}{3-2} = \tan 45^\circ$

$\therefore k - 1 = 1$ $\therefore k = 2$

[b] $\because$ The straight line passes through $(2, 0)$, $(0, 4)$

$\therefore$ Its slope $= \frac{4-0}{0-2} = -2$

$\therefore$ Its equation is : $y = -2X + c$

, $\because$ the straight line intercepts 4 units from the positive part of y-axis

$\therefore$ Its equation is : $y = -2X + 4$

24 South Sinai

1

1 a 2 b 3 c 4 d 5 a 6 c

2

[a] $\because \cos 60° = \frac{1}{2}$ (1)

, $\cos^2 30° - \sin^2 30° = \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = \frac{1}{2}$ (2)

From (1) , (2) : $\therefore \cos 60° = \cos^2 30° - \sin^2 30°$

[b] Let B (X , y)

$\therefore (1 , -3) = \left(\frac{4 + X}{2} , \frac{-3 + y}{2}\right)$

$\therefore \frac{4 + X}{2} = 1$ $\therefore 4 + X = 2$ $\therefore X = -2$

, $\frac{-3 + y}{2} = -3$ $\therefore -3 + y = -6$ $\therefore y = -3$

$\therefore$ B $(-2 , -3)$

3

[a] $\because$ The slope of the straight line $= \frac{-3 - 3}{-1 - 1} = 3$

$\therefore$ Its equation is : $y = 3X + c$

, $\because (1 , 3)$ satisfies the equation.

$\therefore 3 = 3 \times 1 + c$ $\therefore c = 0$

$\therefore$ The equation is : $y = 3X$

[b] $\because AB = \sqrt{(1-3)^2 + (5-3)^2} = \sqrt{4+4}$

$= 2\sqrt{2}$ length units

, $BC = \sqrt{(1-1)^2 + (3-5)^2} = \sqrt{4}$

$= 2$ length units

, $AC = \sqrt{(1-3)^2 + (3-3)^2} = \sqrt{4} = 2$ length units

$\therefore BC = AC$

$\therefore \Delta ABC$ is an isosceles triangle.

4

[a] $\because$ The slope of the straight line $= \tan 45° = 1$

$\therefore$ Its equation is : $y = X + c$

, $\because (-2 , 3)$ satisfies the equation.

$\therefore 3 = -2 + c$ $\therefore c = 5$

$\therefore$ The equation is : $y = X + 5$

[b] $\frac{2 \tan 45°}{1 + \tan^2 45°} = \frac{2 \times 1}{1 + (1)^2} = 1$

5

[a] $\because$ The slope of the straight line is 2 and it intersects 5 units from the positive part of the y-axis

$\therefore$ Its equation is : $y = 2X + 5$

[b] 1 $\because m(\angle B) = 90°$ $\therefore \sin C = \frac{5}{10} = \frac{1}{2}$

$\therefore m(\angle C) = 30°$

2 $\sin^2 C + \cos^2 C = \sin^2 30° + \cos^2 30°$

$= \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = 1$

25 North Sinai

1

1 d 2 c 3 a 4 c 5 d 6 c

2

[a] $\because \cos 60° = \frac{1}{2}$ (1)

, $2\cos^2 30° - 1 = 2 \times \left(\frac{\sqrt{3}}{2}\right)^2 - 1$

$= 2 \times \frac{3}{4} - 1 = \frac{1}{2}$ (2)

From (1) , (2) : $\therefore \cos 60° = 2\cos^2 30° - 1$

[b] $\because AB = \sqrt{(-4-1)^2 + (2+2)^2} = \sqrt{25+16}$

$= \sqrt{41}$ length units

, $BC = \sqrt{(1+4)^2 + (6-2)^2} = \sqrt{25+16}$

$= \sqrt{41}$ length units

, $AC = \sqrt{(1-1)^2 + (6+2)^2} = \sqrt{64}$

$= 8$ length units

$\therefore AB = BC$

$\therefore \Delta ABC$ is an isosceles triangle.

3

[a] $\because$ The slope of the straight line = 2 and it cuts 7 units from the positive part of the y-axis

$\therefore$ Its equation is : $y = 2X + 7$

[b] 1 $\because m(\angle B) = 90°$

$\therefore (AB)^2 = (10)^2 - (8)^2 = 36$

$\therefore AB = 6$ cm.

2 $\sin^2 A + \cos^2 A = \left(\frac{8}{10}\right)^2 + \left(\frac{6}{10}\right)^2$

$= \frac{64}{100} + \frac{36}{100} = 1$

4

[a] $\because \cos X = \frac{\sin 60° \sin 30°}{\sin^2 45°}$ $\therefore \cos X = \frac{\frac{\sqrt{3}}{2} \times \frac{1}{2}}{\left(\frac{1}{\sqrt{2}}\right)^2}$

$\therefore \cos X = \frac{\sqrt{3}}{2}$ $\therefore X = 30°$

[b] $\because$ The slope of the given straight line $= \frac{-4+3}{5-2}$

$= \frac{-1}{3}$

$\therefore$ The slope of the required straight line = 3

$\therefore$ Its equation is : $y = 3X + c$

, $\because$ (1 , 2) satisfies the equation.

$\therefore 2 = 3 \times 1 + c$ $\therefore c = -1$

$\therefore$ The equation is : $y = 3X - 1$

5

1 $\because MA = \sqrt{(-1-3)^2 + (2+1)^2} = \sqrt{16+9}$

= 5 length units

, $MB = \sqrt{(-1+4)^2 + (2-6)^2} = \sqrt{9+16}$

= 5 length units

and $MC = \sqrt{(-1-2)^2 + (2+2)^2} = \sqrt{9+16}$

= 5 length units

$\therefore MA = MB = MC$

$\therefore$ A , B and C lie on the circle M

2 The circumference of the circle $= 2 \times 3.14 \times 5$

= 31.4 length units.

26 Red Sea

1

1 b	2 c	3 a	4 c	5 b	6 d

2

[a] $\because \sin 60° = \frac{\sqrt{3}}{2}$ (1)

, $2 \sin 30° \cos 30° \tan 45° = 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} \times 1$

$= \frac{\sqrt{3}}{2}$ (2)

From (1) , (2) :

$\therefore \sin 60° = 2 \sin 30° \cos 30° \tan 45°$

[b] $\because$ The slope of the straight line $= \frac{-1-2}{-2-4} = \frac{1}{2}$

$\therefore$ Its equation is : $y = \frac{1}{2} X + c$

, $\because$ (4 , 2) satisfies the equation.

$\therefore 2 = \frac{1}{2} \times 4 + c$ $\therefore c = 0$

$\therefore$ The equation is : $y = \frac{1}{2} X$

3

[a] $\because \tan X = 4 \cos 60° \sin 30°$

$\therefore \tan X = 4 \times \frac{1}{2} \times \frac{1}{2}$

$\therefore \tan X = 1$ $\therefore X = 45°$

[b] $\because AB = \sqrt{(-3-2)^2 + (0-4)^2} = \sqrt{25+16}$

$= \sqrt{41}$ length units

, $BC = \sqrt{(-7+3)^2 + (5-0)^2} = \sqrt{16+25}$

$= \sqrt{41}$ length units

, $AC = \sqrt{(-7-2)^2 + (5-4)^2} = \sqrt{81+1}$

$= \sqrt{82}$ length units

$\therefore (AC)^2 = (AB)^2 + (BC)^2$

$\therefore \Delta$ ABC is a right-angled triangle at B

, its area $= \frac{1}{2} \times \sqrt{41} \times \sqrt{41} = 20\frac{1}{2}$ square units.

4

[a] $\because$ The slope of the straight line = 2 and it intercepts 7 units from the positive part of the y-axis

$\therefore$ Its equation is : $y = 2X + 7$

[b] $\because m(\angle B) = 90°$

$\therefore (AB)^2 = (13)^2 - (5)^2 = 144$

$\therefore AB = 12$ cm.

$\therefore \sin A \cos C + \cos A \sin C$

$= \frac{5}{13} \times \frac{5}{13} + \frac{12}{13} \times \frac{12}{13} = 1$

5

[a] $\because \sqrt{(X+2)^2 + (7-3)^2} = 5$ (Squaring both sides)

$\therefore (X+2)^2 + (7-3)^2 = 25$

$\therefore X^2 + 4X + 4 + 16 - 25 = 0$

$\therefore X^2 + 4X - 5 = 0$ $\therefore (X+5)(X-1) = 0$

$\therefore X = -5$ or $X = 1$

[b] $\because L_1 // L_2$ $\therefore m_1 = m_2$

$\therefore \frac{k-1}{2-3} = \tan 45°$ $\therefore -k + 1 = 1$

$\therefore k = 0$

27 Matrouh

1

1 b 2 a 3 a 4 c 5 a 6 d

2

[a] $\because \tan 60° = \sqrt{3}$ (1)

$, \frac{2 \tan 30°}{1 - \tan^2 30°} = \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}} = \sqrt{3}$ (2)

From (1) , (2) :

$\therefore \tan 60° = \frac{2 \tan 30°}{1 - \tan^2 30°}$

[b] $\because$ The slope of $\overleftrightarrow{AB} = m_1 = \frac{-4-0}{2-6} = 1$

, the slope of $\overleftrightarrow{BC} = m_2 = \frac{2+4}{-4-2} = -1$

$\therefore m_1 \times m_2 = 1 \times -1 = -1$

$\therefore \overleftrightarrow{AB} \perp \overleftrightarrow{BC}$

$\therefore \Delta$ ABC is a right-angled triangle at B

3

[a] $\because \sqrt{(a+2)^2 + (7-3)^2} = 5$ (Squaring both sides)

$\therefore (a+2)^2 + (7-3)^2 = 25$

$\therefore a^2 + 4a + 4 + 16 - 25 = 0$

$\therefore a^2 + 4a - 5 = 0$ $\therefore (a+5)(a-1) = 0$

$\therefore a = -5$ or $a = 1$

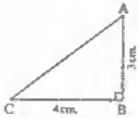
[b] $\because m(\angle B) = 90°$

$\therefore (AC)^2 = (3)^2 + (4)^2 = 25$

$\therefore AC = 5$ cm.

$\therefore \sin A \cos C + \cos A \sin C$

$= \frac{4}{5} \times \frac{4}{5} + \frac{3}{5} \times \frac{3}{5} = 1$



4

[a] Let $A = x°$, $B = 2x°$

$\therefore x + 2x = 90°$ $\therefore 3x = 90°$

$\therefore x = 30°$ $\therefore A = 30°$, $B = 60°$

$\therefore \sin A + \cos B = \sin 30° + \cos 60°$

$= \frac{1}{2} + \frac{1}{2} = 1$

[b] $\because \frac{x}{2} + \frac{y}{2} = 1$ (Multiplying by 2)

$\therefore x + y = 2$

$\therefore$ The slope $= -1$

, the intercepted part = 2 units from the positive part of y-axis.

5

[a] $\because (-3 , y) = \left(\frac{x+9}{2} , \frac{-6-12}{2}\right)$

$\therefore y = -9$

$, \frac{x+9}{2} = -3$ $\therefore x + 9 = -6$

$\therefore x = -15$

[b] $\because$ The slope of the given straight line $= \frac{-1}{2}$

$\therefore$ The slope of the required straight line $= \frac{-1}{2}$

$\therefore$ Its equation is : $y = \frac{-1}{2} x + c$

, $\because (3 , -5)$ satisfies the equation.

$\therefore -5 = \frac{-1}{2} \times 3 + c$ $\therefore c = \frac{-7}{2}$

$\therefore$ The equation is : $y = \frac{-1}{2} x - \frac{7}{2}$